Chapter 1

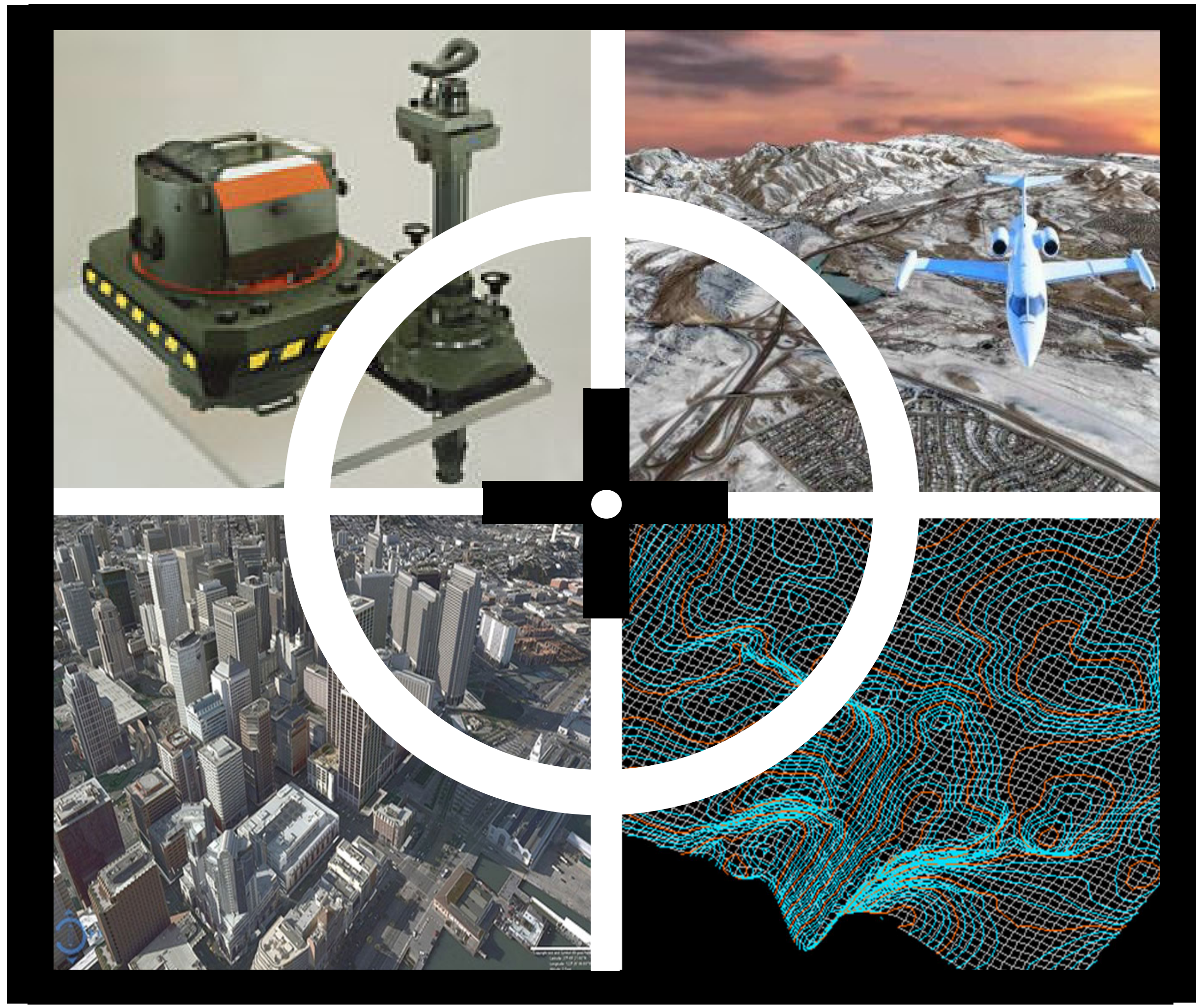
PRINCIPLES OF PHOTOGRAMMETRIC MAPPING

Overview

- Photogrammetry: Definition and applications
- Photogrammetric tools:
  - Rotation matrices
  - Photogrammetric orientation: interior and exterior orientation
- Photogrammetric point positioning
  - Collinearity equations/conditions (single camera systems)
  - GNSS/INS-assisted photogrammetric systems
  - Multi-camera photogrammetric systems
- Photogrammetric bundle adjustment
  - Structure of the design and normal matrices

Photogrammetry

- Objective: Derive the positions and shapes of objects from imagery



Photogrammetry

- Classical Definitions:
  - The art and science of determining the position and shape of objects from photography
  - The process of reconstructing objects without touching them
  - Non-contact positioning method
- Contemporary Definition:
  - The art and science of tool development for automatic generation of spatial and descriptive information from multi-sensory data and/or systems

Data Acquisition Systems

Traditional Mapping Cameras

Large Format Imaging Systems

Low-Cost Digital Cameras

Medium and Small Format Digital Imaging Systems

Data Acquisition Systems

Traditional mapping cameras

- ↑ accurate lab calibration
- ↑ large image format
- ↑ low distortion lens system
- ↑ stable IOP
- ↑ extremely-high geometric image quality
- ↓ high initial procurement cost
- ↓ not easy to integrate with other systems on the same platform (e.g., LiDAR)

Medium-format digital cameras

- ↑ low-cost/off-the-shelf
- ↑ easy to integrate with other systems on the same platform (e.g., LiDAR)
- ↑ convenient for small area coverage & UAV systems
- ↓ should be calibrated by the end user
- ↓ inferior geometric quality and lens system
- ↓ stability of IOP is not guaranteed
- ↓ limited array size

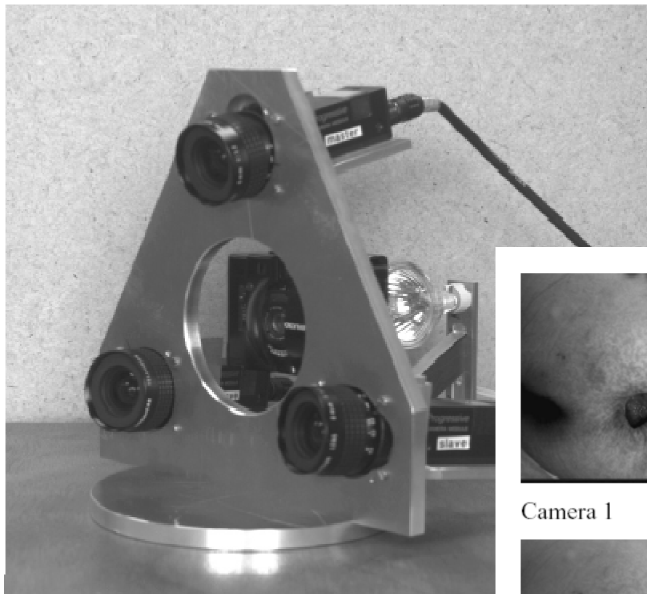
Data Acquisition Systems

WILD RC10

Data Acquisition Systems

SONY DSC F717

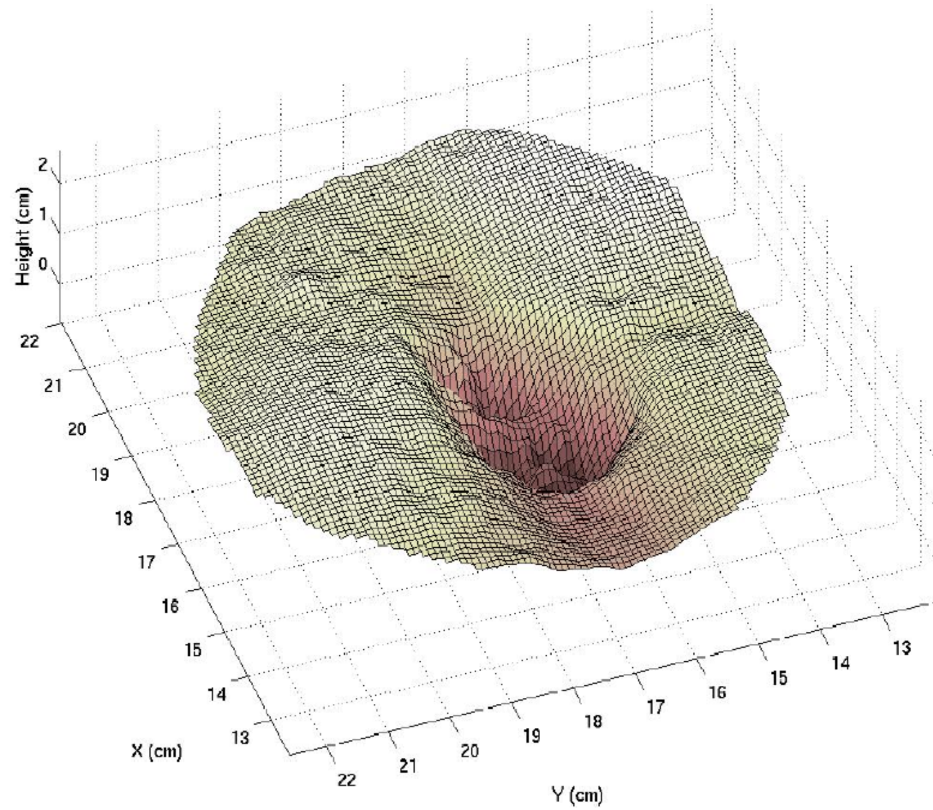
Terrestrial (Close Range) Imagery



Camera 1

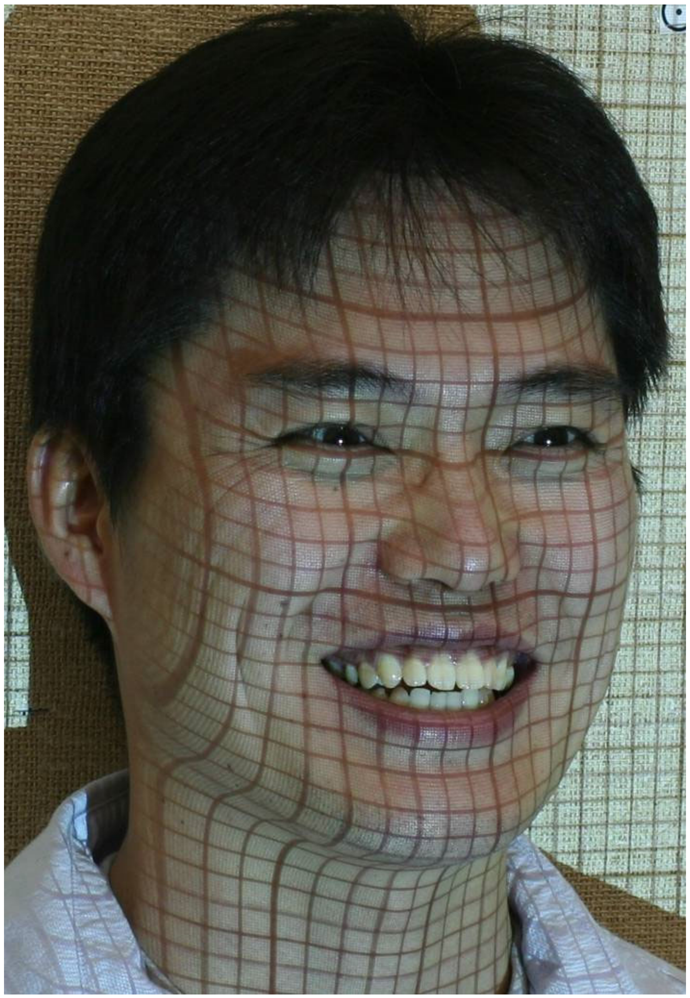
Camera 2

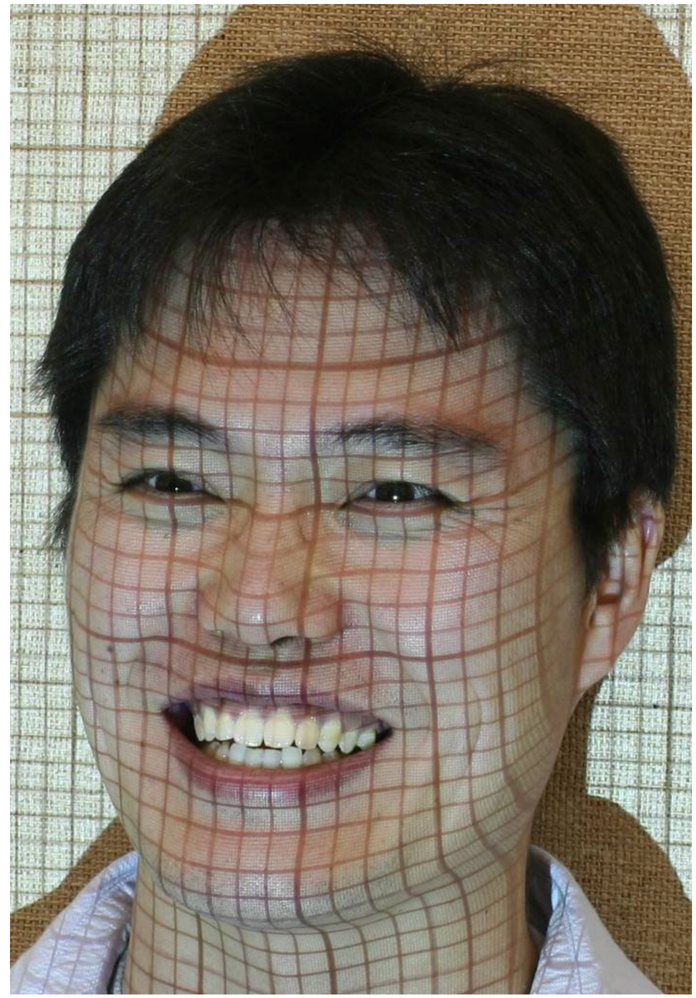
Camera 3



Terrestrial (Close Range) Imagery

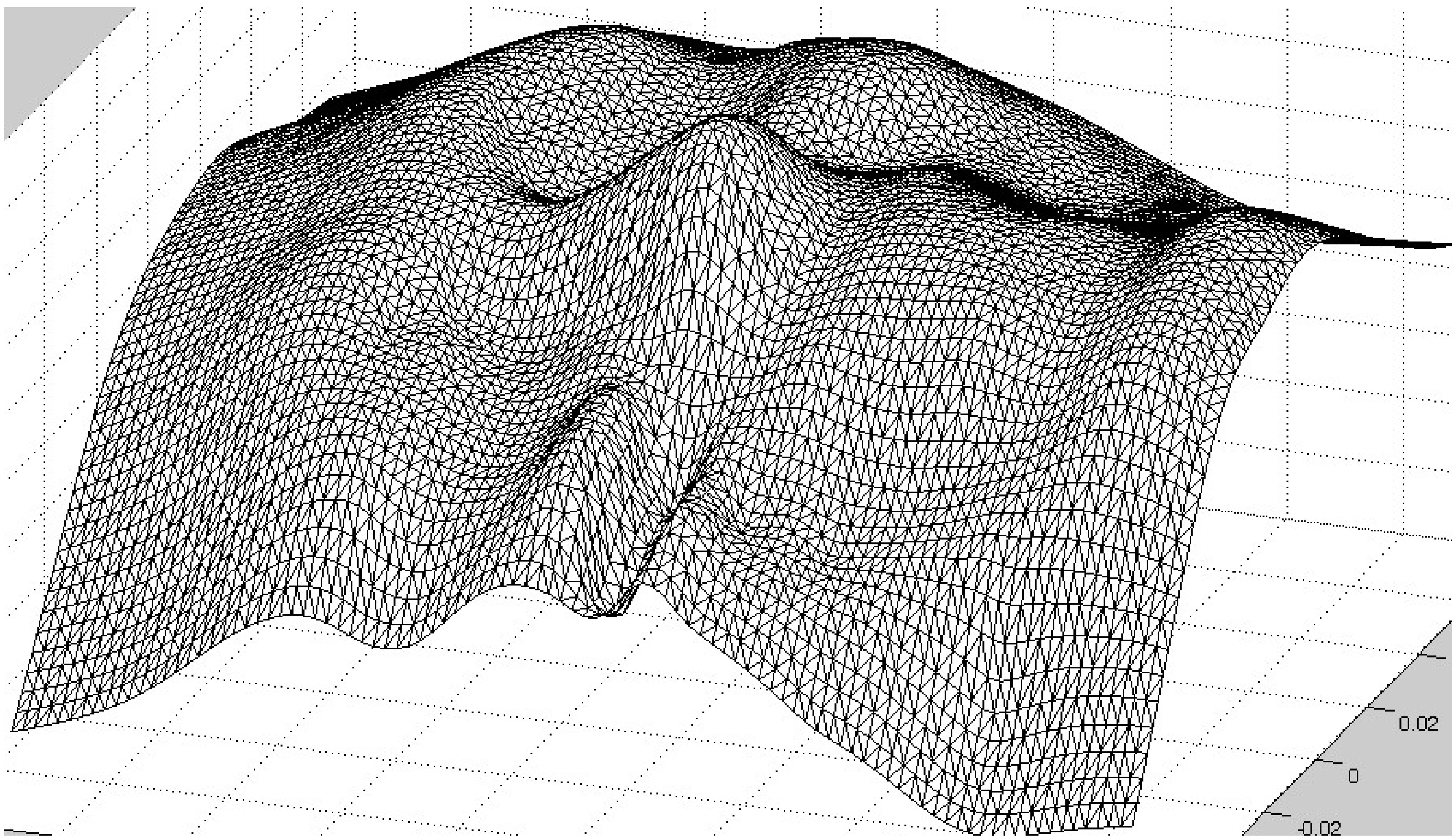
Terrestrial (Close Range) Imagery





Input Stereo-Imagery

Terrestrial (Close Range) Imagery



Output Three-Dimensional Model

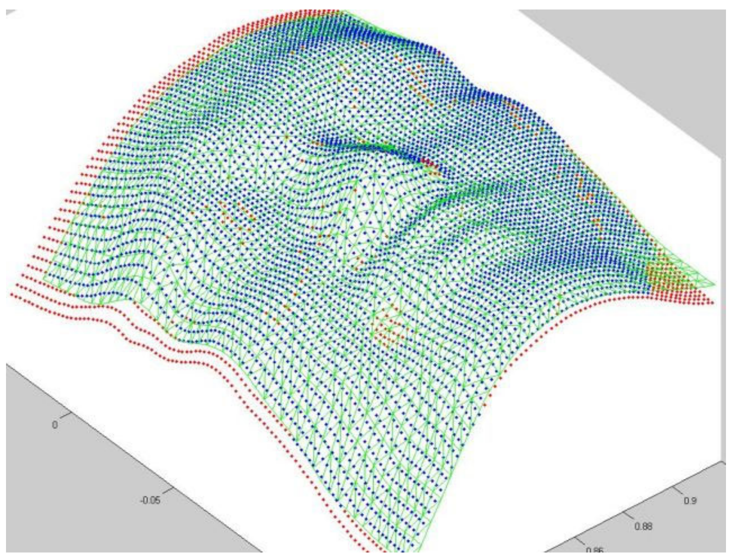
Terrestrial (Close Range) Imagery

- Experiments:

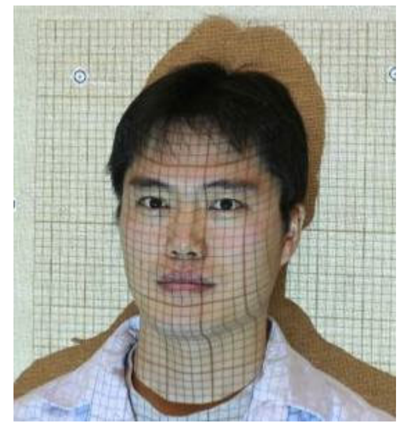
Test	Descriptions
1	Subject 1: Time 1 & Time 2
2	Subject 1: No Smile & Smile
3	Subject 2 & Subject 3

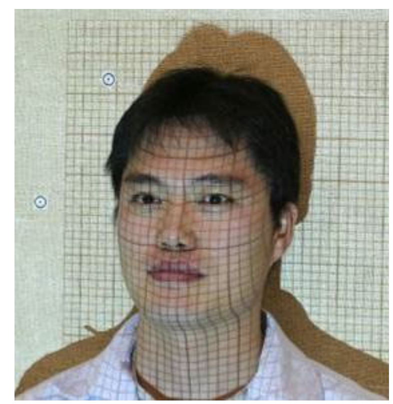
- Results:

Test 1



Green: Reference
Blue: Matches
Red: Non-matches





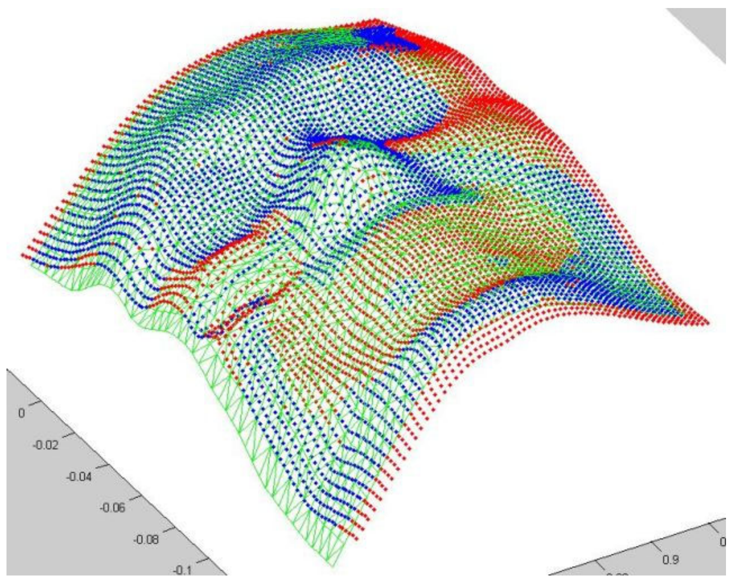
Terrestrial (Close Range) Imagery

- Experiments:

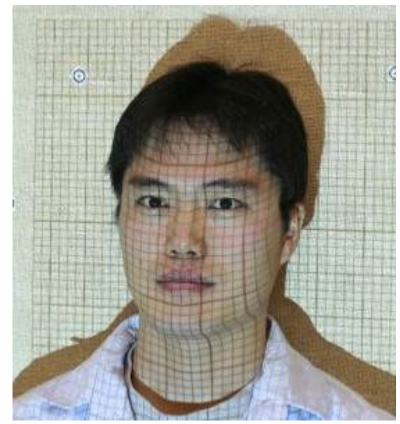
Test	Descriptions
1	Subject 1: Time 1 & Time 2
2	Subject 1: No Smile & Smile
3	Subject 2 & Subject 3

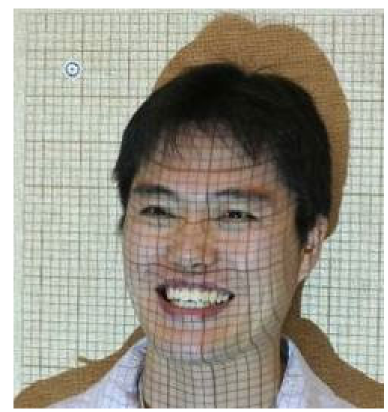
- Results:

Test 2



Green: Reference
Blue: Matches
Red: Non-matches





Terrestrial (Close Range) Imagery

- Experiments:

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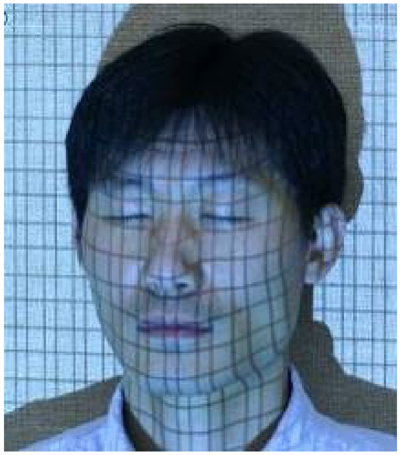
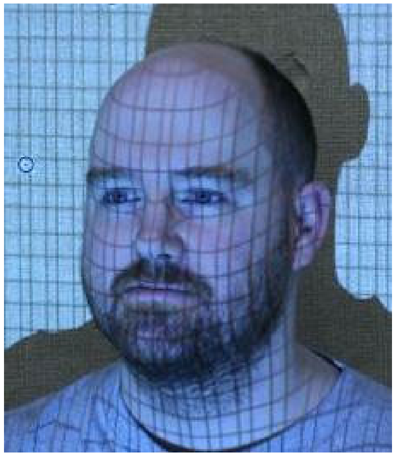
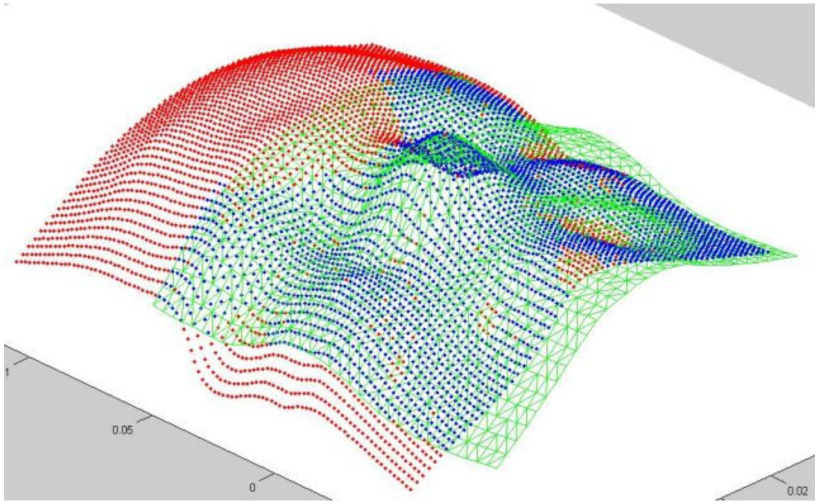




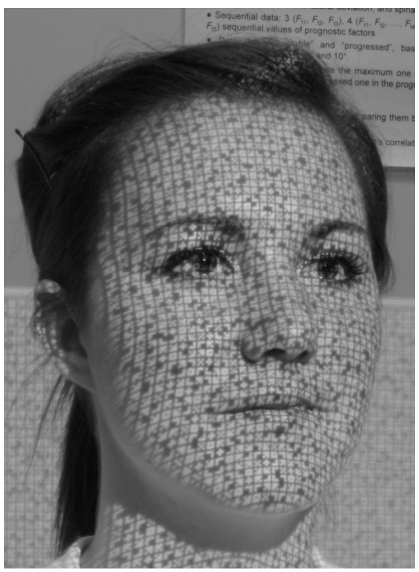
- Results:

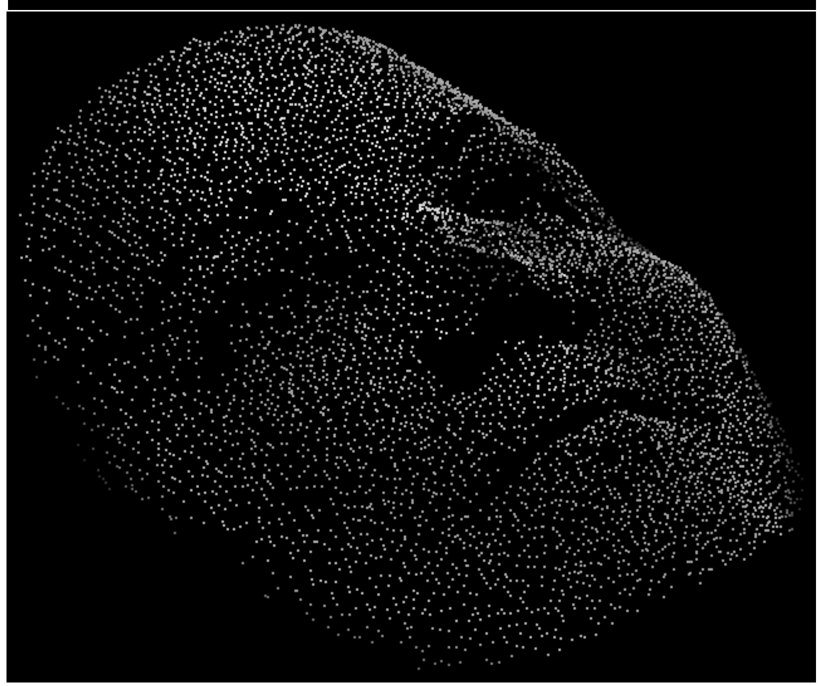
Test 3



Green: Reference
Blue: Matches
Red: Non-matches

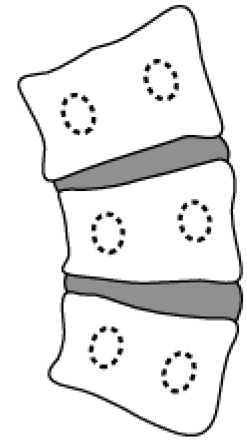
Terrestrial (Close Range) Imagery



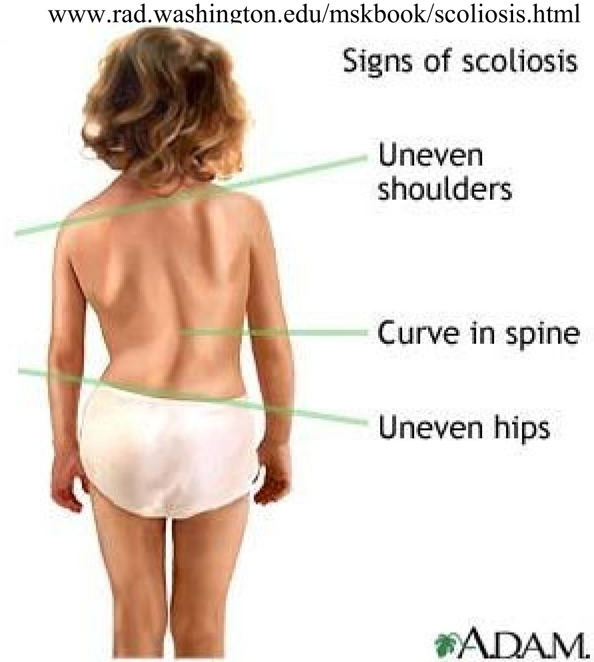


Terrestrial (Close Range) Imagery

- **<u>Scoliosis</u>**
- 3D deformity of the human spine
- Affects 2-3% of the population
- Impacts the quality of life
- Early detection is vital



www.rad.washington.edu/mskbook/scoliosis.html



www.nlm.nih.gov/MEDLINEPLUS/ency/images/ency/fullsize/19466.jpg

Terrestrial (Close Range) Imagery

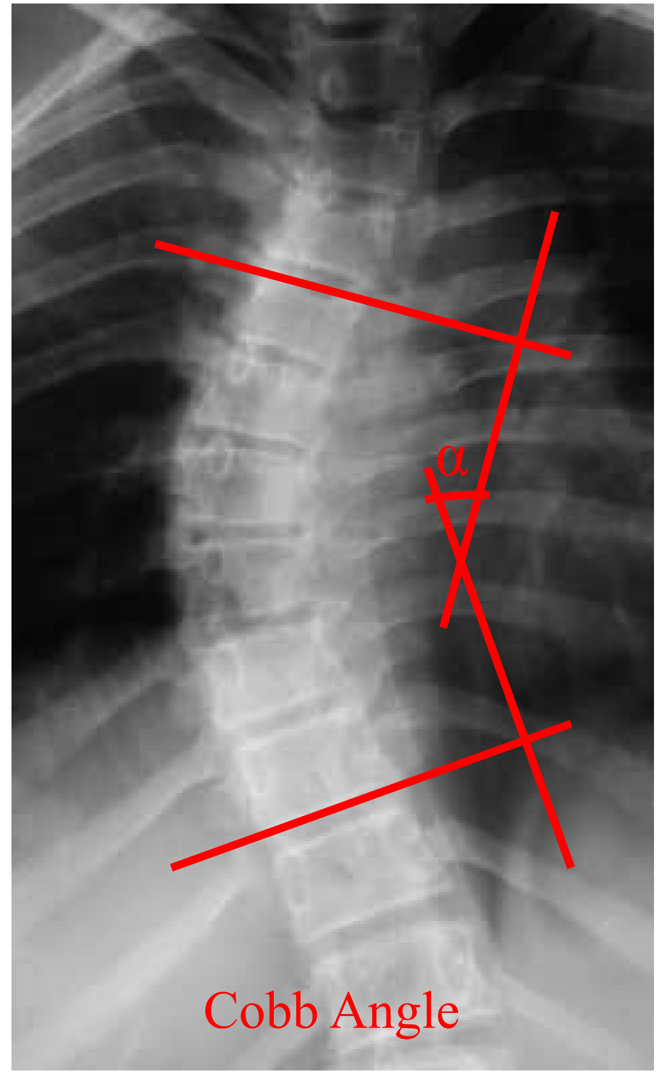
- **<u>Scoliosis Detection & Monitoring</u>**

➢ Traditional method:

- Full-length spinal x-ray in a standing position

➢ Consequences:

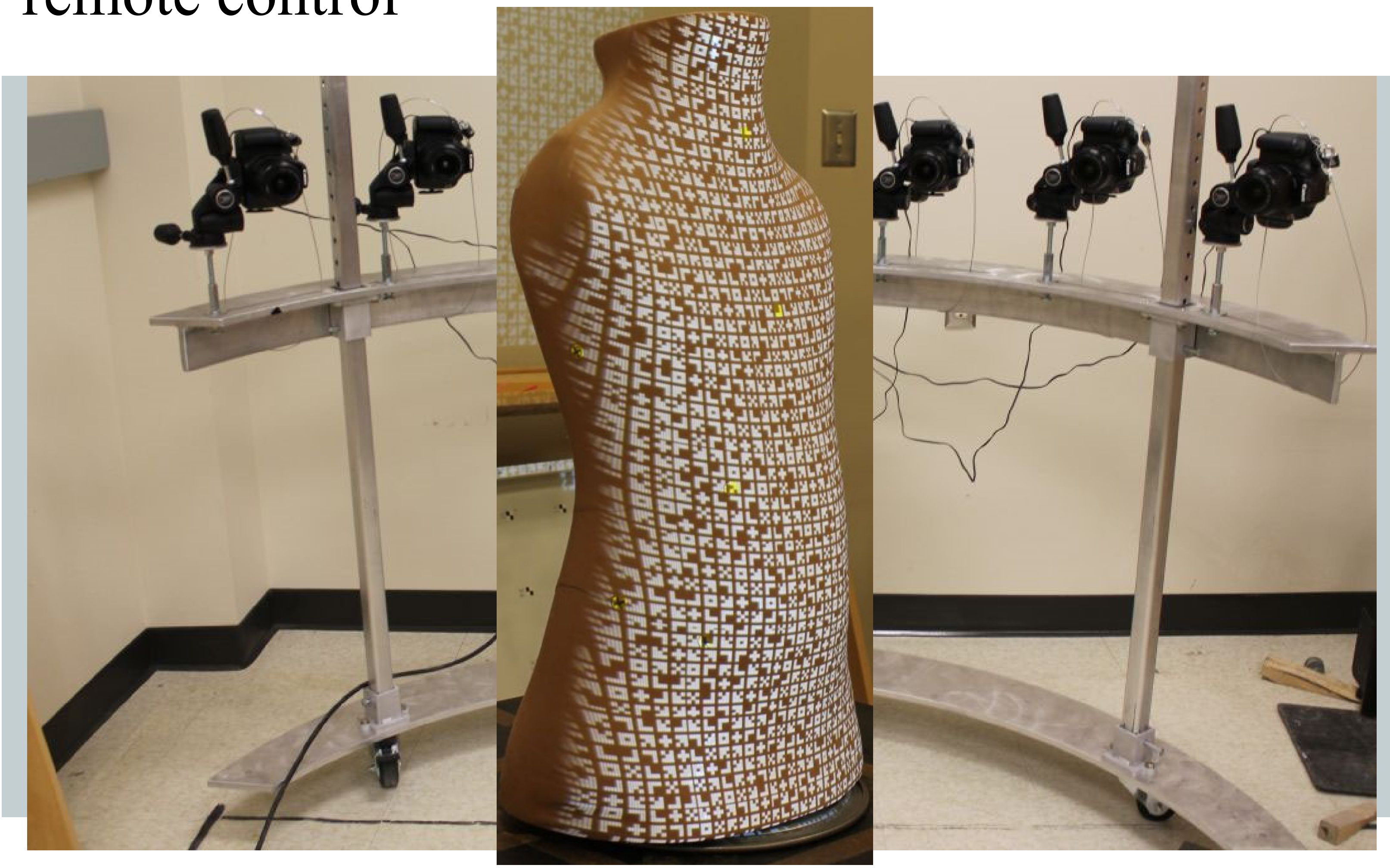
- Frequent exposure to radiation (4-5 times a year, for 3-5 years)
- Increased risk of cancer



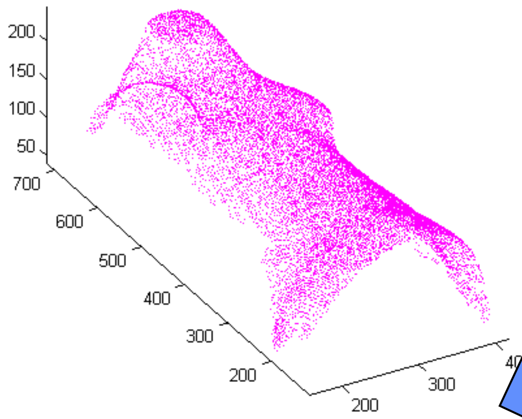
http://www.e-radiography.net/radpath/c/cobb-angle.jpg

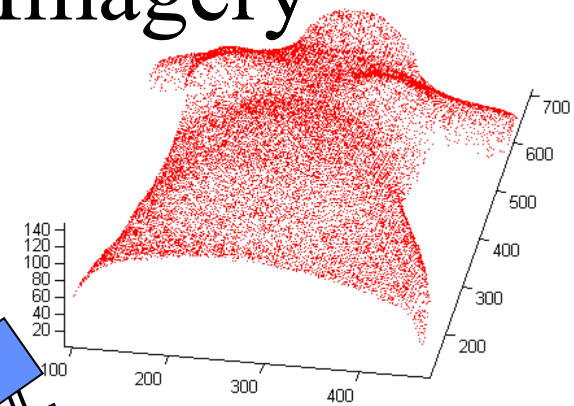
Terrestrial (Close Range) Imagery

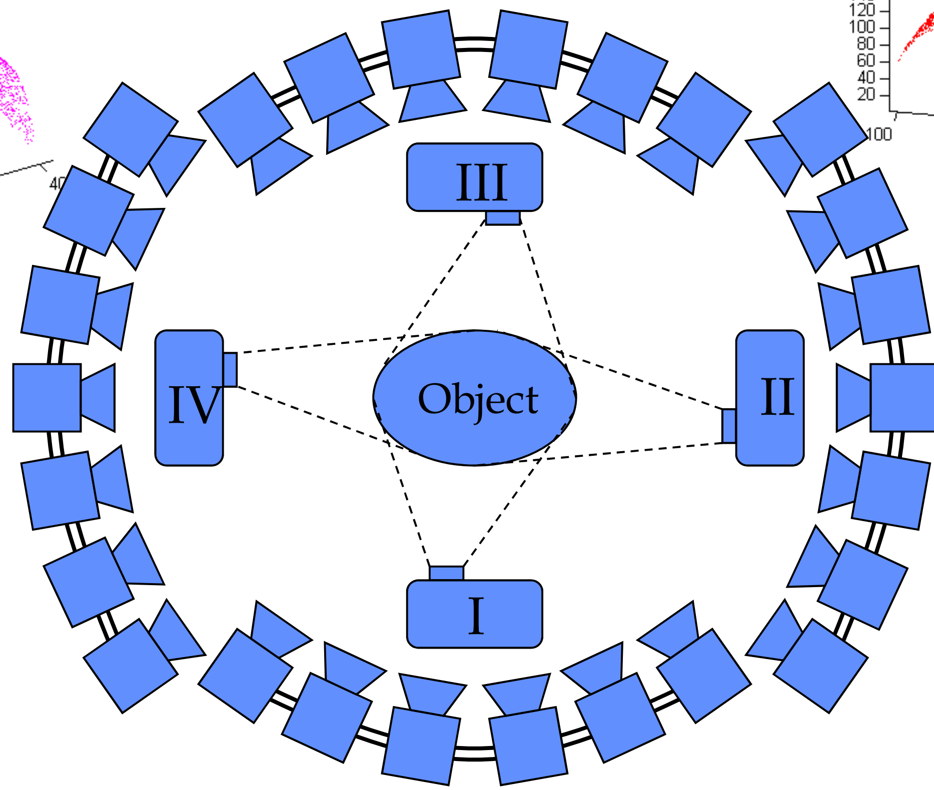
Cameras, projectors, frame, target board, computer(s), remote control

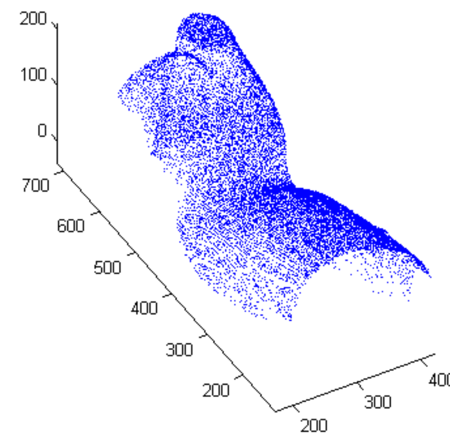


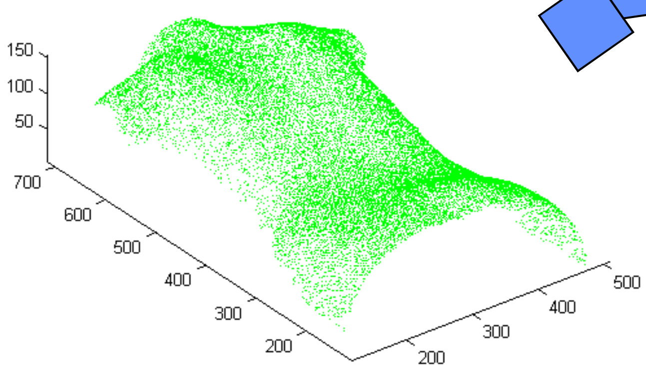
Terrestrial (Close Range) Imagery







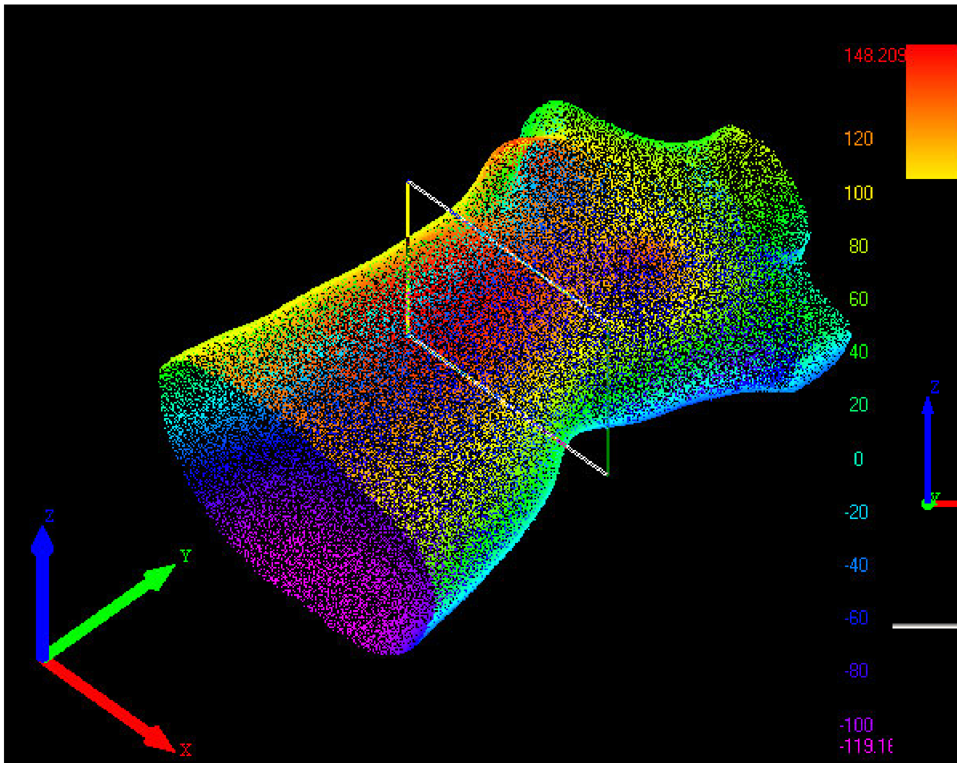




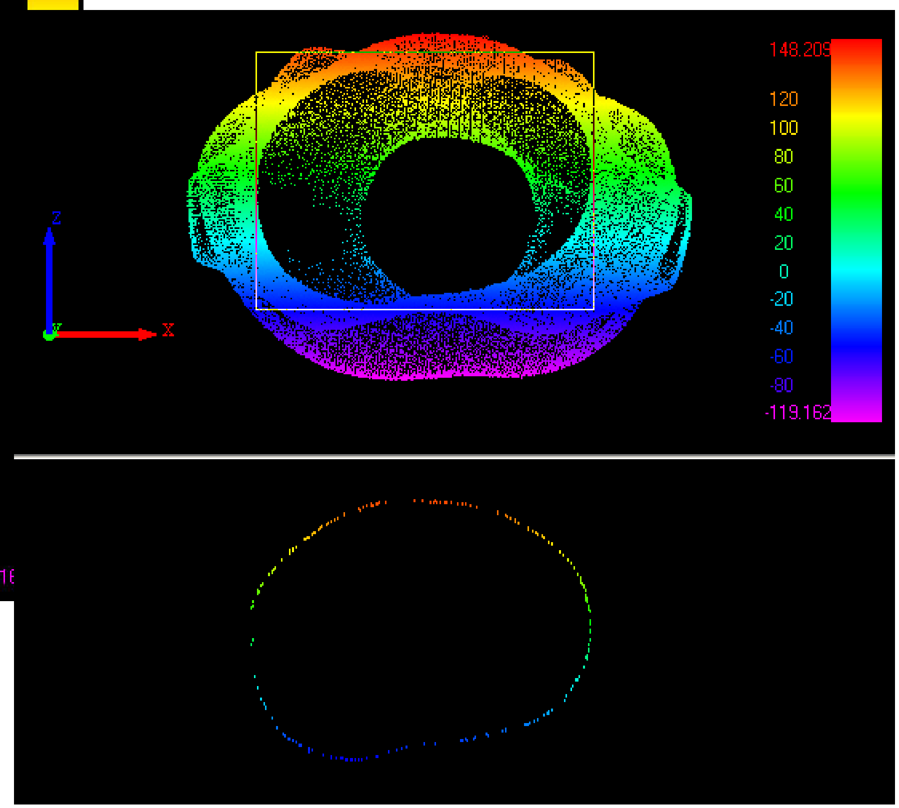
Four point clouds in four different reference frames → necessity for a registration to a common reference frame

Terrestrial (Close Range) Imagery

- Multiple surface registration: complete 3D torso model



Qualitative assessment



Terrestrial (Close Range) Imagery

Terrestrial (Close Range) Imagery

Terrestrial (Close Range) Imagery

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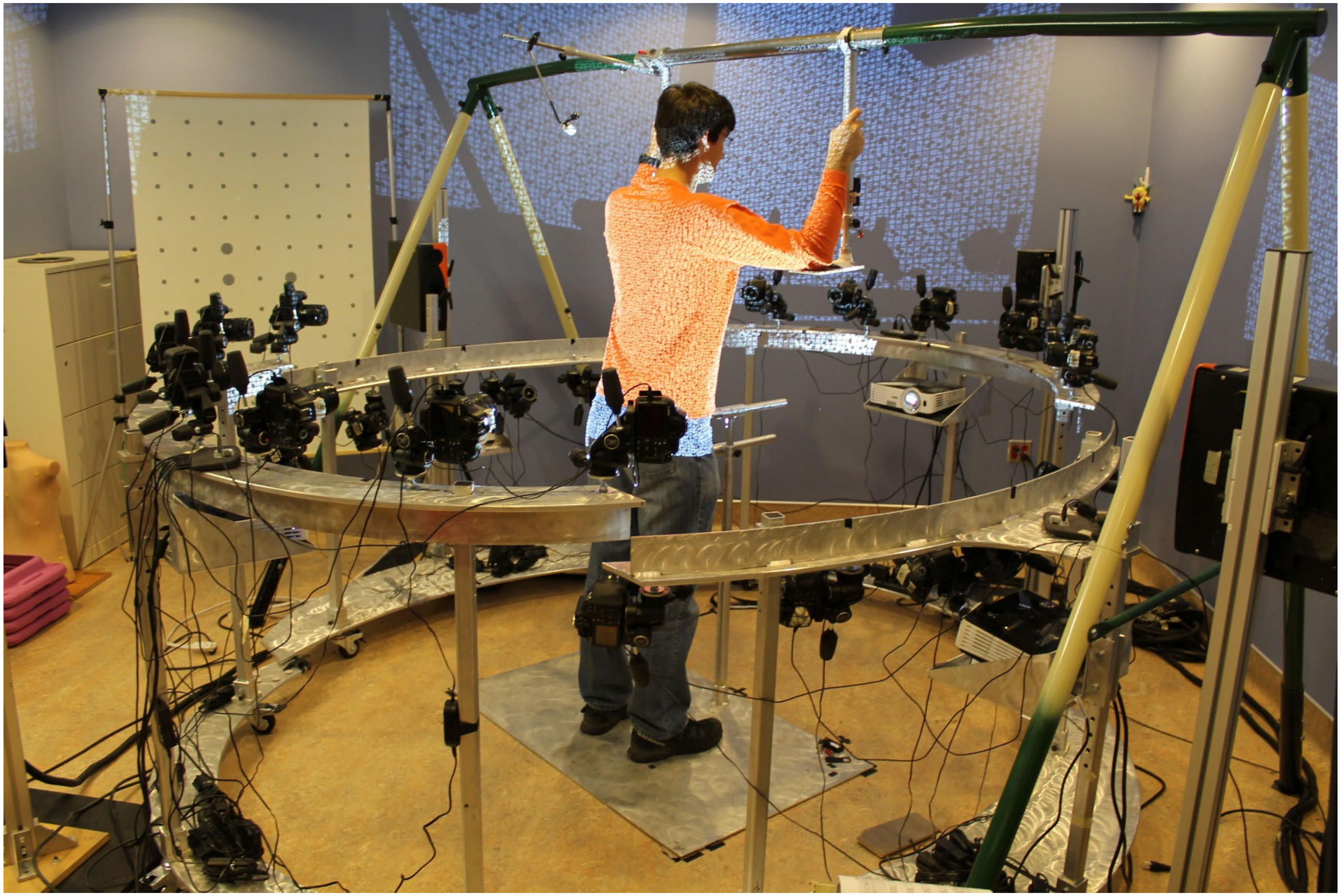
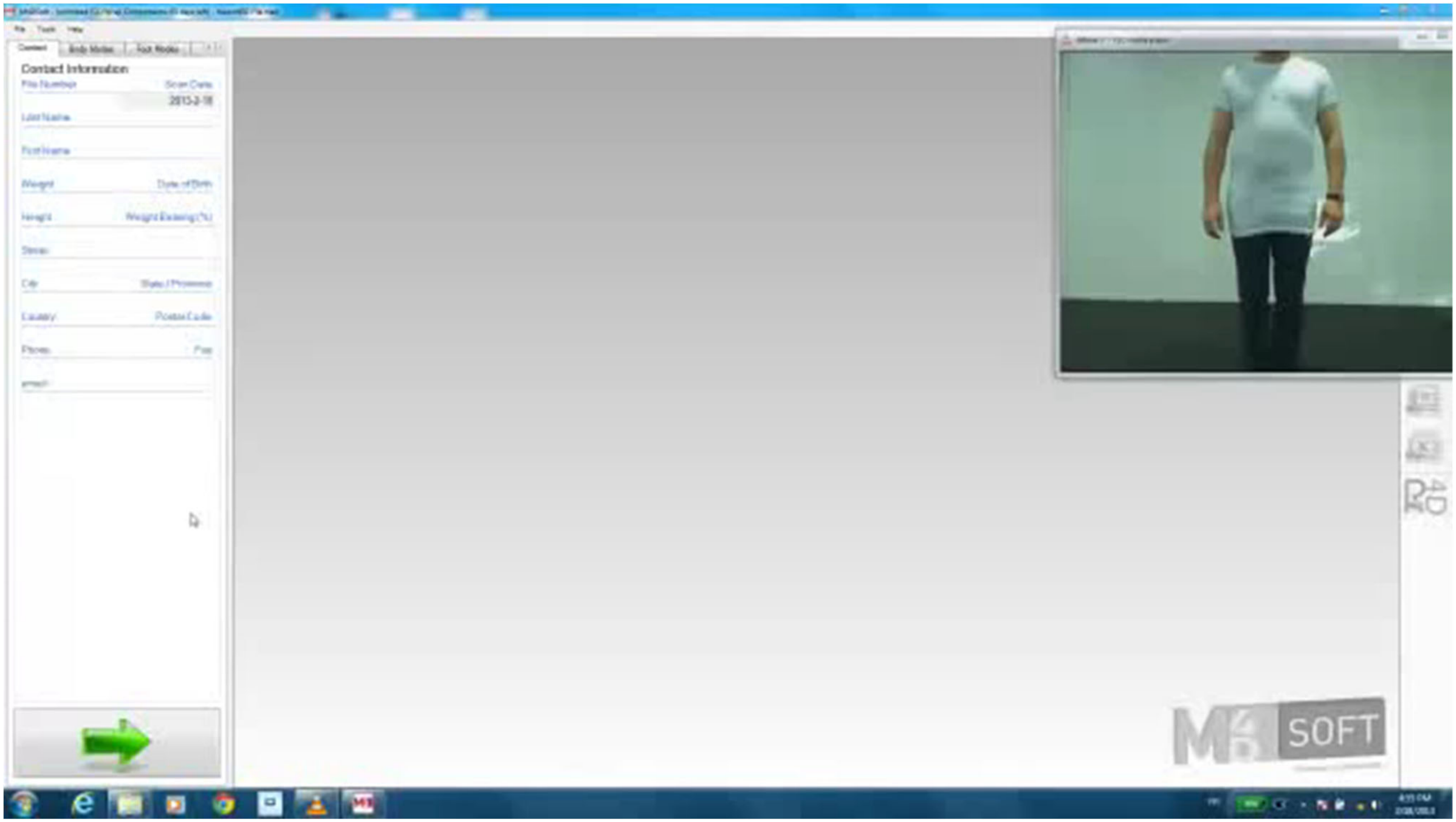


Terrestrial (Close Range) Imagery

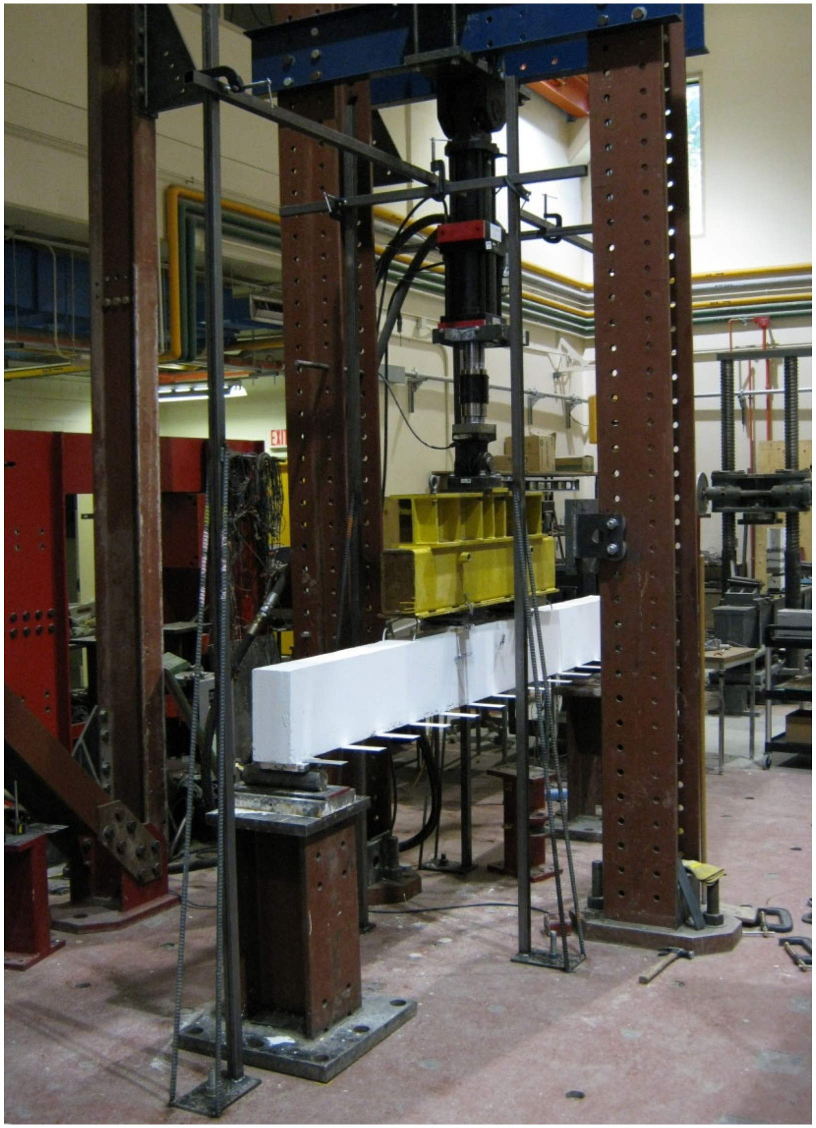
Terrestrial (Close Range) Imagery

Laser-Based Torso Reconstruction

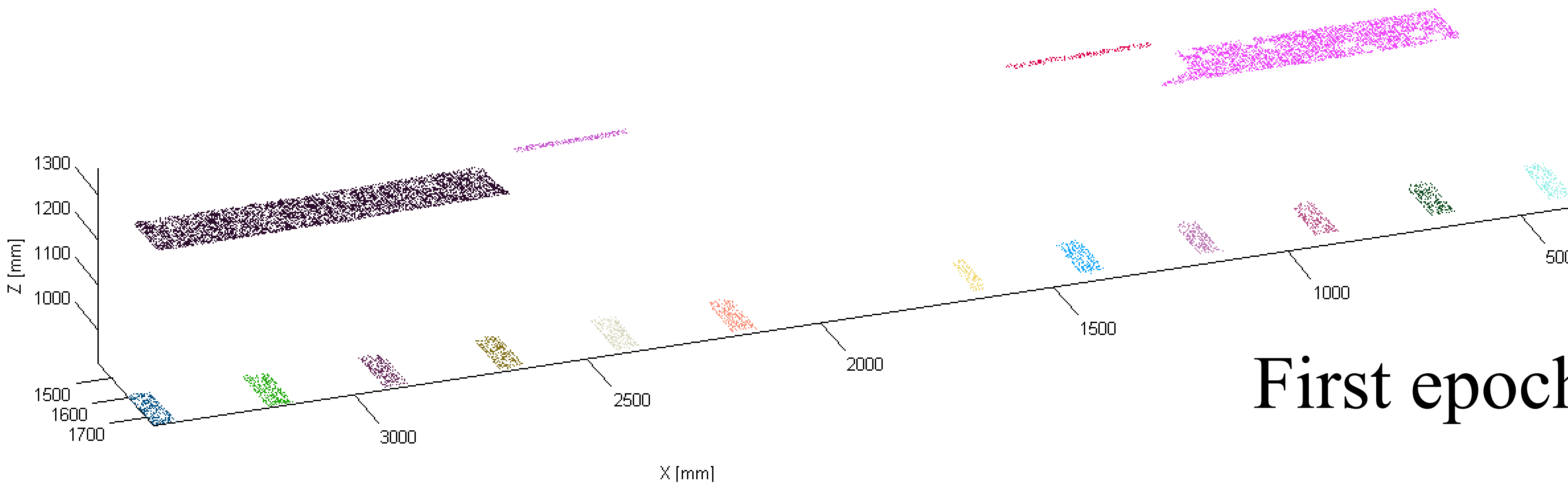


Terrestrial (Close Range) Imagery

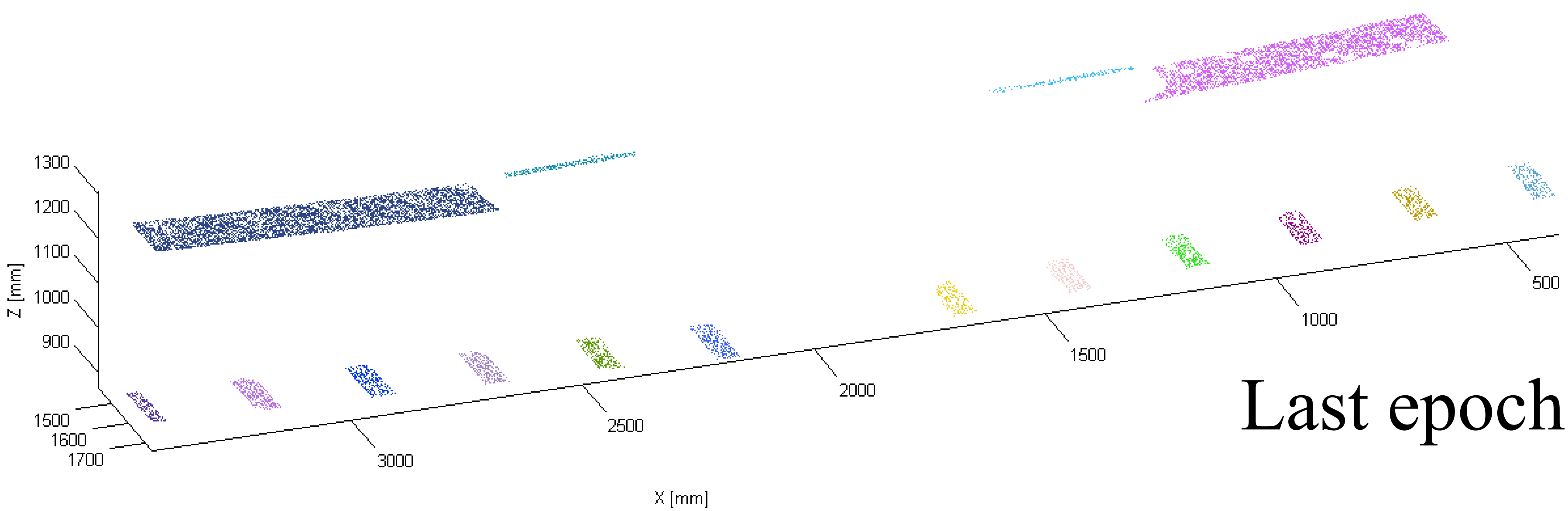
- **Objective:**
  - Develop a system that can evaluate the deflection along the beam under static and dynamic loading conditions
- **Design target function:**
  - Low cost
  - Non-contact
  - Accurate
  - Reusable
  - Continuous evaluation of the deflection along the beam



Terrestrial (Close Range) Imagery

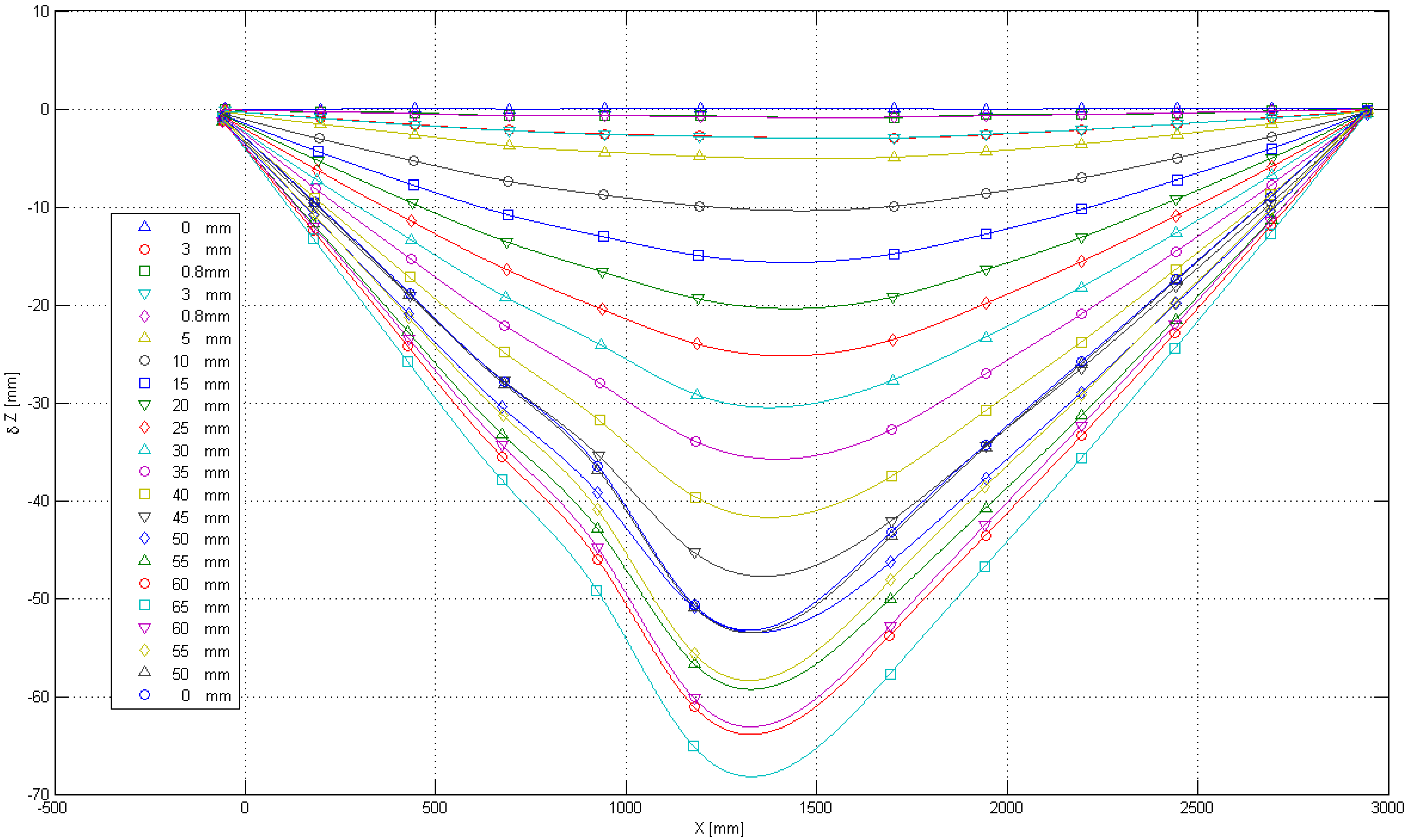


First epoch



Last epoch

Terrestrial (Close Range) Imagery



Terrestrial (Close Range) Imagery

Mobile Mapping Systems (MMS)

Terrestrial (Close Range) Imagery

Mobile Mapping Systems (MMS)

Terrestrial (Close Range) Imagery

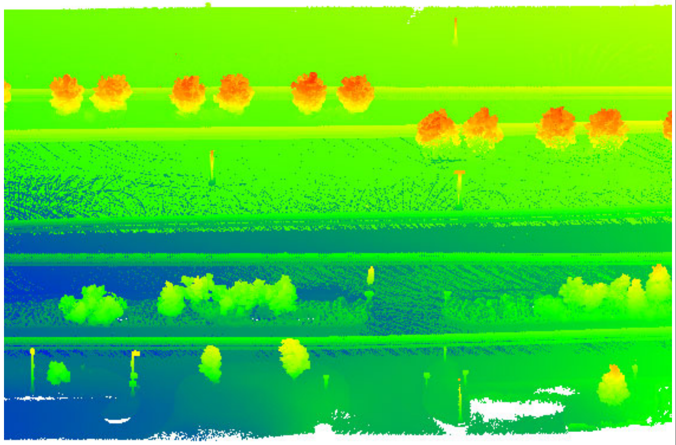
Mobile Mapping Systems (MMS)

University of Calgary

Terrestrial Mobile Mapping Systems

Platform: Truck

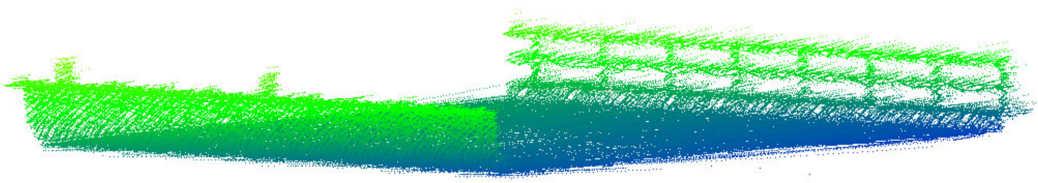
Test Area: Stadium



Collected point cloud
(Colored by height)

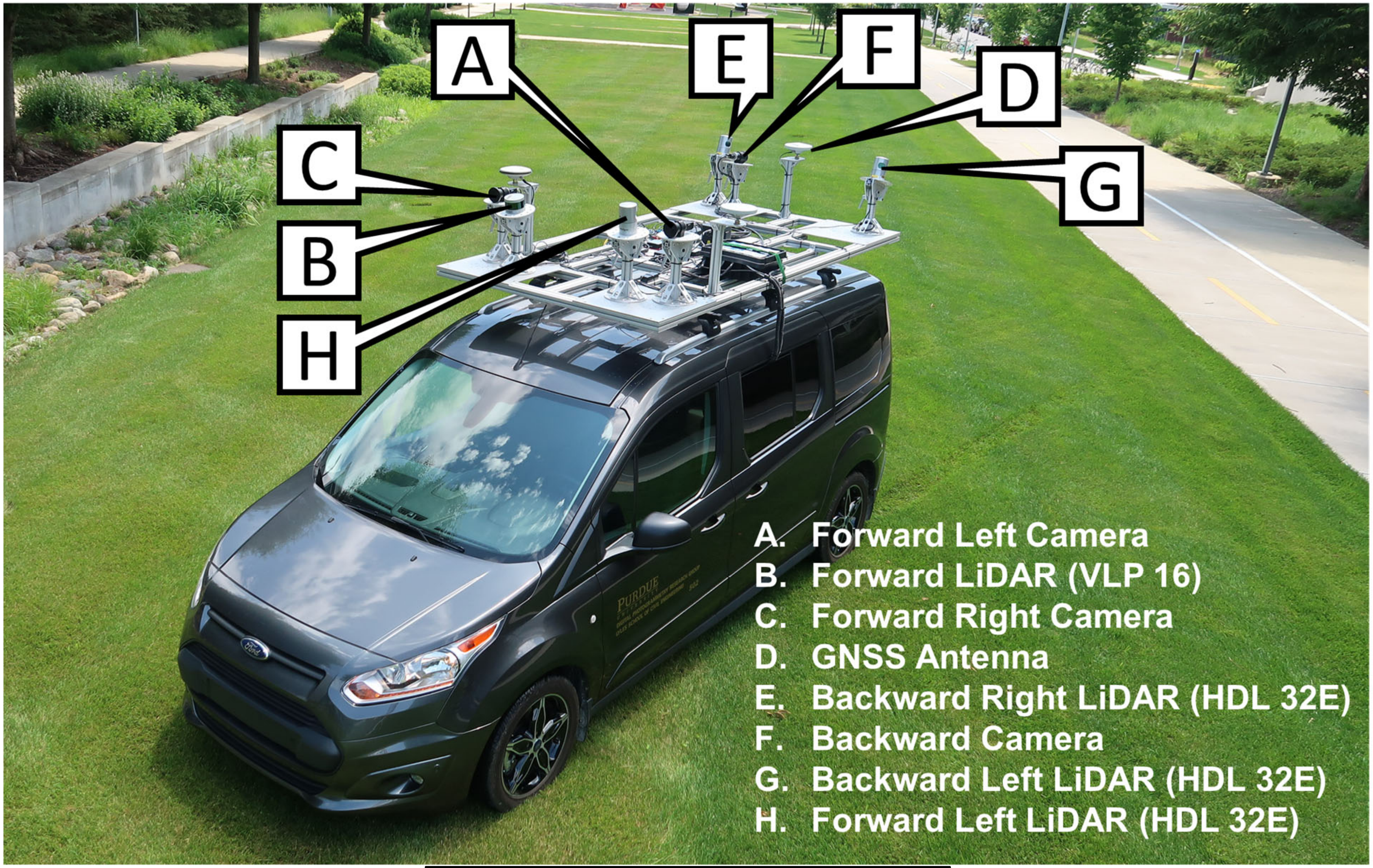
Purdue University

Terrestrial Mobile Mapping Systems



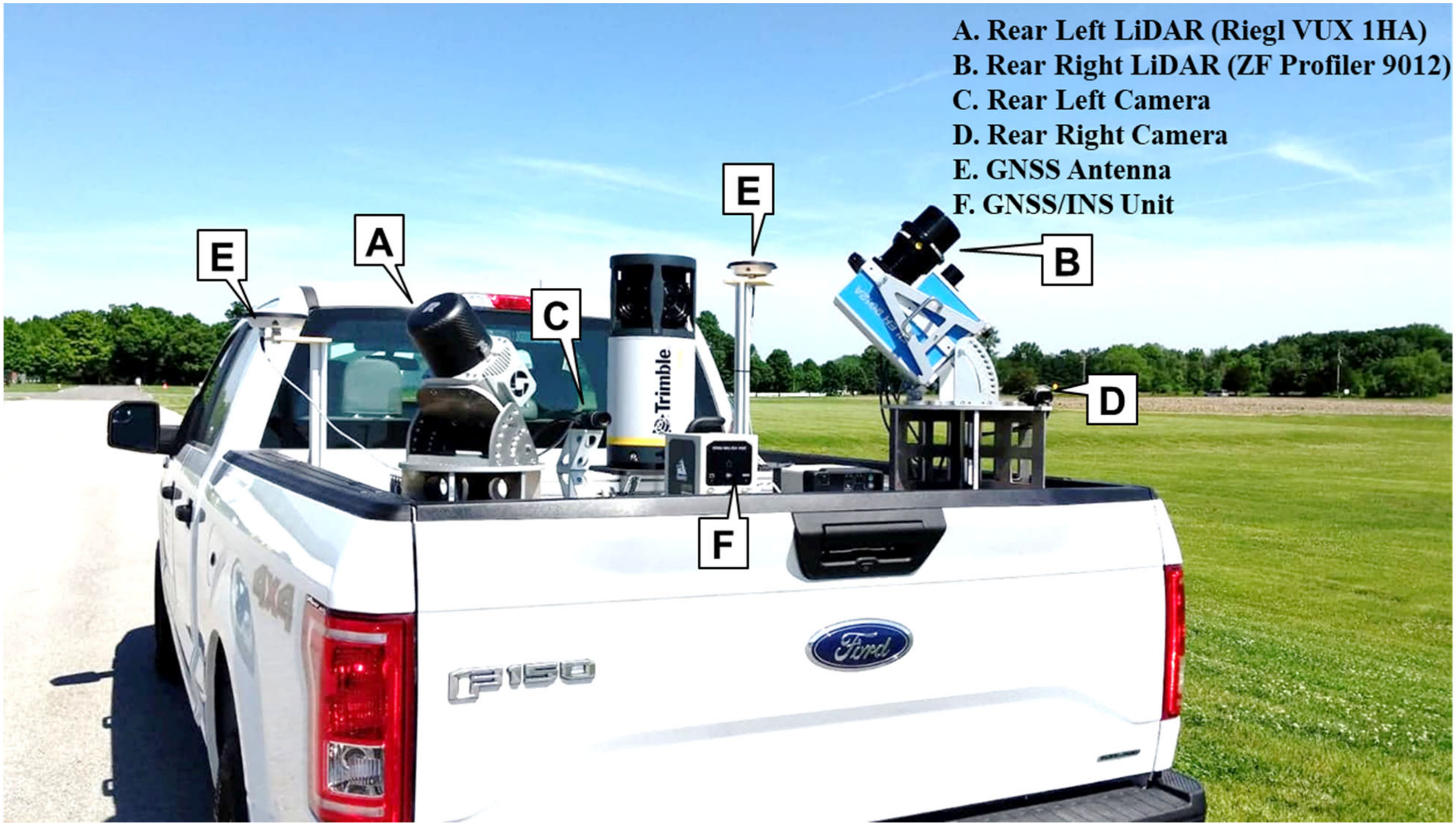
Purdue University

Terrestrial Mobile Mapping Systems



Purdue University

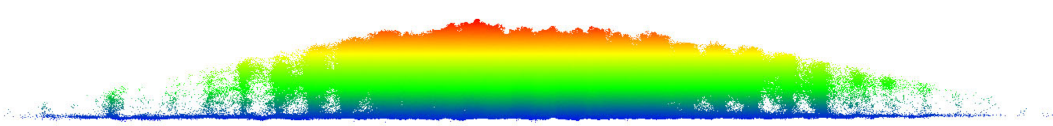
Terrestrial Mobile Mapping Systems



Purdue University

Terrestrial Mobile Mapping Systems

Phenomobile: RGB, Hyperspectral, and LiDAR



Purdue Univesrity

Terrestrial (Close Range) Imagery

Mobile Mapping Systems (MMS)

Sign Type

Height above Pavement

Offset from Road Edge

Coordinate Locations

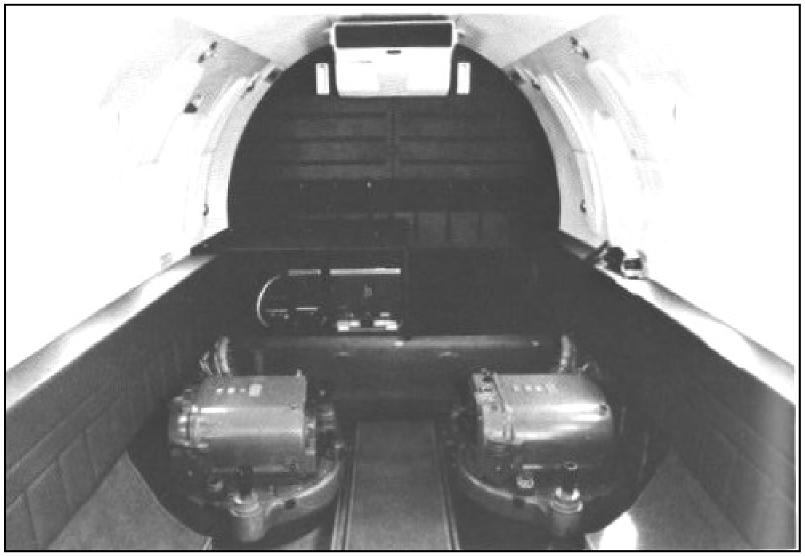
Size of Sign

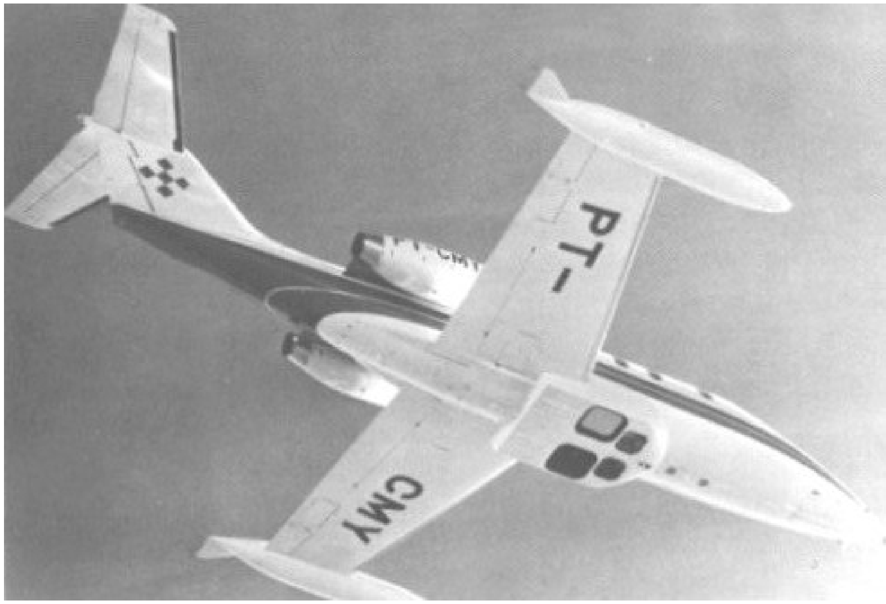
Terrestrial (Close Range) Imagery

Mobile Mapping Systems (MMS)

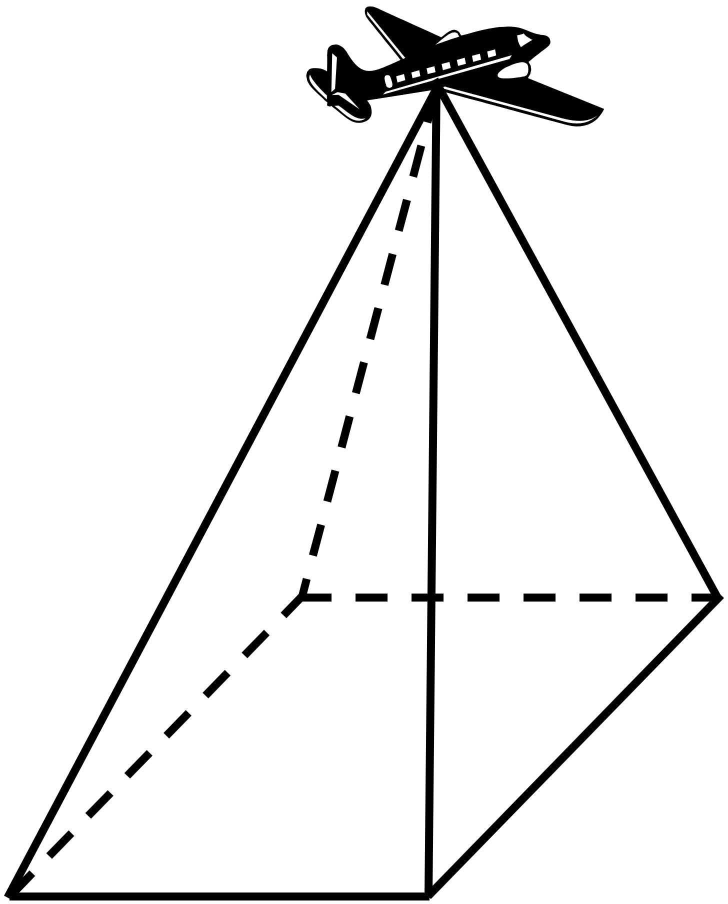
- **Collecting Inventories**
- **Database Integration**
- **On-going Maintenance**

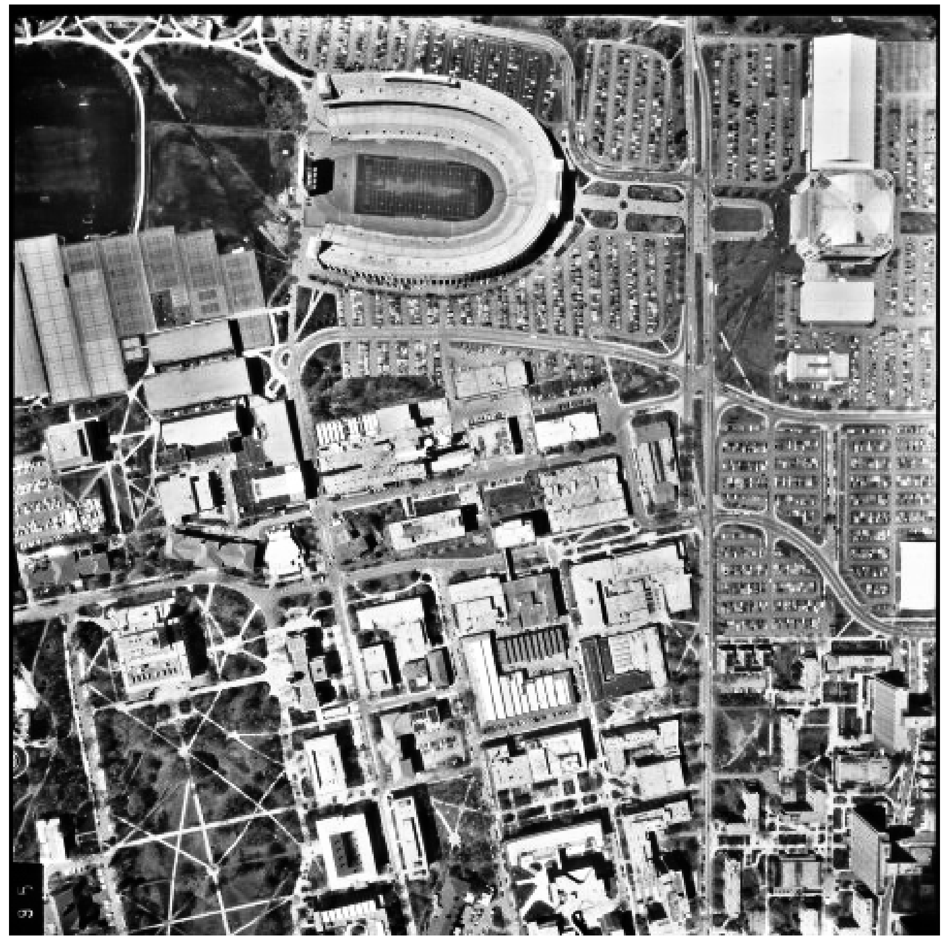
Aerial Imagery



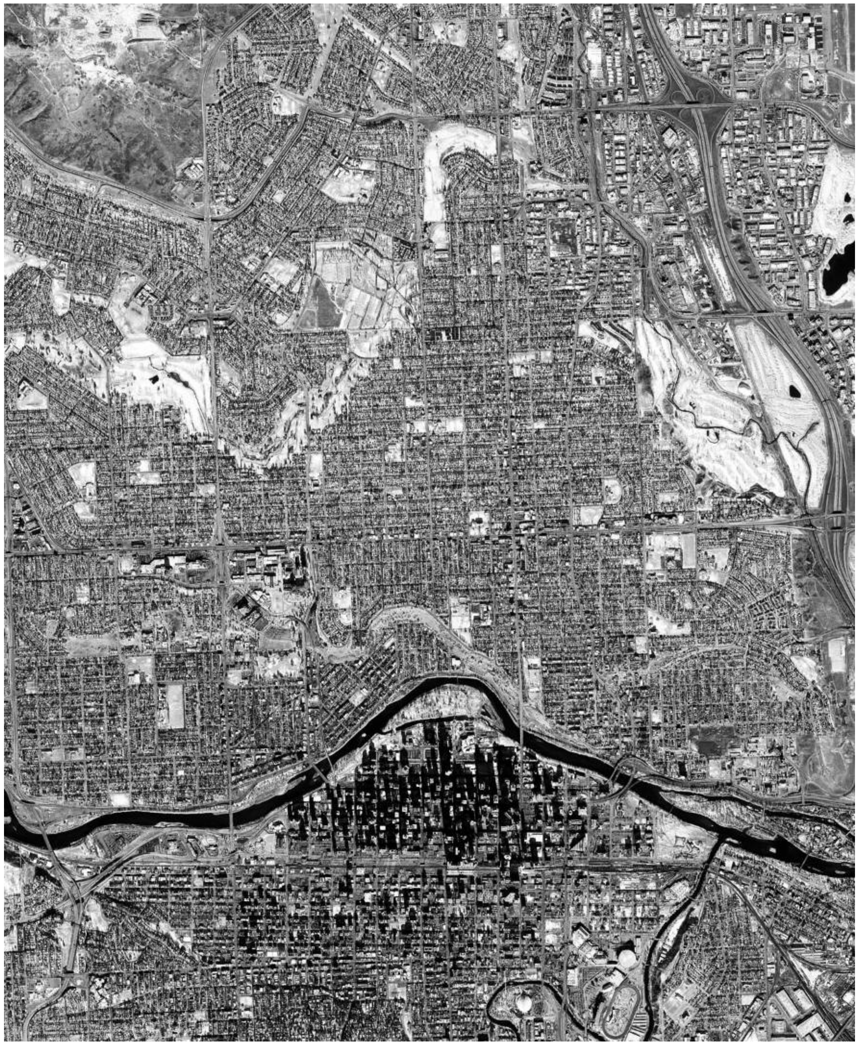


Aerial Imagery

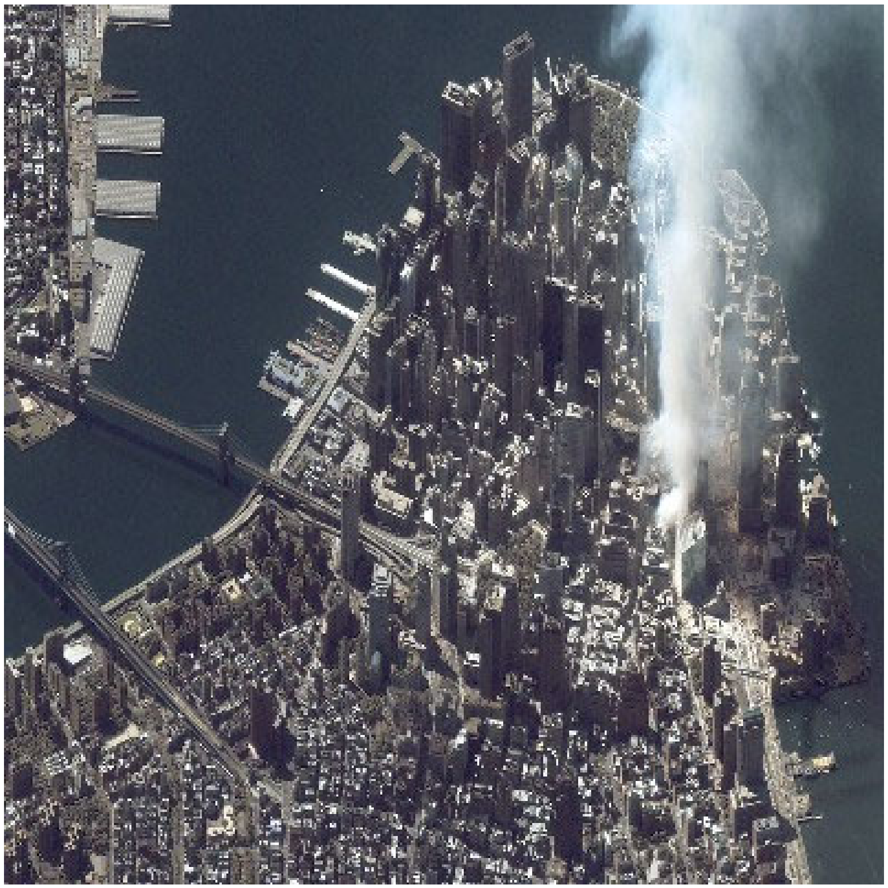




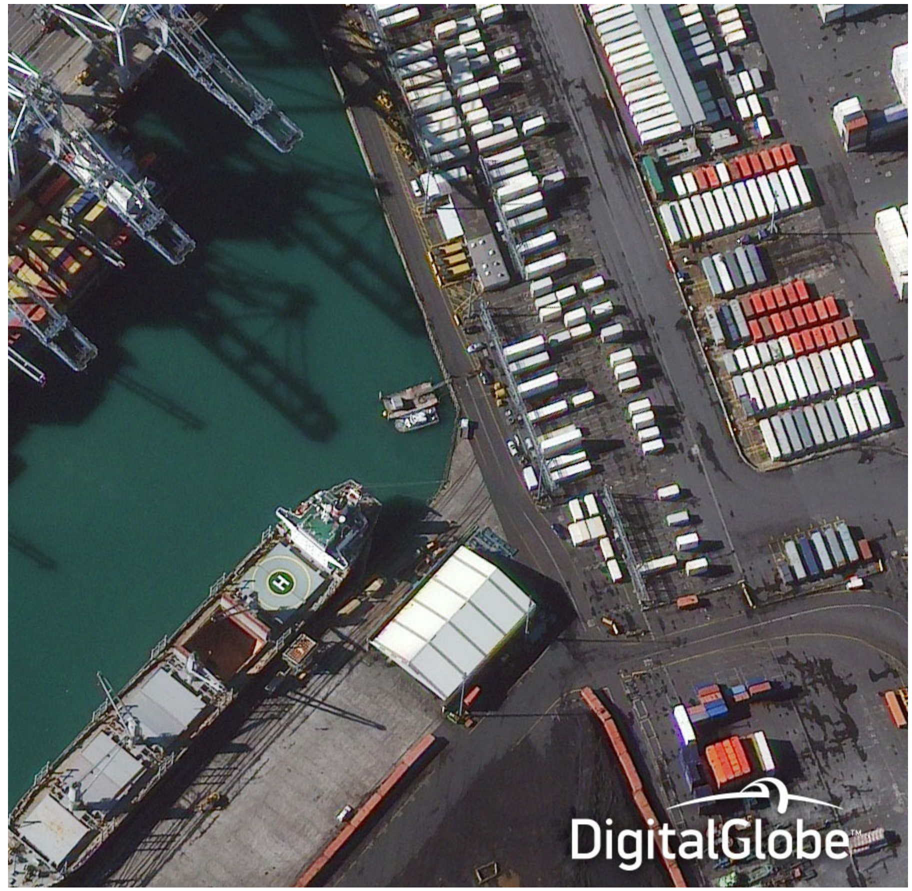
Satellite Imagery



Satellite Imagery



IKONOS



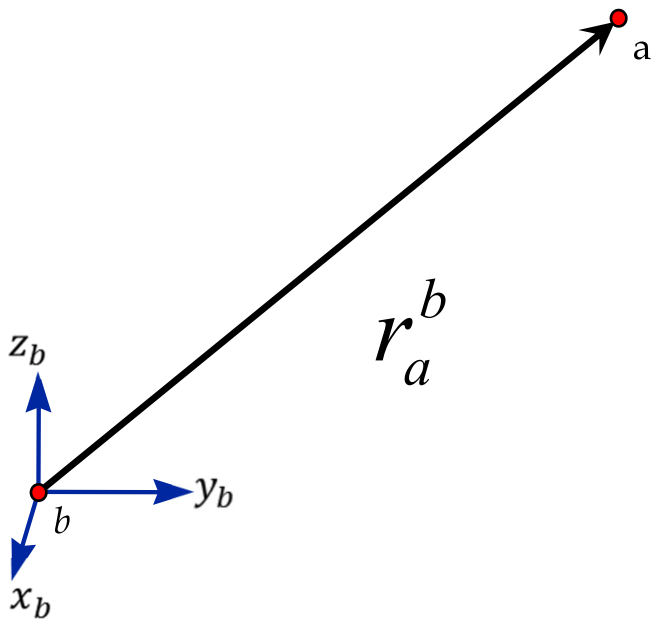
Digital Globe – WorldView 3 (30cm GSD)

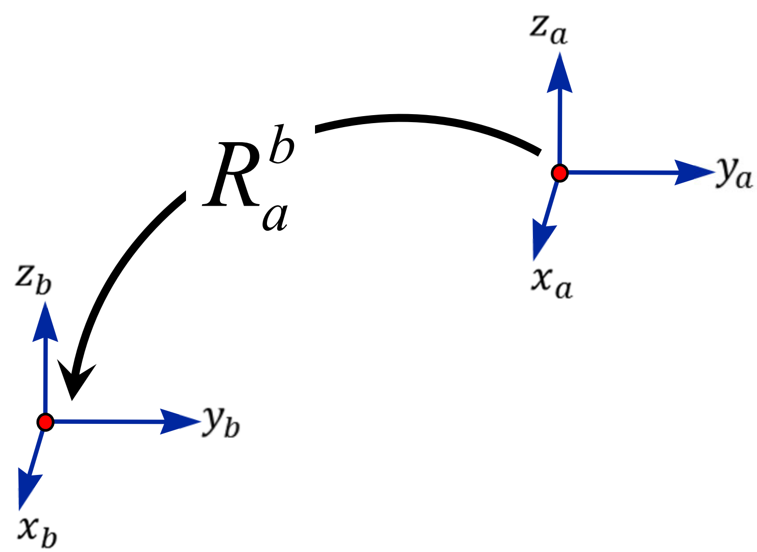
Notations

- r_a^b Stands for the coordinates of point ***a*** relative to point ***b*** – this vector is defined relative to the coordinate system associated with point ***b***.

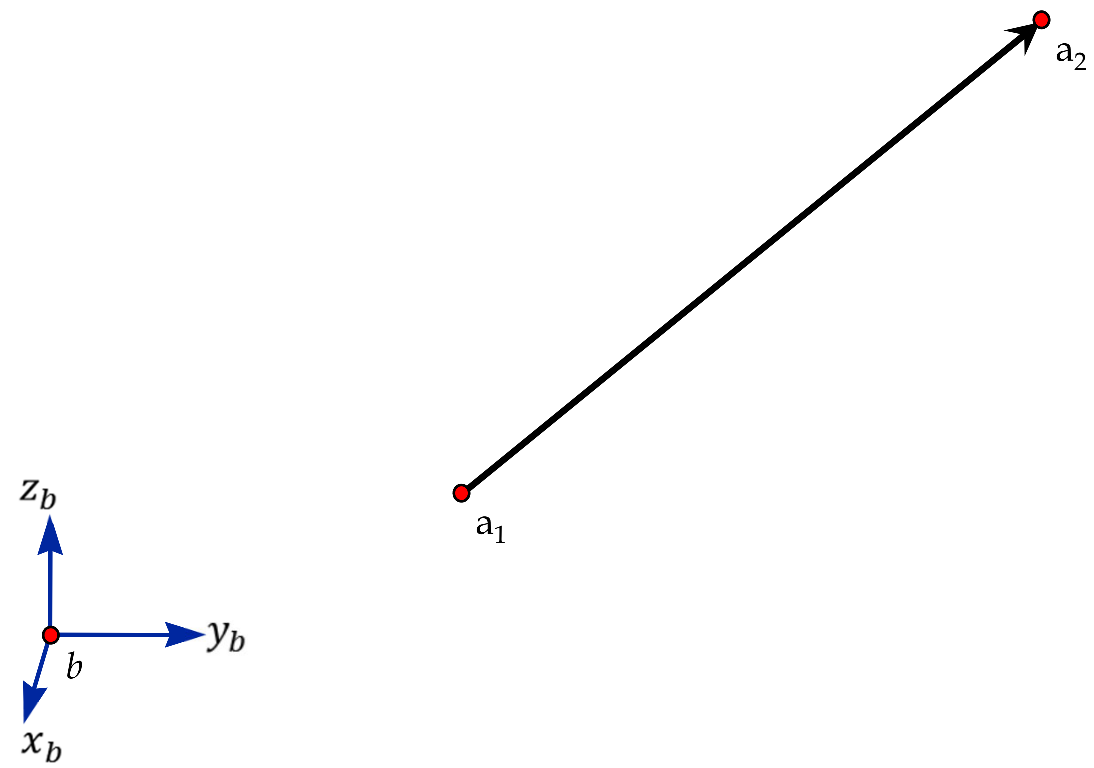
- R_a^b Stands for the rotation matrix that transforms a vector defined relative to the coordinate system denoted by ***a*** into a vector defined relative to the coordinate system denoted by ***b***.

Notations



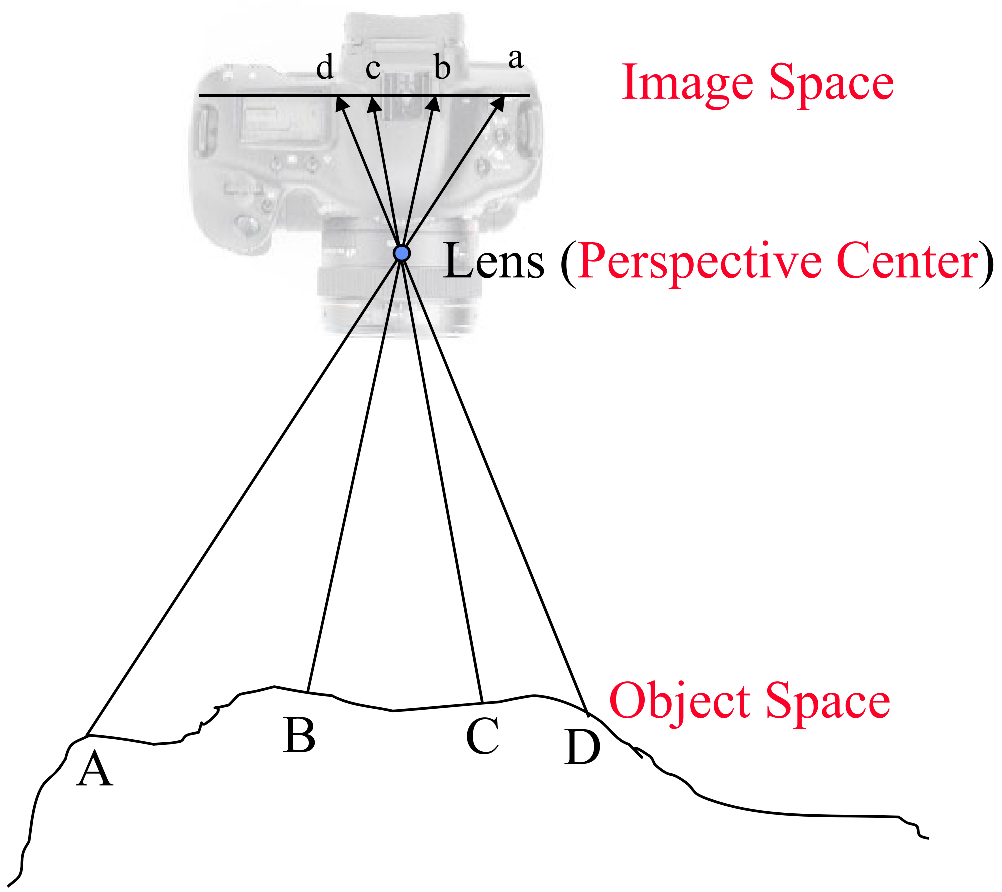


Notations

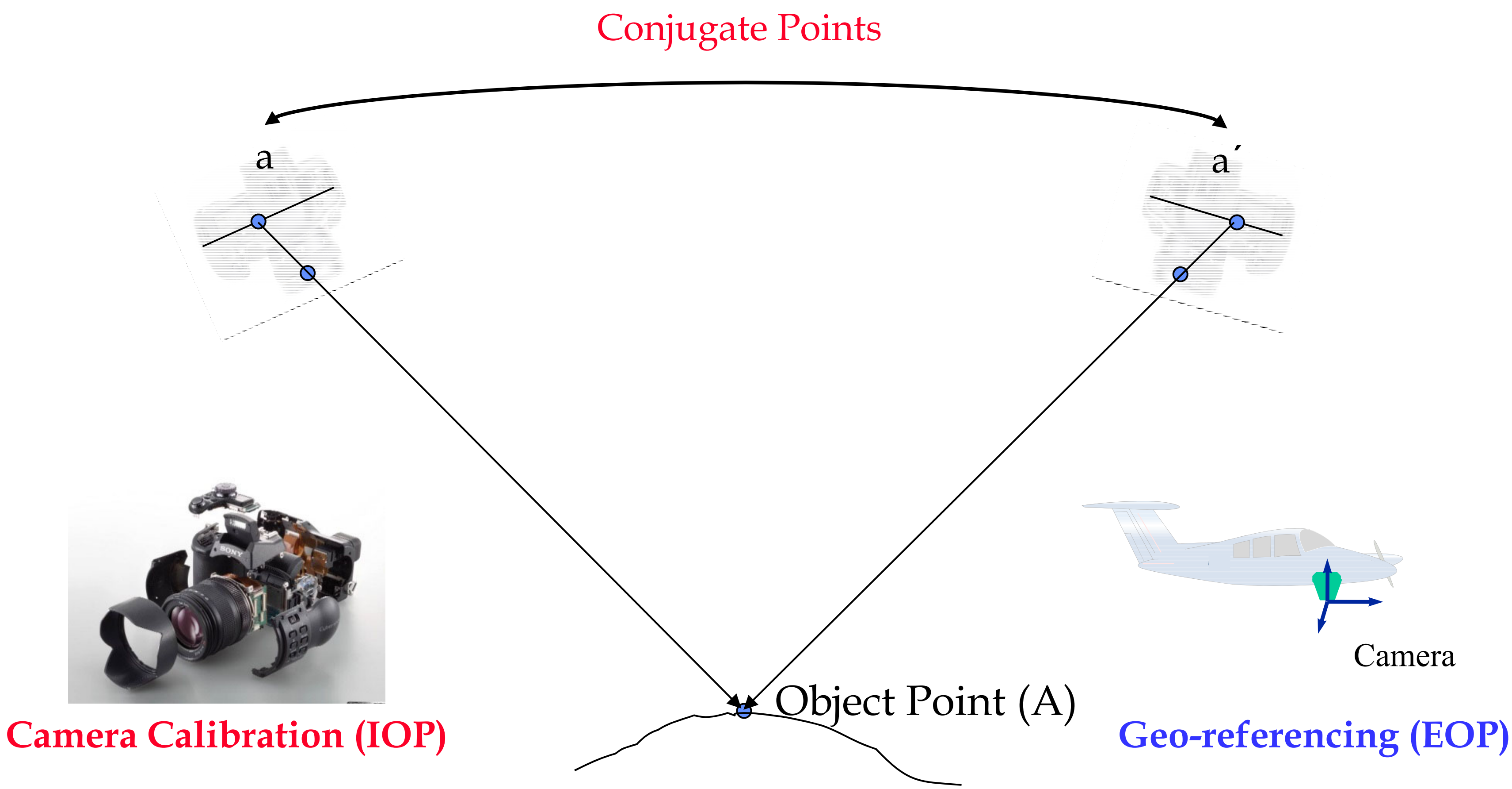


$$r^{b}_{a_1 a_2}$$

Photography



Photogrammetry



- **The interior orientation parameters of the involved cameras have to be known.**
- **The position and the orientation of the camera stations have to be known.**

Camera Calibration

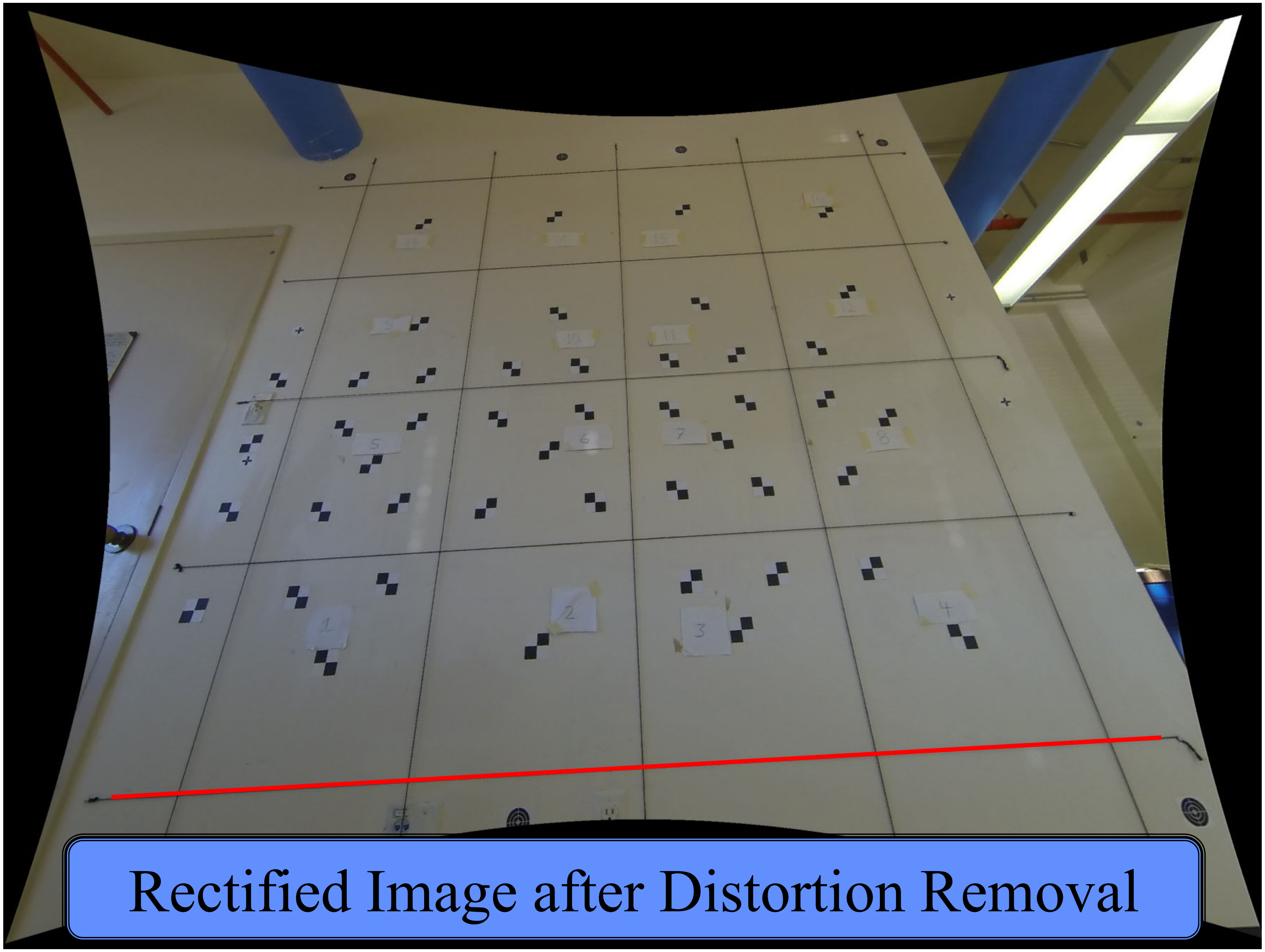
- Alternative procedures for camera calibration are well established.
  - Laboratory camera calibration (Multi-collimators)
  - Indoor camera calibration
  - In-situ camera calibration

Analytical camera calibration

Camera Calibration

Excessive Radial Lens Distortion

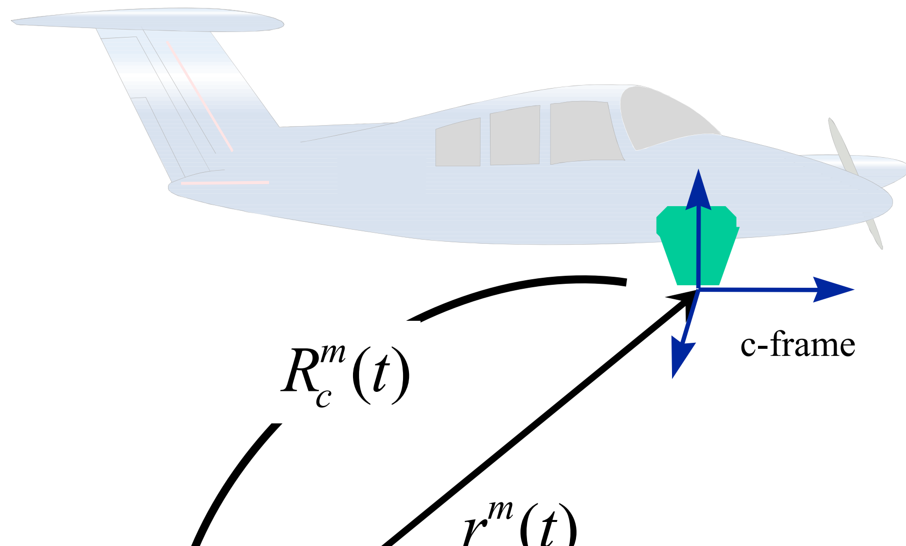
Camera Calibration



Rectified Image after Distortion Removal

Georeferencing

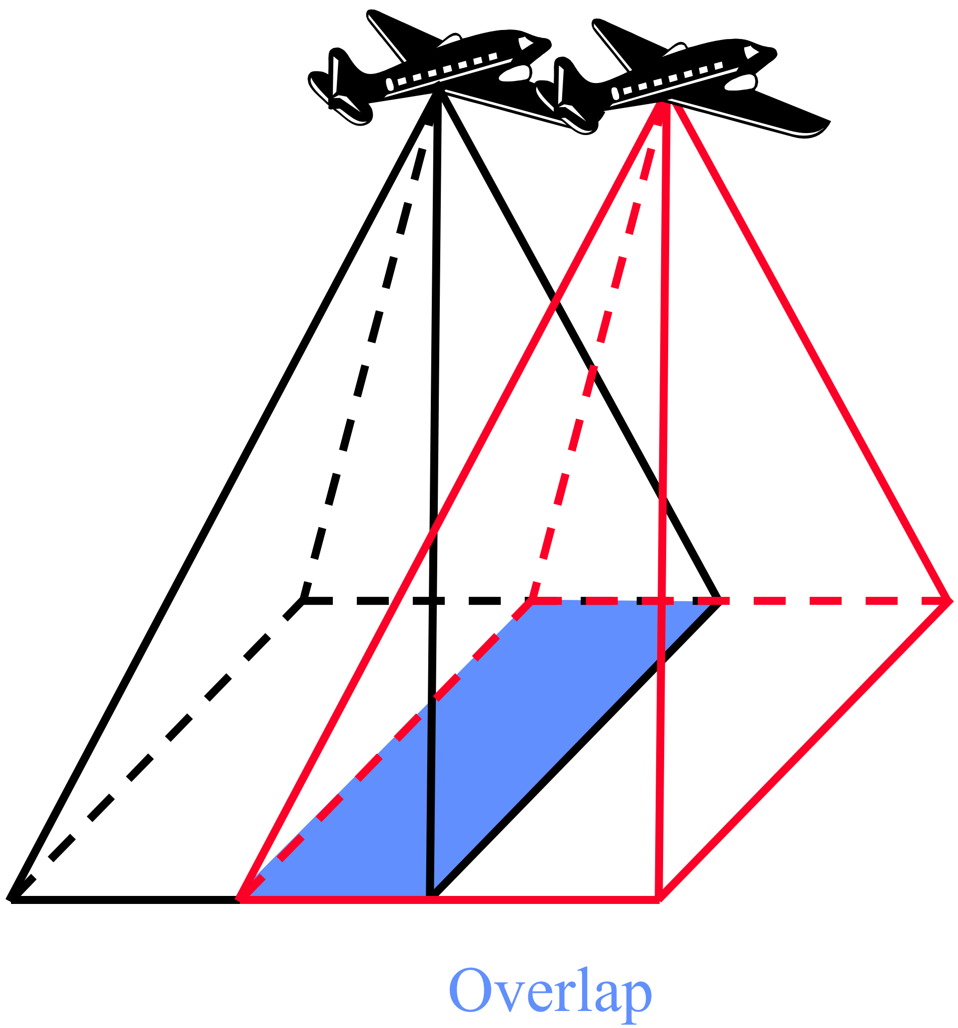
- Exterior Orientation Parameters (EOPs) define the position, $r_c^m(t)$, and orientation $R_c^m(t)$, of the camera coordinate system relative to the mapping reference frame at the moment of exposure.



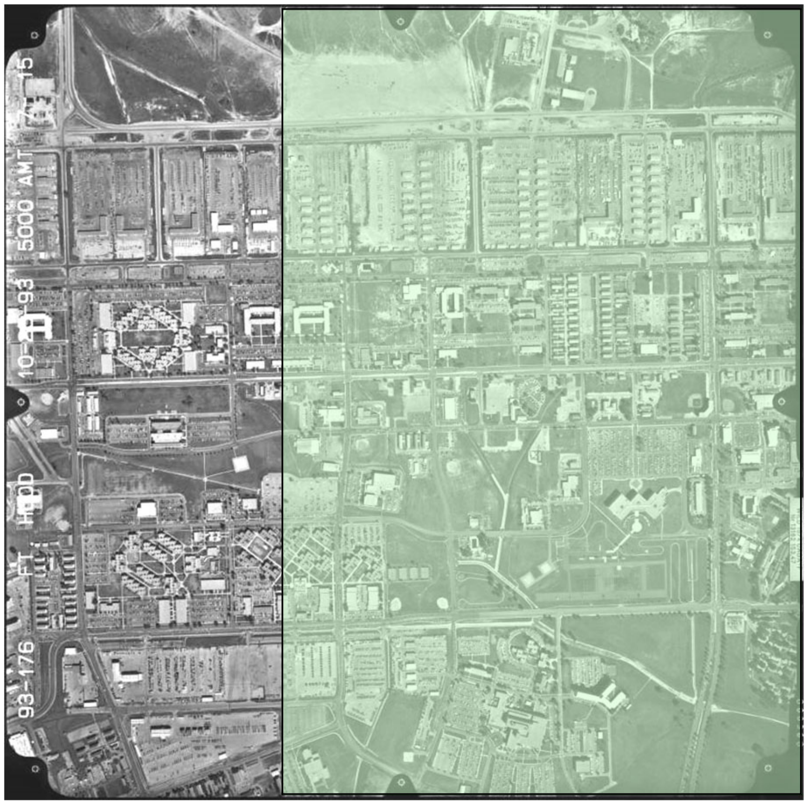
EOPs can be either:

- Indirectly estimated using Ground Control Points (GCPs), or
- Directly derived using GNSS/INS units onboard the imaging platform.

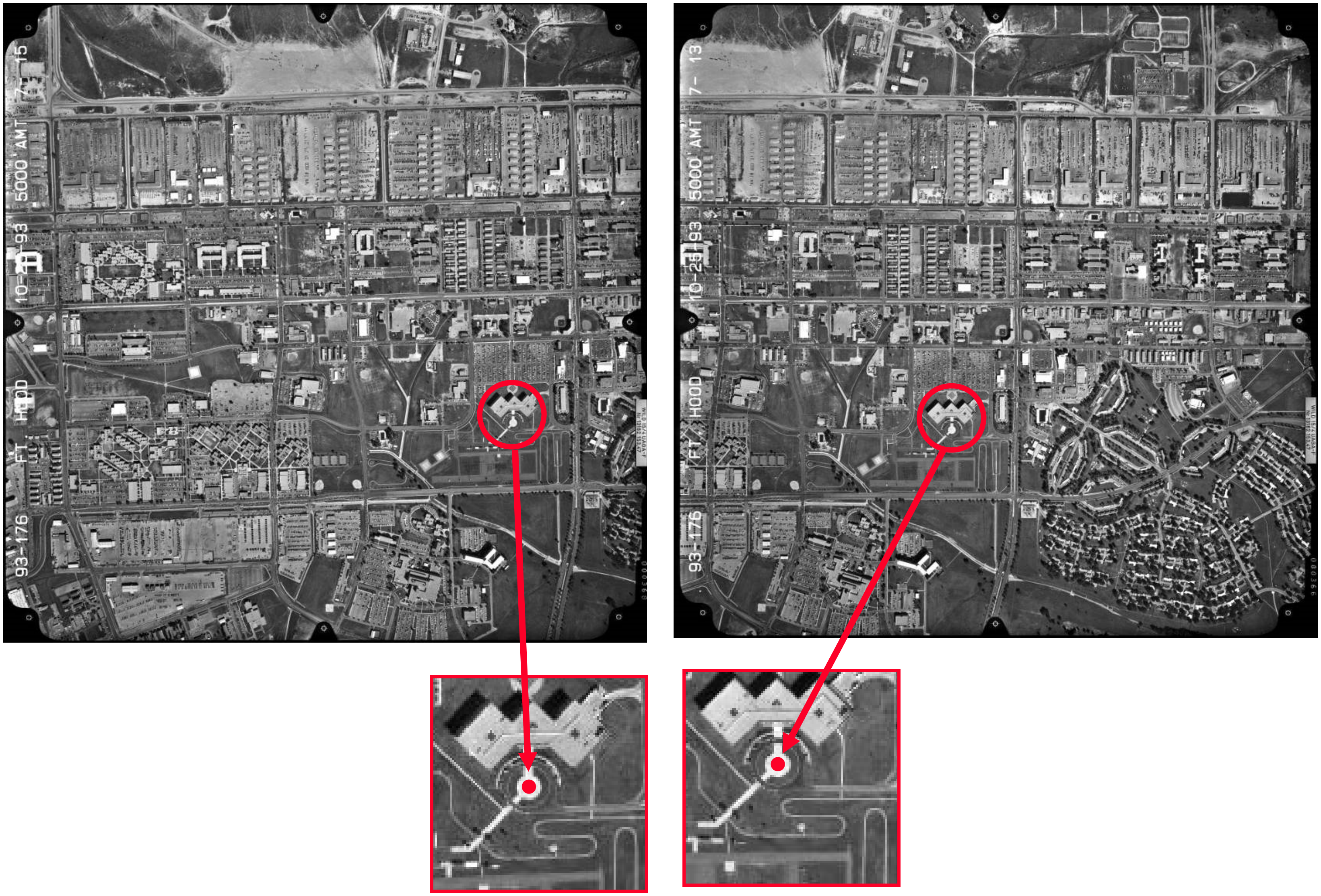
Photogrammetry



Photogrammetry



Photogrammetry

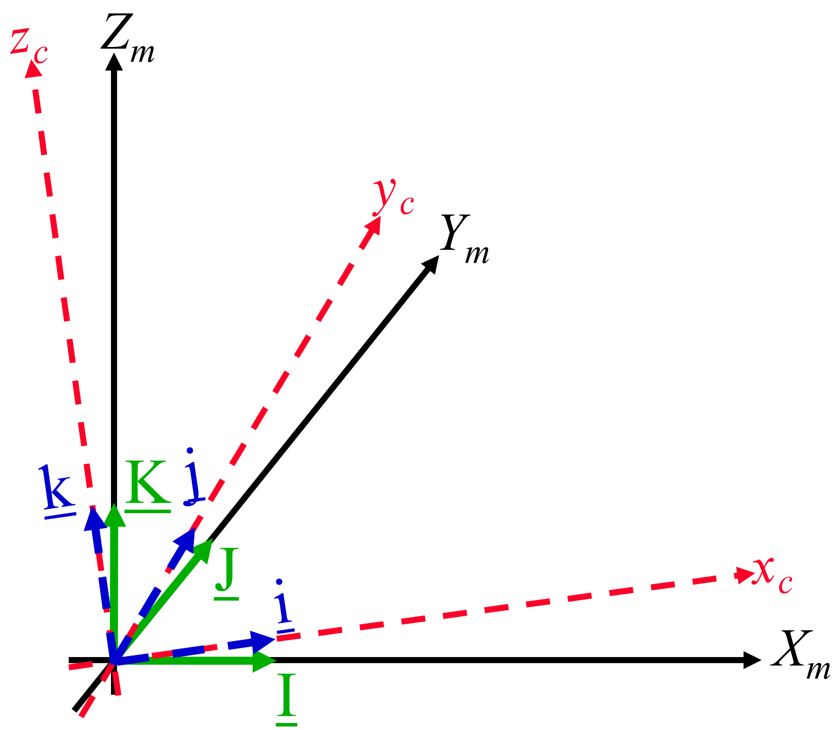


Photogrammetry: Necessary Tools

- Rotation matrices:
  - Express the mathematical relationship between two coordinate systems
  - In a three-dimensional space, a rotation matrix involves at most three independent rotation angles.
- Photogrammetric orientation:
  - Internal characteristics: Interior Orientation Parameters (IOPs)
  - External characteristics: Exterior Orientation Parameters (EOPs)
- Collinearity conditions:
  - The general mathematical model relating the image and ground coordinates of corresponding points

Rotation Matrix

- A rotation matrix transforms a vector from one coordinate system to another.



$$r_a^m = R_c^m \, r_a^c$$

$$R_c^m = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Rotation Matrix

- Let's consider the transformation of a unit vector along the x-axis of the camera coordinate system

$$r_a^m = R_c^m \, r_a^c$$

$$\begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix} = R_c^m \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

  - The first column of the rotation matrix represents the components of a unit vector along the x-axis of the camera coordinate system w.r.t. the mapping reference frame.
  - The norm of the first column is unity.

$$r_{11}^2 + r_{21}^2 + r_{31}^2 = 1$$

1

Rotation Matrix

- Let's consider the transformation of a unit vector along the y-axis of the camera coordinate system

$$r_a^m = R_c^m \, r_a^c$$

$$\begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \end{bmatrix} = R_c^m \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- The second column of the rotation matrix represents the components of a unit vector along the y-axis of the camera coordinate system w.r.t. the mapping reference frame.
- The norm of the second column is unity.

$$r_{12}^2 + r_{22}^2 + r_{32}^2 = 1 \qquad 2$$

Rotation Matrix

- Let's consider the transformation of a unit vector along the z-axis of the camera coordinate system

$$r_a^m = R_c^m \, r_a^c$$

$$\begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix} = R_c^m \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

  - The third column of the rotation matrix represents the components of a unit vector along the z-axis of the camera coordinate system w.r.t. the mapping reference frame.
  - The norm of the third column is unity.

$$r_{13}^2 + r_{23}^2 + r_{33}^2 = 1$$

3

Rotation Matrix

- Since the x and y axes of the camera coordinate system are orthogonal to each other, then

$$\begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix} \bullet \begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \end{bmatrix} = 0 \qquad r_{11}\, r_{12} + r_{21}\, r_{22} + r_{31}\, r_{32} = 0 \qquad 4$$

  - Since the x and z axes of the camera coordinate system are orthogonal to each other, then

$$\begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix} \bullet \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix} = 0 \qquad r_{11}\, r_{13} + r_{21}\, r_{23} + r_{31}\, r_{33} = 0 \qquad 5$$

Rotation Matrix

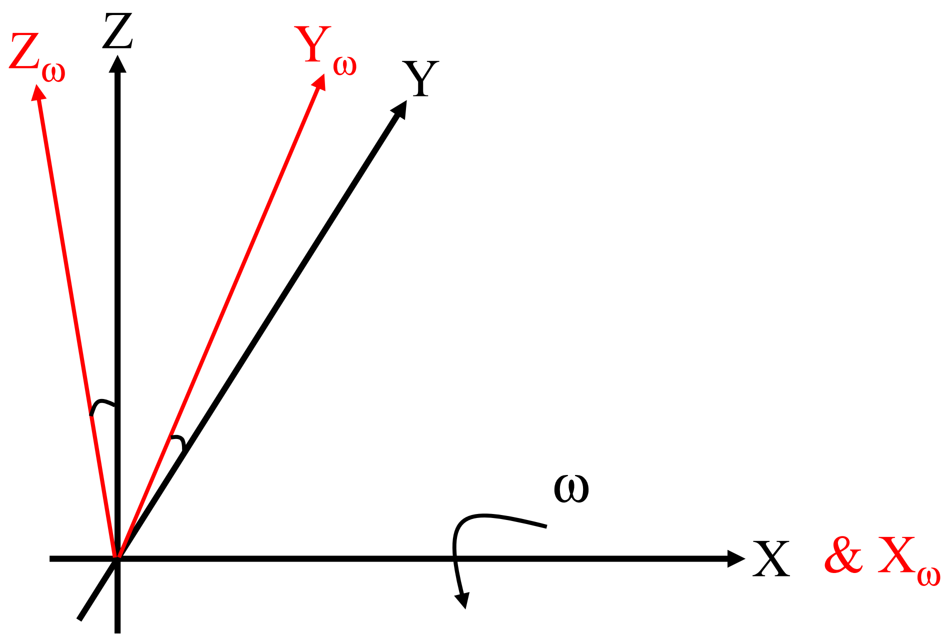
- Since the y and z axes of the camera coordinate system are orthogonal to each other, then

$$\begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \end{bmatrix} \bullet \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix} = 0 \qquad r_{12}\, r_{13} + r_{22}\, r_{23} + r_{32}\, r_{33} = 0$$

6

  - Since the nine elements of a rotation matrix must satisfy six constraints (orthogonality constraints), a 3D rotation matrix is defined by a maximum of three independent parameters/rotation angles.
  - In photogrammetry, the rotation matrix is defined by the angles (ω, ϕ, and κ).

Primary Rotation (ω)



Primary Rotation (ω)

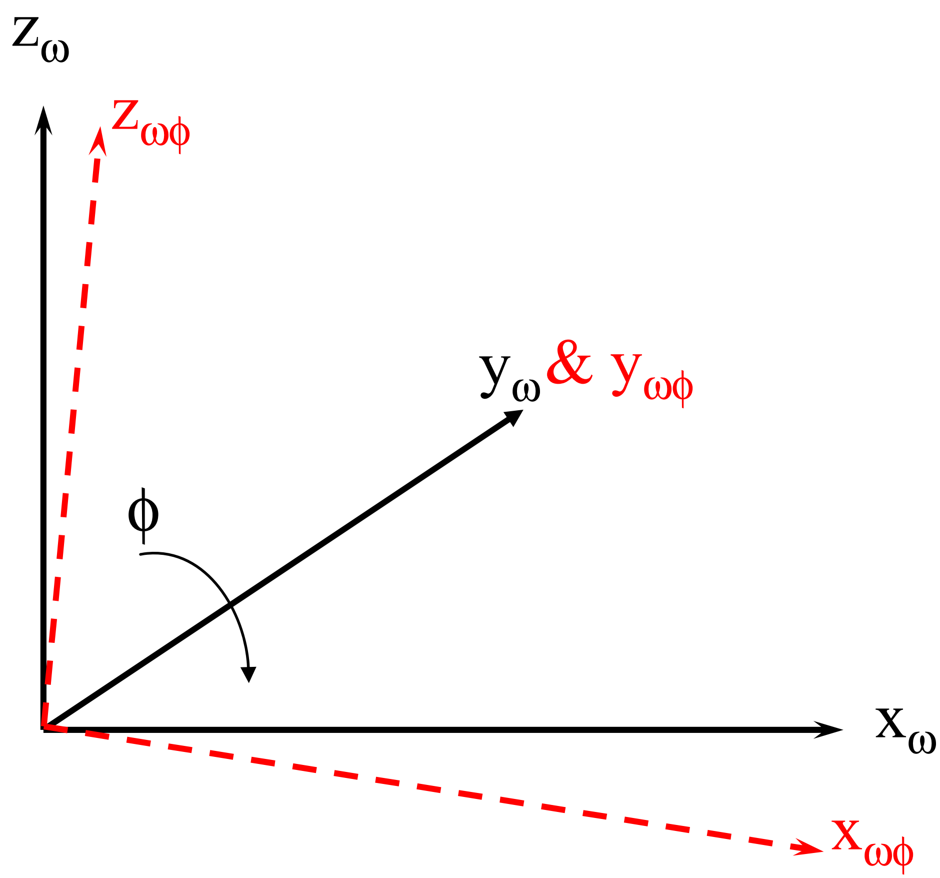
$$\begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix} = M_\omega \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_\omega \begin{bmatrix} x_\omega \\ y_\omega \\ z_\omega \end{bmatrix}$$

Secondary Rotation (ϕ)



Secondary Rotation (ϕ)

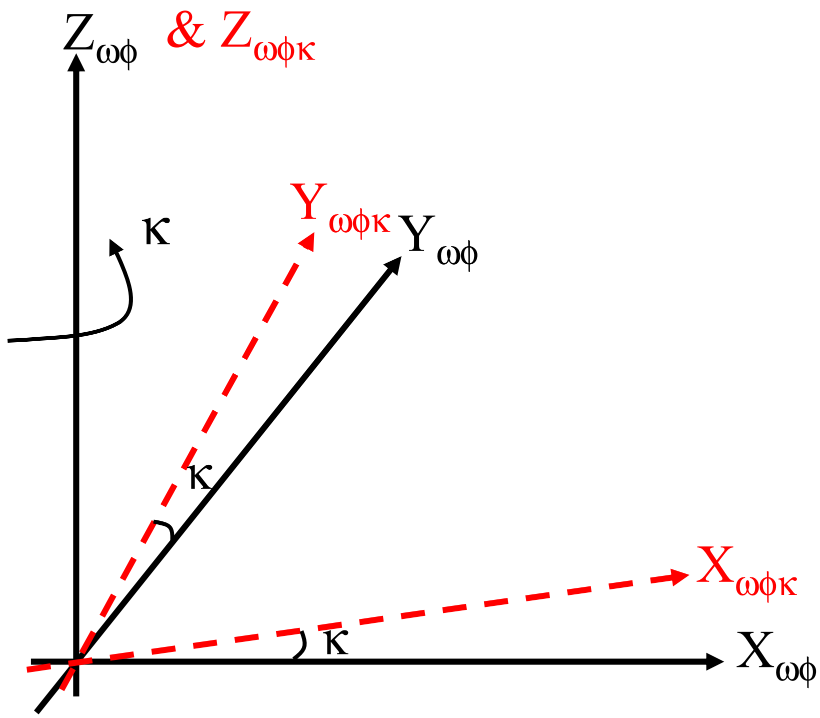
$$\begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x_{\omega} \\ y_{\omega} \\ z_{\omega} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix} = M_{\phi} \begin{bmatrix} x_{\omega} \\ y_{\omega} \\ z_{\omega} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega} \\ y_{\omega} \\ z_{\omega} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega} \\ y_{\omega} \\ z_{\omega} \end{bmatrix} = R_{\phi} \begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix}$$

Tertiary Rotation (κ)



Tertiary Rotation (κ)

$$\begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix} = \begin{bmatrix} \cos\kappa & \sin\kappa & 0 \\ -\sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix} = M_{\kappa} \begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix} = \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix}$$

$$\begin{bmatrix} x_{\omega\phi} \\ y_{\omega\phi} \\ z_{\omega\phi} \end{bmatrix} = R_{\kappa} \begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix}$$

Rotation in Space

$$\begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix} = M_{\kappa} M_{\phi} M_{\omega} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

// to the image coordinate system // to the ground coordinate system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_{\omega} R_{\phi} R_{\kappa} \begin{bmatrix} x_{\omega\phi\kappa} \\ y_{\omega\phi\kappa} \\ z_{\omega\phi\kappa} \end{bmatrix}$$

// to the ground coordinate system // to the image coordinate system

Rotation in Space

$$M_\kappa M_\phi M_\omega = M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

where :

$$m_{11} = \cos\phi \cos\kappa$$
$$m_{12} = \cos\omega \sin\kappa + \sin\omega \sin\phi \cos\kappa$$
$$m_{13} = \sin\omega \sin\kappa - \cos\omega \sin\phi \cos\kappa$$
$$m_{21} = -\cos\phi \sin\kappa$$
$$m_{22} = \cos\omega \cos\kappa - \sin\omega \sin\phi \sin\kappa$$
$$m_{23} = \sin\omega \cos\kappa + \cos\omega \sin\phi \sin\kappa$$
$$m_{31} = \sin\phi$$
$$m_{32} = -\sin\omega \cos\phi$$
$$m_{33} = \cos\omega \cos\phi$$

Rotation in Space

$$R_\omega R_\phi R_\kappa = R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

where :

$$r_{11} = \cos\phi \cos\kappa$$

$$r_{12} = -\cos\phi \sin\kappa$$

$$r_{13} = \sin\phi$$

$$r_{21} = \cos\omega \sin\kappa + \sin\omega \sin\phi \cos\kappa$$

$$r_{22} = \cos\omega \cos\kappa - \sin\omega \sin\phi \sin\kappa$$

$$r_{23} = -\sin\omega \cos\phi$$

$$r_{31} = \sin\omega \sin\kappa - \cos\omega \sin\phi \cos\kappa$$

$$r_{32} = \sin\omega \cos\kappa + \cos\omega \sin\phi \sin\kappa$$

$$r_{33} = \cos\omega \cos\phi$$

Orthogonality Conditions

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$r_{11}^2 + r_{21}^2 + r_{31}^2 = 1$$

$$r_{12}^2 + r_{22}^2 + r_{32}^2 = 1$$

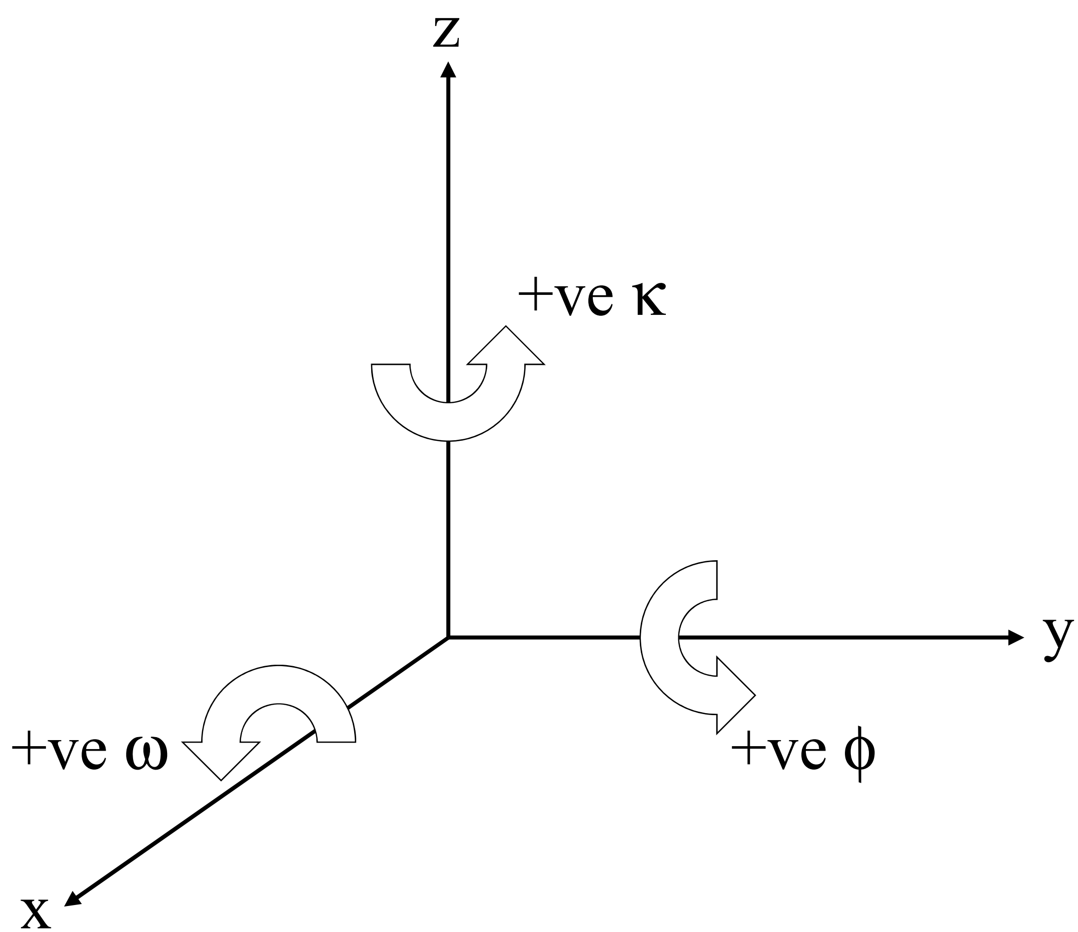
$$r_{13}^2 + r_{23}^2 + r_{33}^2 = 1$$

$$r_{11} \; r_{12} + r_{21} \; r_{22} + r_{31} \; r_{32} = 0$$

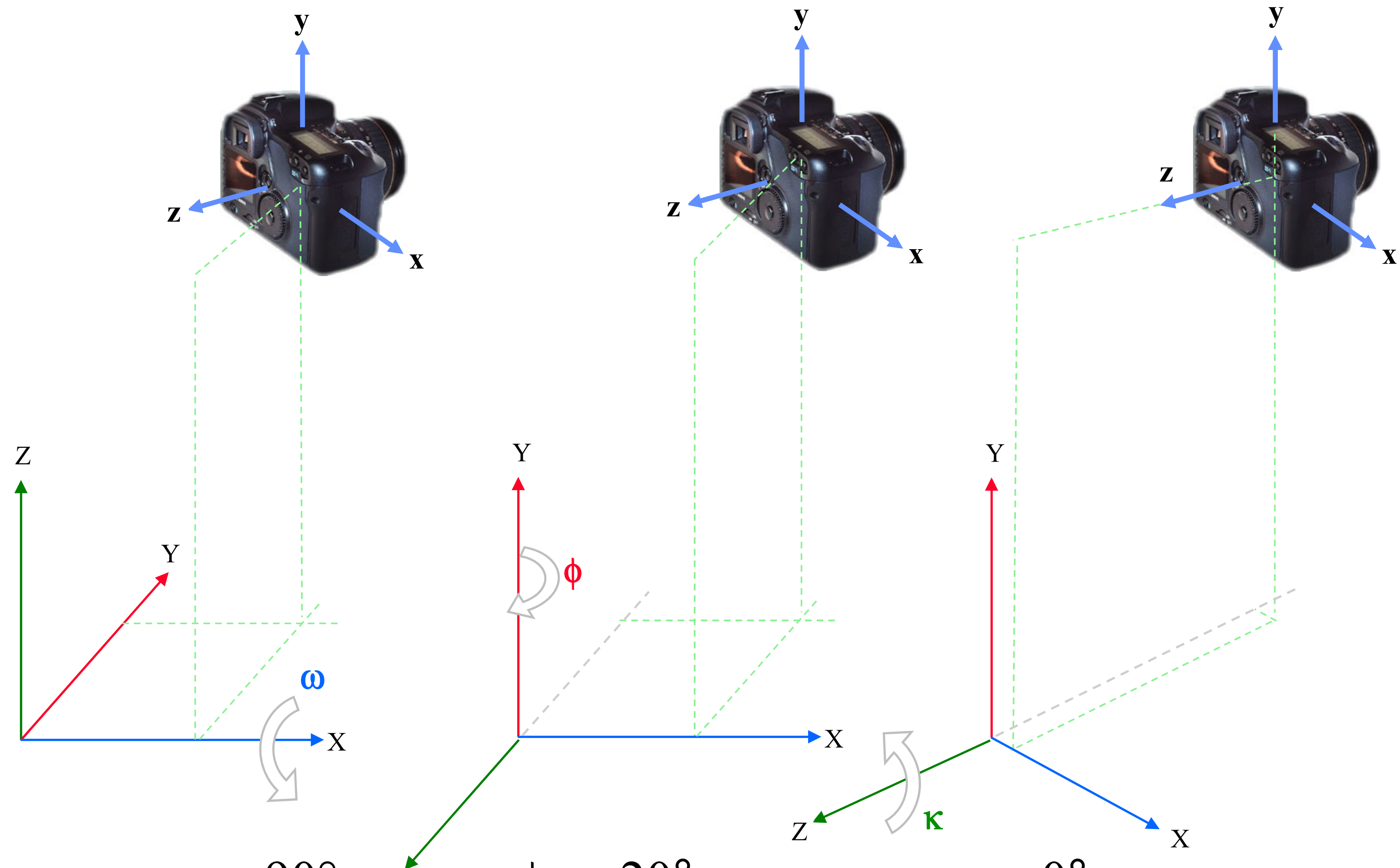
$$r_{11} \; r_{13} + r_{21} \; r_{23} + r_{31} \; r_{33} = 0$$

$$r_{12} \; r_{13} + r_{22} \; r_{23} + r_{32} \; r_{33} = 0$$

Positive Rotation Angles: (Right Handed System)



Rotation Angles (ω, ϕ, κ)

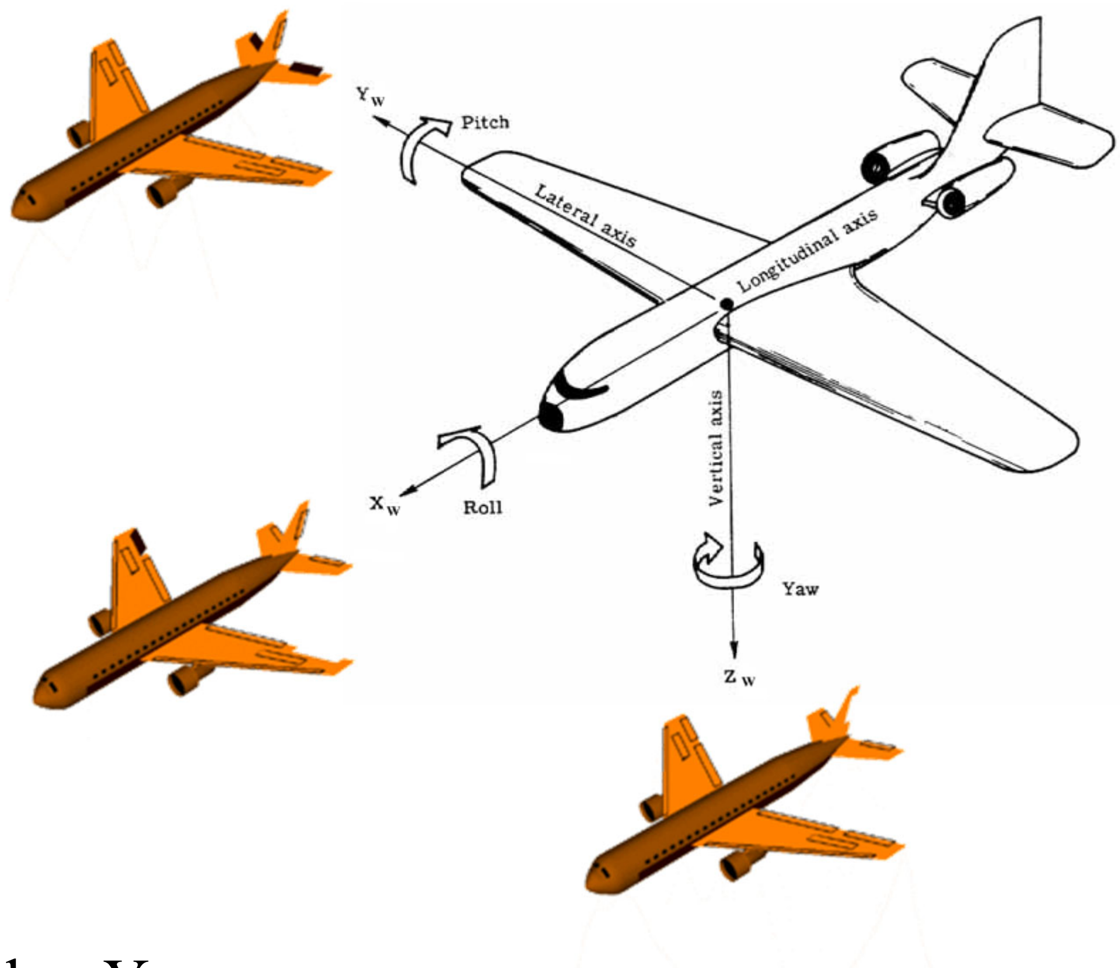


$\omega \approx 90°$

$\phi \approx -20°$

$\kappa \approx 0°$

Rotation Angles (Azimuth, Pitch, Roll)



Azimuth ≡ Yaw

Photogrammetric Orientation

Interior Orientation

Interior Orientation Parameters

- Interior Orientation Parameters (IOPs) describe the internal characteristics of the implemented camera.
  - IOPs include the principal distance, principal point coordinates, and distortion parameters.
  - IOPs are determined using a calibration procedure.

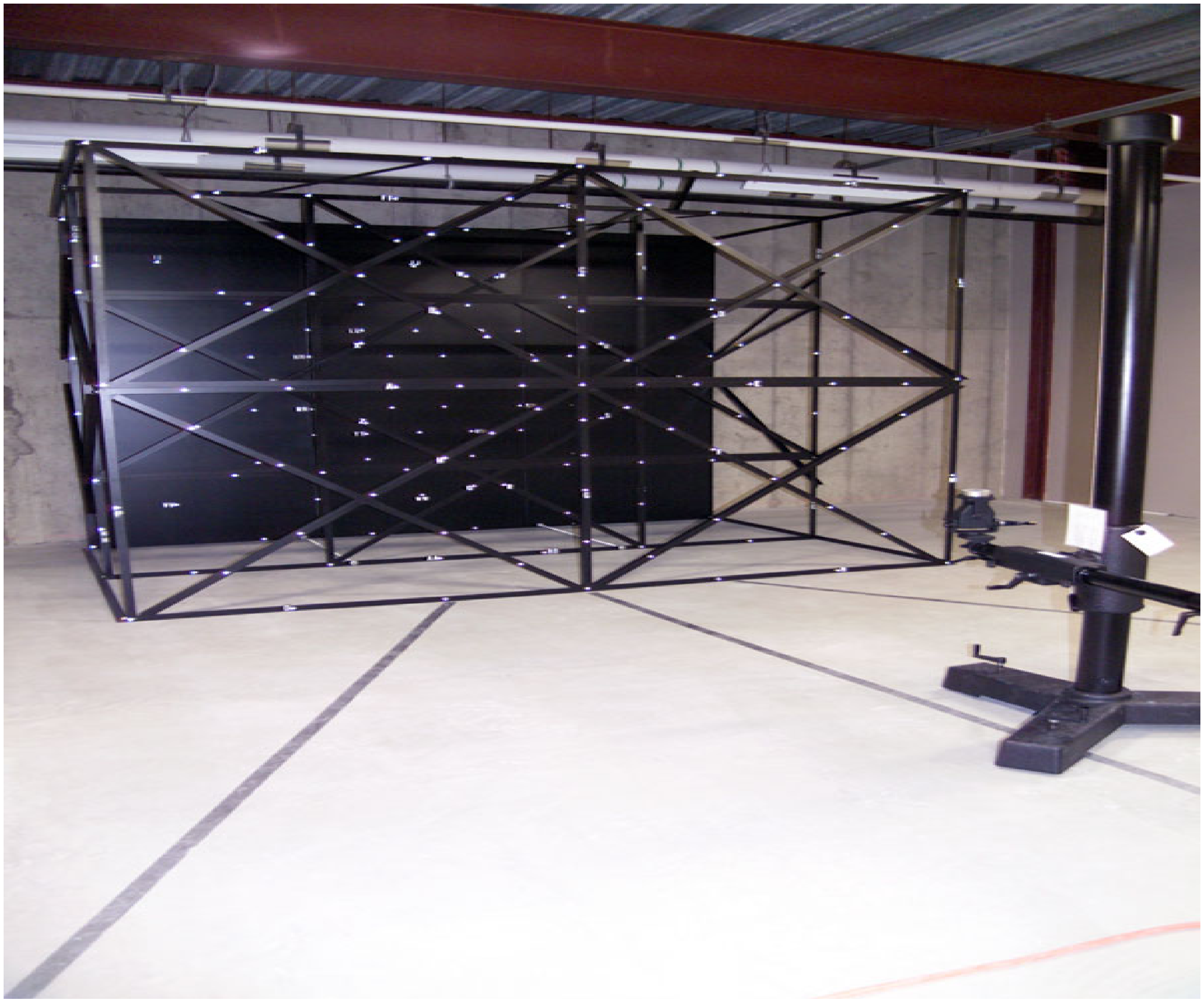
Interior Orientation Parameters

- Alternative procedures for camera calibration are well established.
  - Laboratory camera calibration (Multi-collimators)
  - Indoor camera calibration
  - In-situ camera calibration

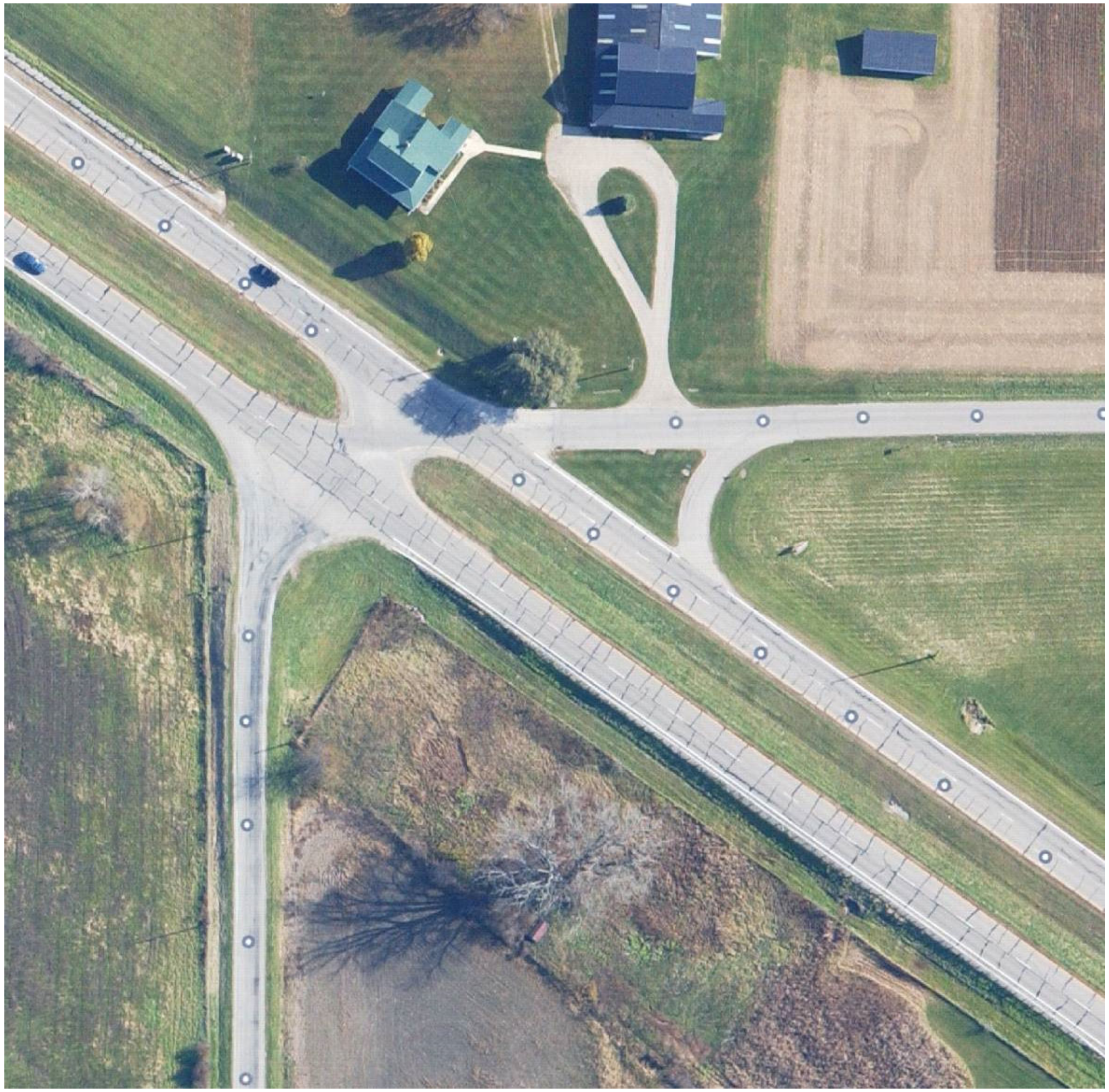
Analytical camera calibration

Laboratory Calibration: Multi-Collimators

Indoor Camera Calibration

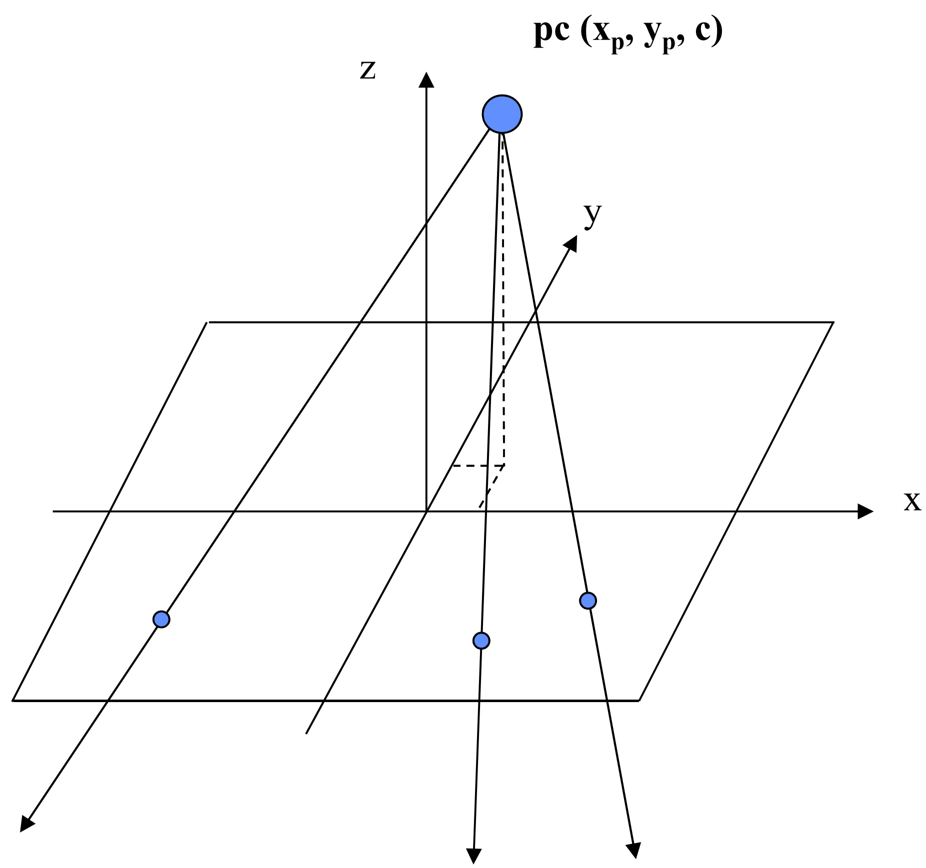


In-Situ Camera Calibration



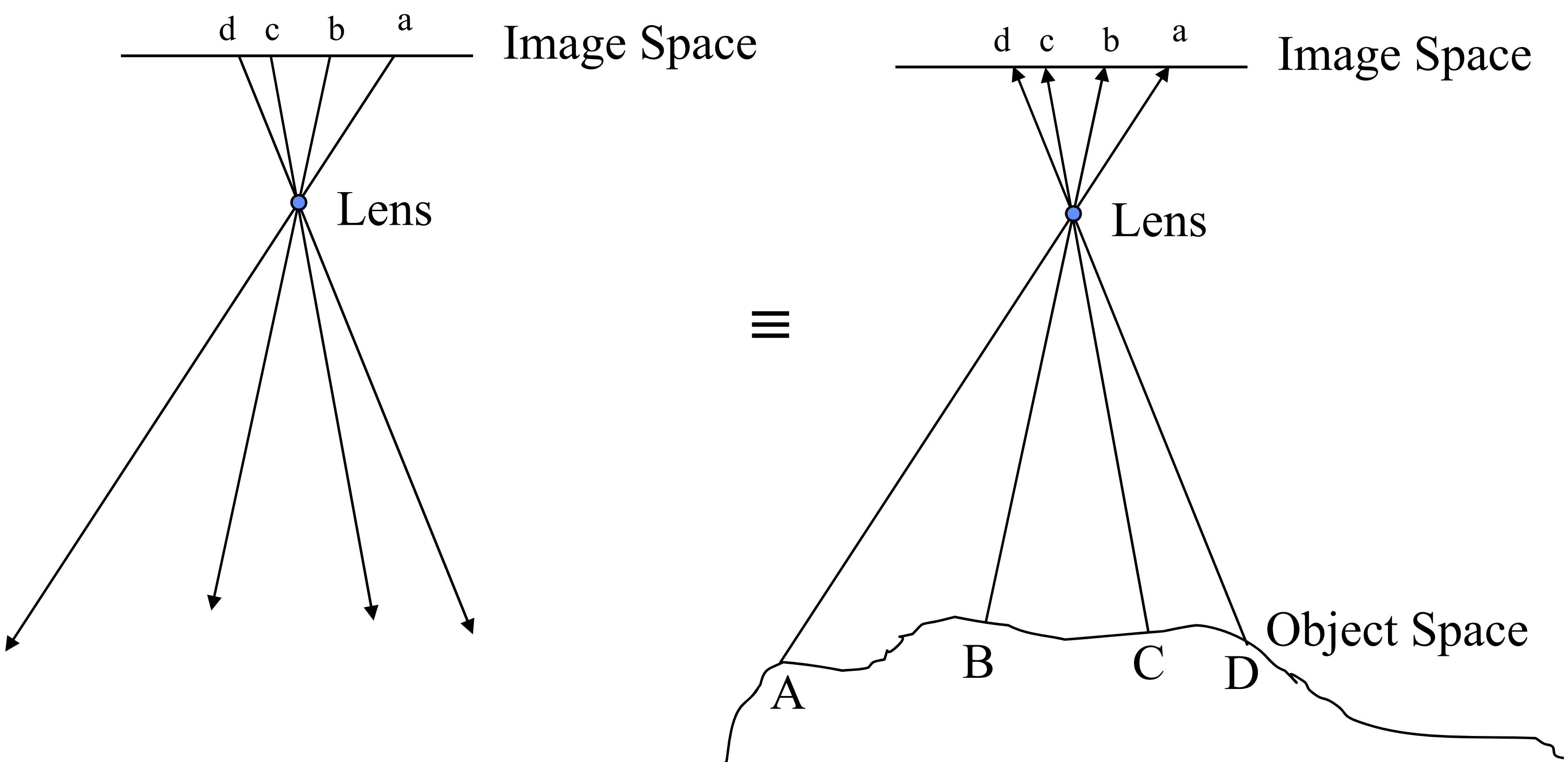
Interior Orientation Parameters

- IOPs together with the image coordinates of selected features define a bundle of light rays (image bundle).



Interior Orientation Parameters

- **Target function of the Interior Orientation:**
  - The defined bundle by the IOPs should be as similar as possible to the incident bundle onto the camera at the moment of exposure.



Establishing IO

Photography

Photogrammetric Orientation

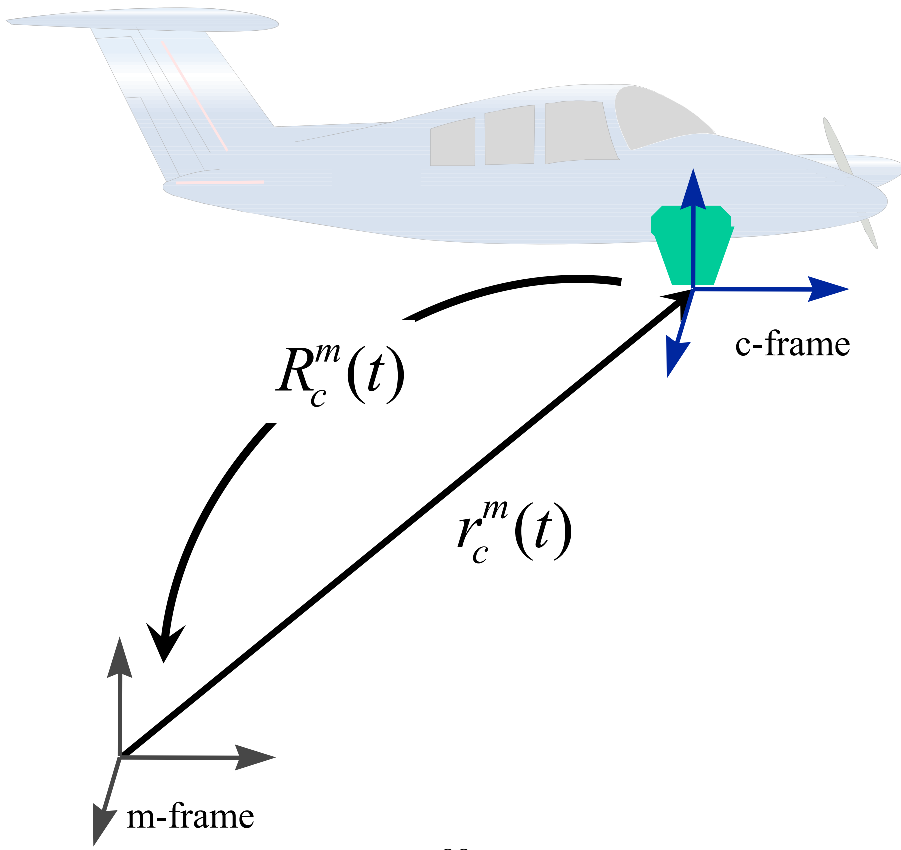
Exterior Orientation

Exterior Orientation Parameters

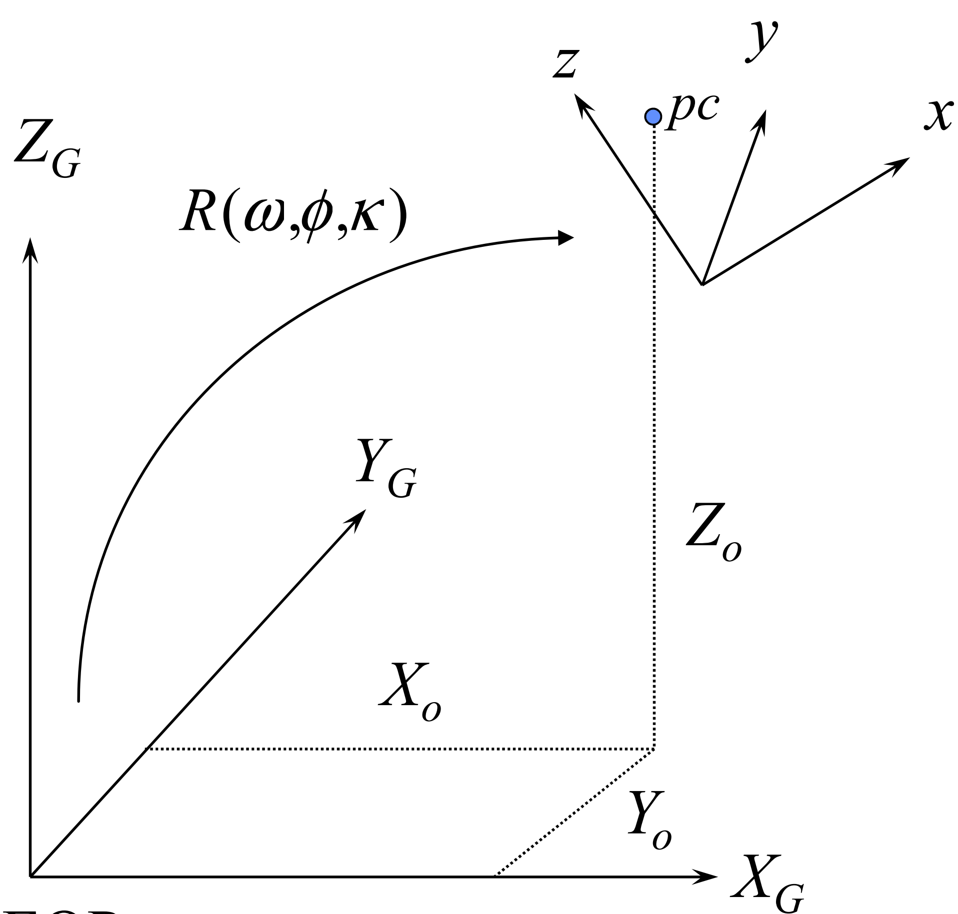
- Exterior Orientation Parameters (EOPs) – georeferencing parameters – define the position and the attitude of the image bundle relative to the ground coordinate system.
  - The position of the bundle is defined by (X_o, Y_o, Z_o).
  - The attitude of the bundle (camera/image coordinate system) relative to the ground coordinate system is defined by the rotation angles (ω, ϕ, κ).
- EOPs can be either:
  - Indirectly estimated using Ground Control Points (GCPs), or
  - Directly derived using GNSS/INS units onboard the imaging platform.

Exterior Orientation Parameters

- Exterior Orientation Parameters (EOPs) define the position, $r_c^m(t)$, and orientation $R_c^m(t)$, of the camera coordinate system relative to the mapping reference frame at the moment of exposure.

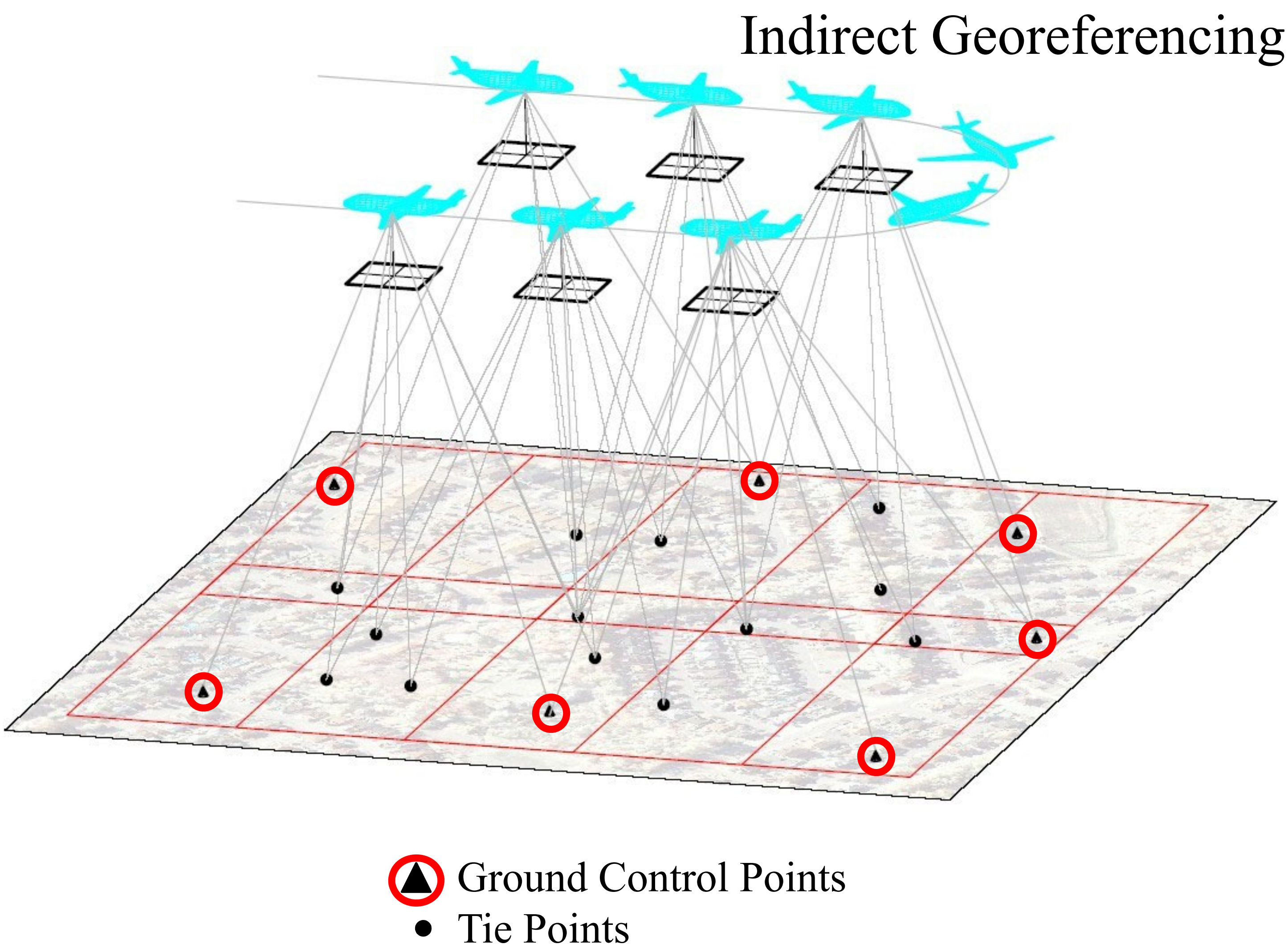


Exterior Orientation Parameters



- EOPs:
  - Indirectly estimated (indirect georeferencing), or
  - Directly derived (direct georeferencing)

Exterior Orientation Parameters



Exterior Orientation Parameters

Indirect Georeferencing

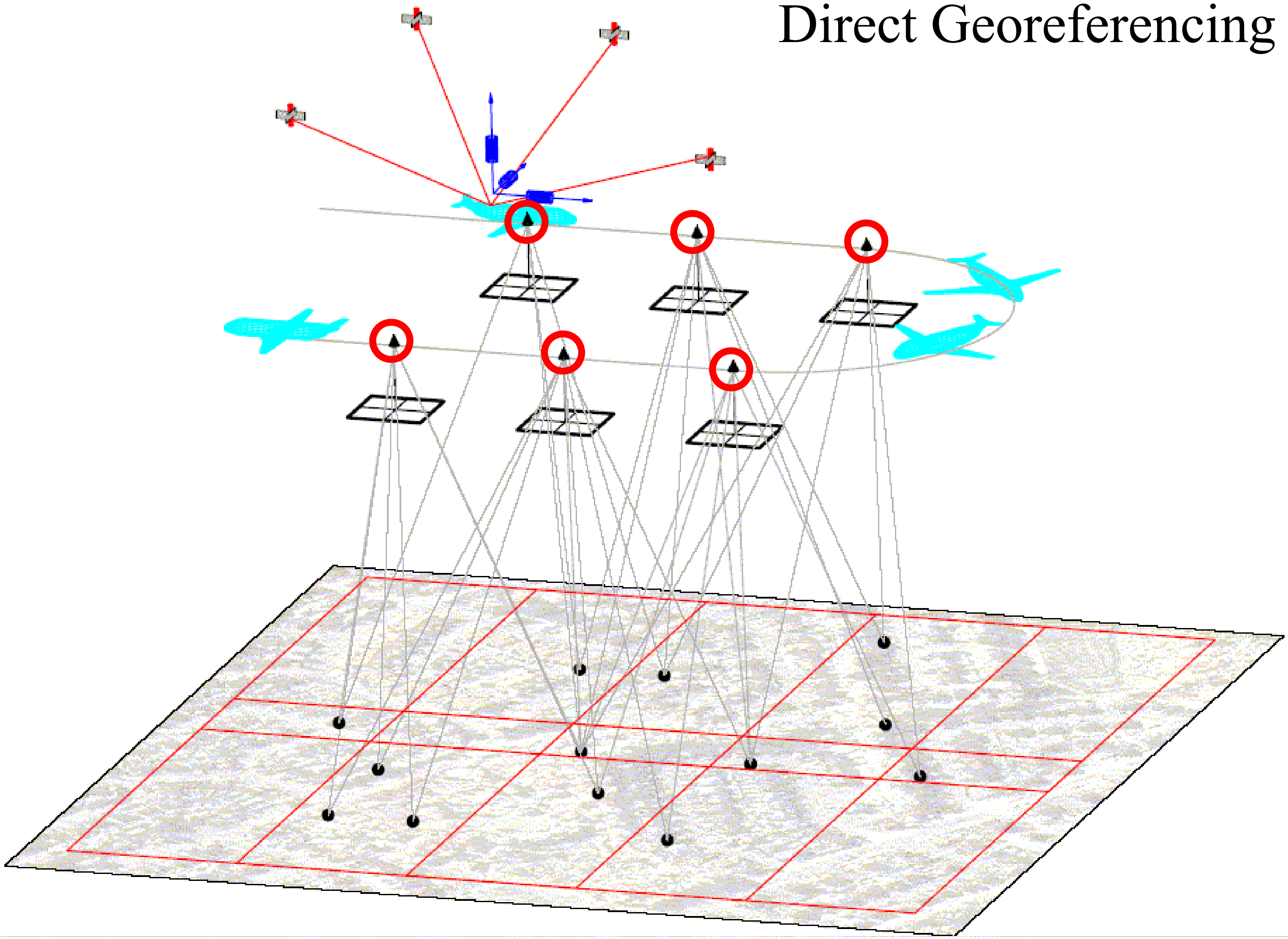
Signalized Targets

Exterior Orientation Parameters

Indirect Georeferencing

Natural Targets

Exterior Orientation Parameters



Exterior Orientation Parameters

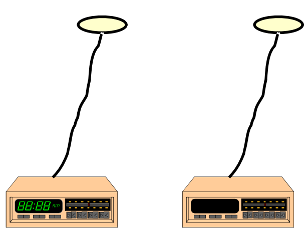
Direct Georeferencing

GNSS Antenna

INS

PC

Two Base Stations



Camera

GNSS Receiver

Direct georeferencing in practice

Exterior Orientation Parameters

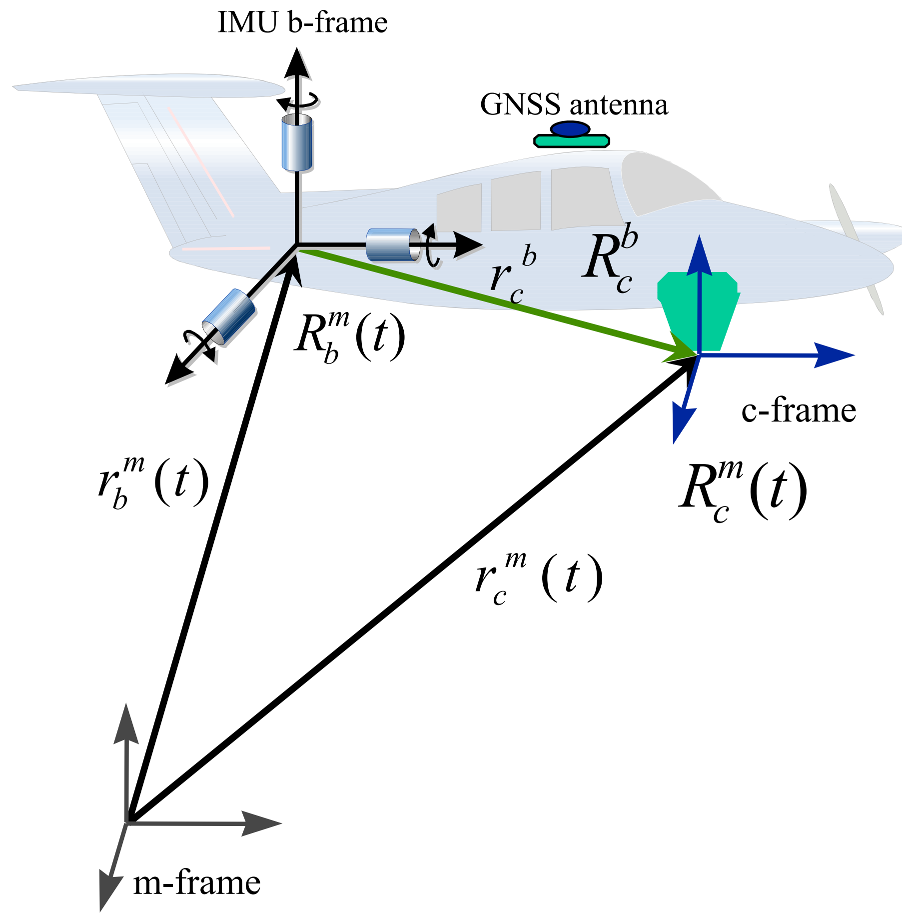
$$r_c^m(t) = r_b^m(t) + R_b^m(t)\, r_c^b$$

- $r_c^m(t)$: Camera position
- $r_b^m(t)$: GNSS/INS position
- $R_b^m(t)$: GNSS/INS attitude
- r_c^b: Calibration

$$R_c^m(t) = R_b^m(t)\; R_c^b$$

- $R_c^m(t)$: Camera attitude
- $R_b^m(t)$: GNSS/INS attitude
- R_c^b: Calibration

Direct Georeferencing



Photogrammetric Mathematical Model

Collinearity Equations

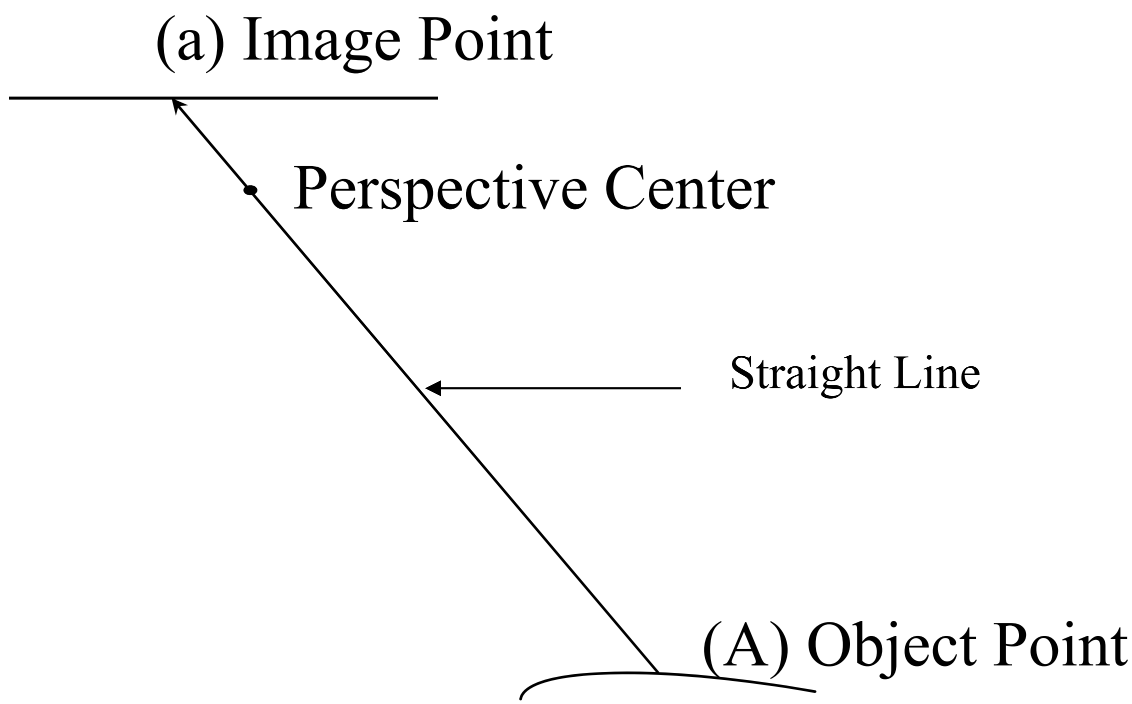
Vector Summation Based Point Positioning

Collinearity Equations

- Objective:
  - Mathematically represent the general relationship between image and ground coordinates

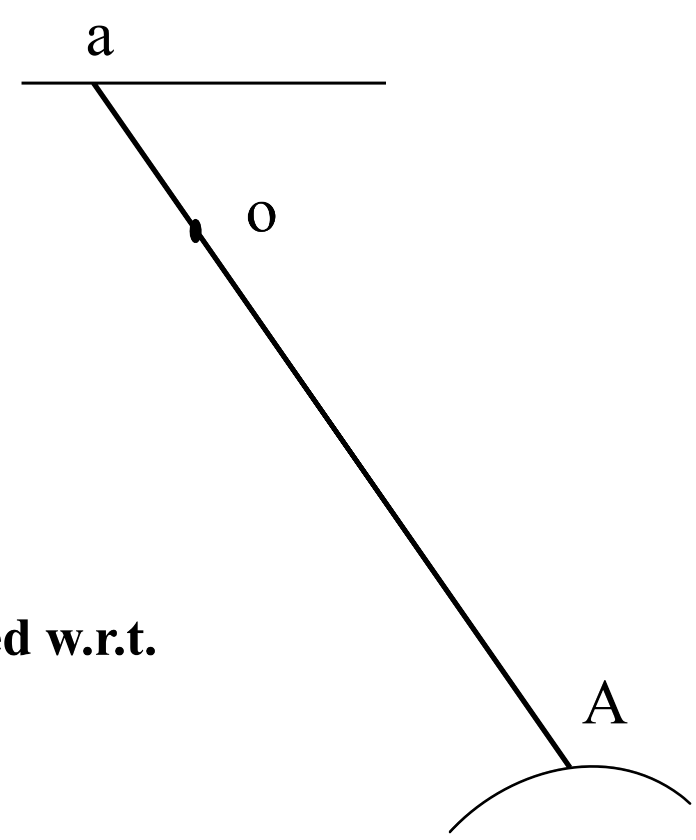
- Concept:
  - Image Point, Object Point, and the Perspective Center are collinear

Collinearity Equations



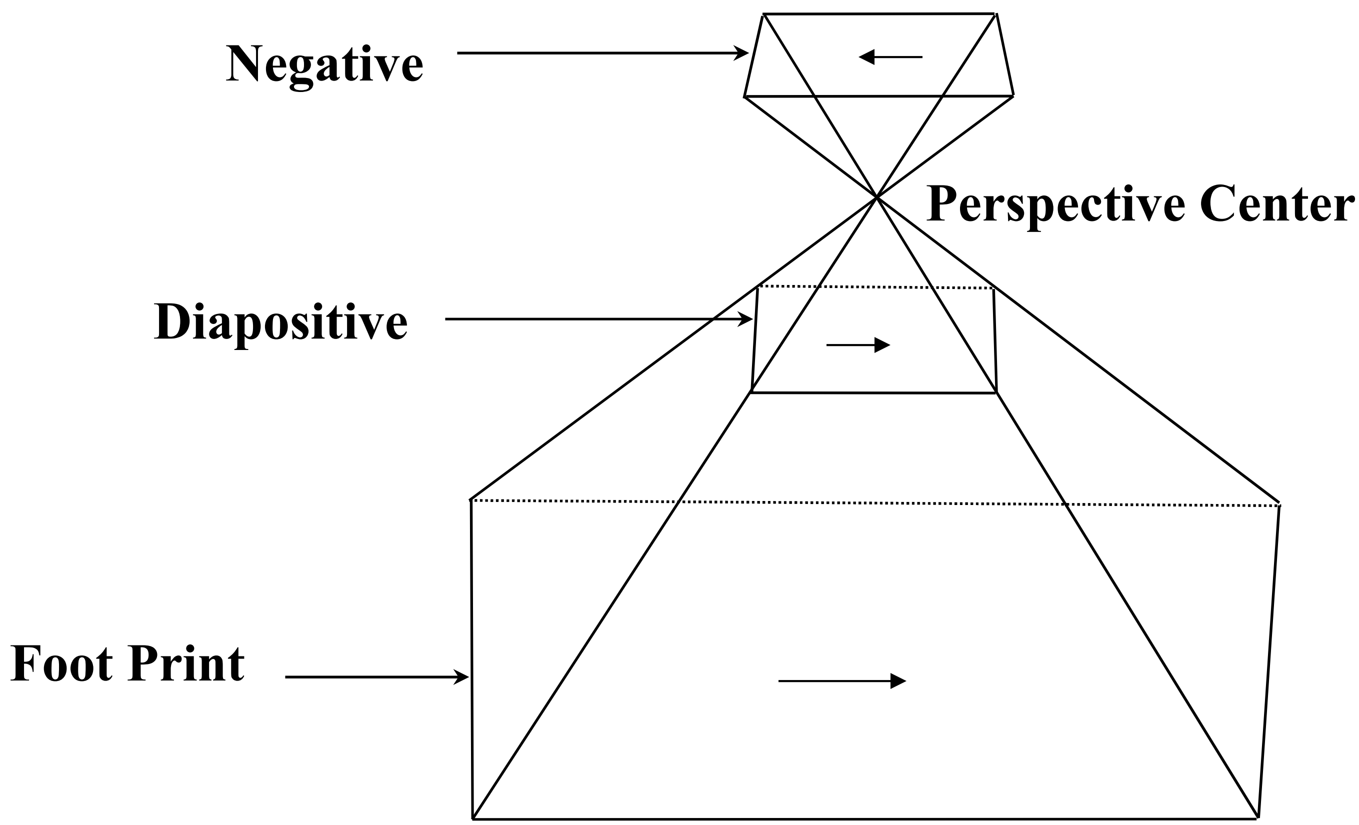
Collinearity Equations

$$\vec{\mathbf{oa}} = \lambda \; \vec{\mathbf{oA}}$$



These vectors should be defined w.r.t. the same coordinate system

Frame Camera

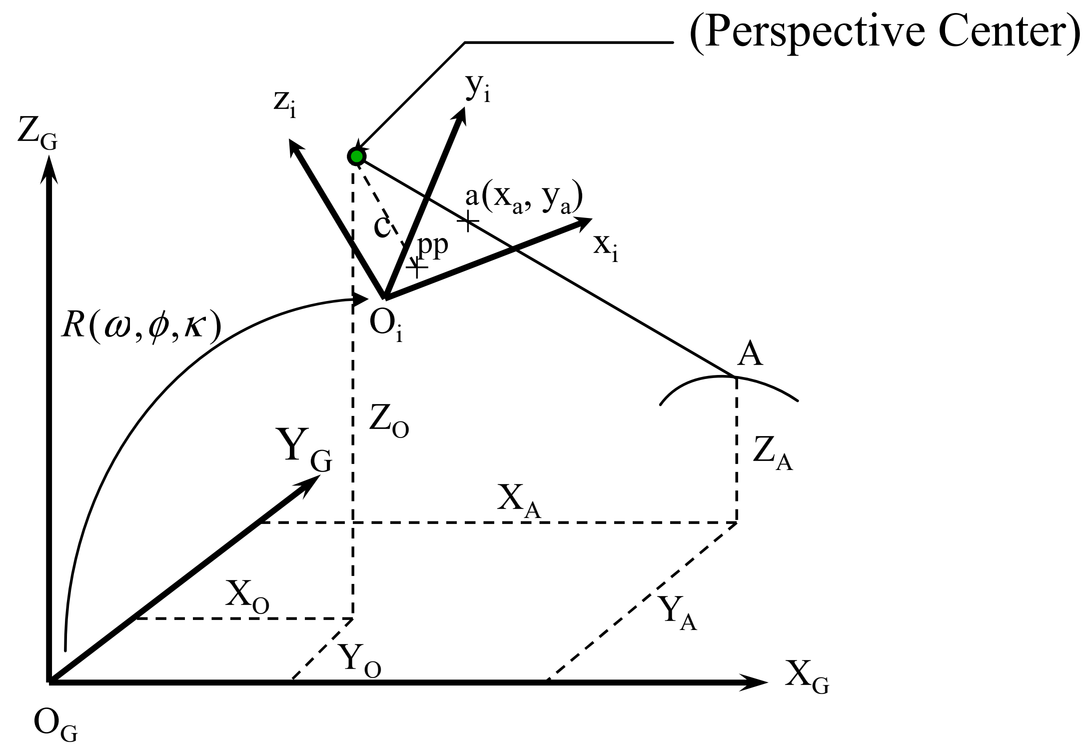


Negative Versus Diapositive Films

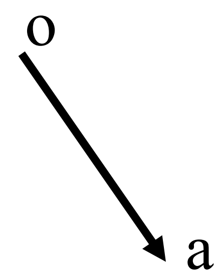
Negative Film

Diapositive

Collinearity Equations



The Vector Connecting the Perspective Center to the Image Point

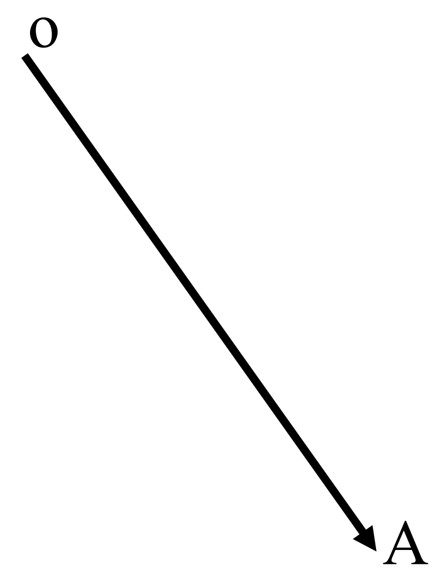


$$\vec{v}_i = r^c_{oa} = \begin{bmatrix} x_a - dist_x \\ y_a - dist_y \\ 0 \end{bmatrix} - \begin{bmatrix} x_p \\ y_p \\ c \end{bmatrix} = \begin{bmatrix} x_a - x_p - dist_x \\ y_a - y_p - dist_y \\ -c \end{bmatrix}$$

w.r.t. the image coordinate system

The Vector Connecting the Perspective Center to the Object Point

$$\vec{V}_o = r_{oA}^m = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} - \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} X_A - X_o \\ Y_A - Y_o \\ Z_A - Z_o \end{bmatrix}$$



w.r.t. the ground coordinate system

Collinearity Equations

$$\overrightarrow{oa} = \lambda \overrightarrow{oA}$$

$$\vec{v}_i = r_{oa}^c = \lambda \; M(\omega, \varphi, \kappa) \; \vec{V}_O = \lambda \; R_m^c \, r_{oA}^m$$

$$\begin{bmatrix} x_a - x_p - dist_x \\ y_a - y_p - dist_y \\ -c \end{bmatrix} = \lambda \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_A - X_o \\ Y_A - Y_o \\ Z_A - Z_o \end{bmatrix}$$

Where: λ is a scale factor

- Questions:
  - Can you come up with an average estimate of λ?
  - Is λ constant for a given image? Why?

Collinearity Equations

$$M = R_m^c$$

$$x_a = x_p - c\frac{m_{11}(X_A - X_o) + m_{12}(Y_A - Y_o) + m_{13}(Z_A - Z_o)}{m_{31}(X_A - X_o) + m_{32}(Y_A - Y_o) + m_{33}(Z_A - Z_o)} + dist_x$$

$$y_a = y_p - c\frac{m_{21}(X_A - X_o) + m_{22}(Y_A - Y_o) + m_{23}(Z_A - Z_o)}{m_{31}(X_A - X_o) + m_{32}(Y_A - Y_o) + m_{33}(Z_A - Z_o)} + dist_y$$

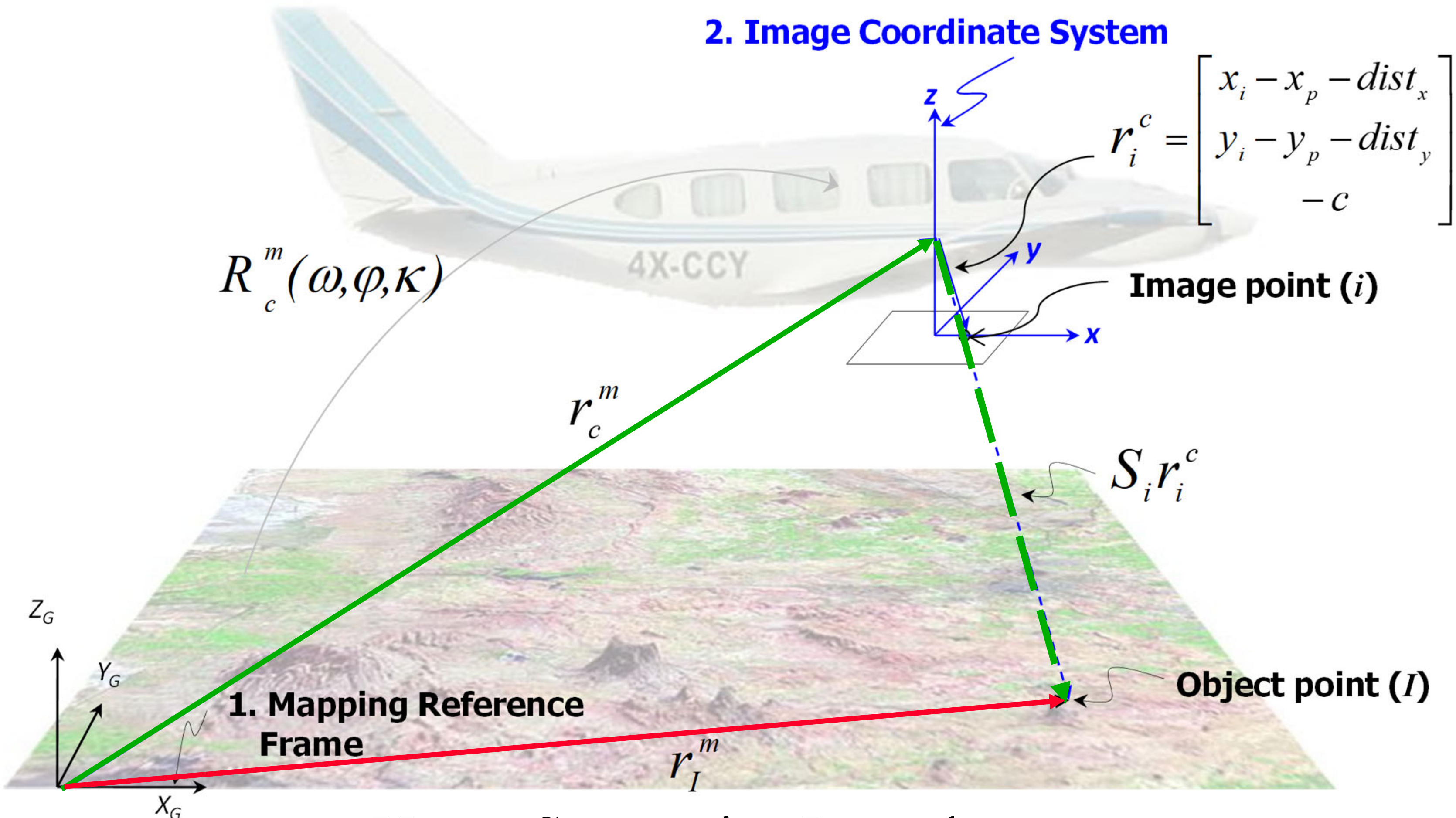
$$R = R_c^m$$

$$x_a = x_p - c\frac{r_{11}(X_A - X_o) + r_{21}(Y_A - Y_o) + r_{31}(Z_A - Z_o)}{r_{13}(X_A - X_o) + r_{23}(Y_A - Y_o) + r_{33}(Z_A - Z_o)} + dist_x$$

$$y_a = y_p - c\frac{r_{12}(X_A - X_o) + r_{22}(Y_A - Y_o) + r_{32}(Z_A - Z_o)}{r_{13}(X_A - X_o) + r_{23}(Y_A - Y_o) + r_{33}(Z_A - Z_o)} + dist_y$$

Collinearity Equations

$$r_I^m = r_c^m + S_i\, R_c^m(\omega,\phi,\kappa)\; r_i^c$$



Vector Summation Procedure

Collinearity Equations

$$r_I^m = r_c^m + S_i\, R_c^m(\omega,\phi,\kappa)\; r_i^c$$

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} + S_i\, R_c^m(\omega,\phi,\kappa) \begin{bmatrix} x_i - x_p - dist_{x_i} \\ y_i - y_p - dist_{y_i} \\ -c \end{bmatrix}$$

$$\begin{bmatrix} x_i - x_p - dist_{x_i} \\ y_i - y_p - dist_{y_i} \\ -c \end{bmatrix} = {}^{1}\!/_{S_i}\, R_m^c(\omega,\phi,\kappa)\left[\vec{X}_G - \vec{X}_o\right] = {}^{1}\!/_{S_i} \begin{bmatrix} N_x \\ N_y \\ D \end{bmatrix}$$

$$x_i = x_p - c\, {}^{N_x}\!/_{D} + dist_{x_i}$$

$$y_i = y_p - c\, {}^{N_y}\!/_{D} + dist_{y_i}$$

Vector Summation Procedure

Collinearity Equations

$$R = R_c^m$$

$$x_a = x_p - c\frac{r_{11}(X_A - X_O) + r_{21}(Y_A - Y_O) + r_{31}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)}$$

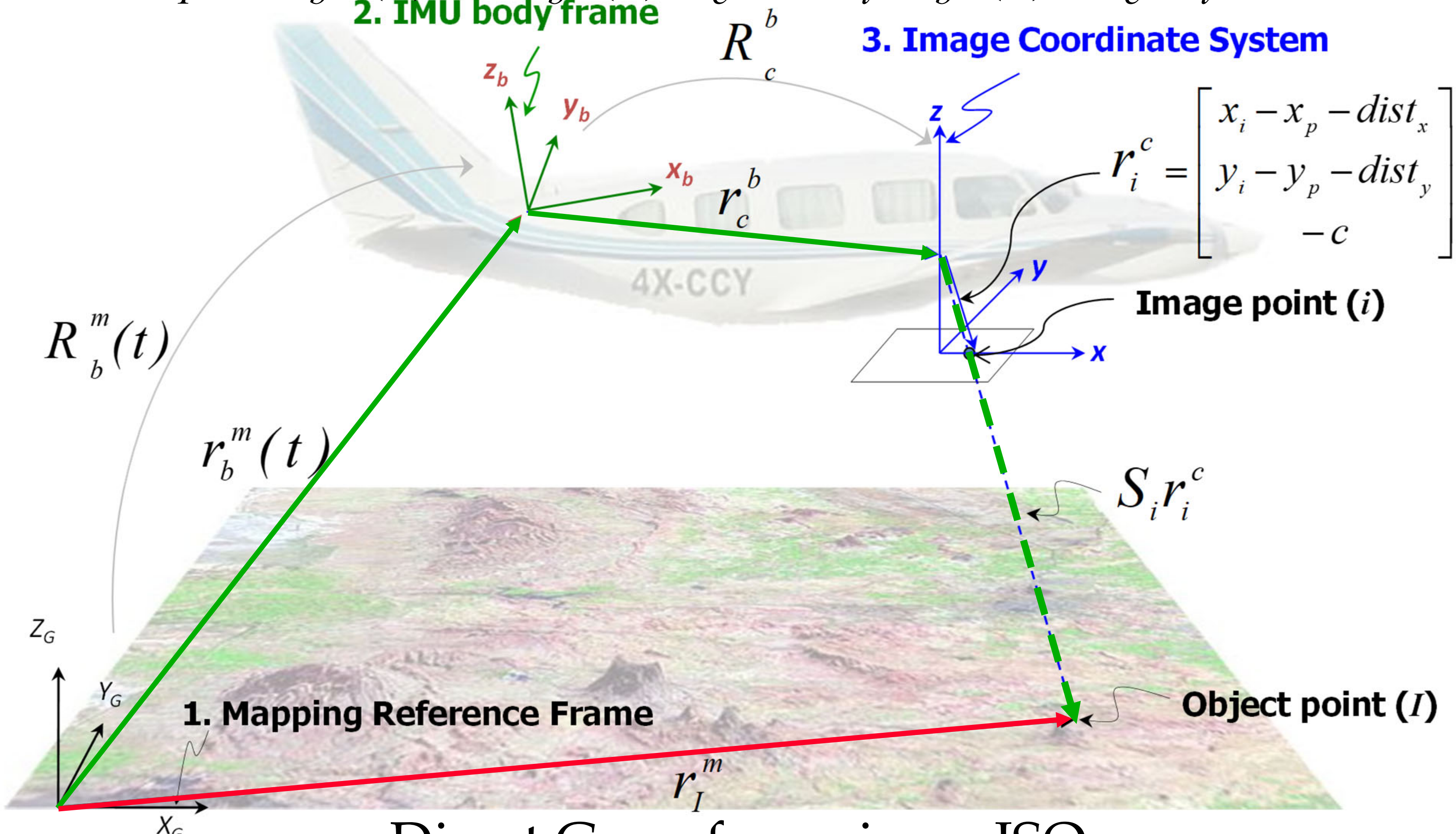
$$y_a = y_p - c\frac{r_{12}(X_A - X_O) + r_{22}(Y_A - Y_O) + r_{32}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)}$$

- Involved parameters:
  - Image coordinates (x_a, y_a)
  - Ground coordinates (X_A, Y_A, Z_A)
  - Exterior Orientation Parameters (X_O, Y_O, Z_O, ω, ϕ, κ)
  - Interior Orientation Parameters (x_p, y_p, c, distortion parameters)

Photogrammetric Point Positioning

GNSS/INS-Assisted Photogrammetric System:

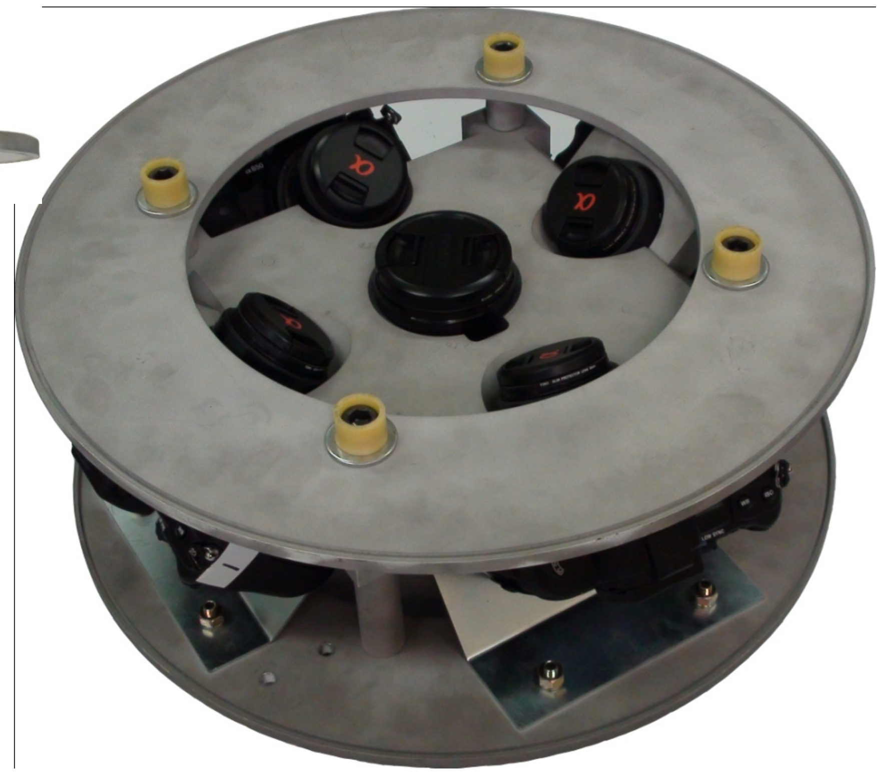
$$r_I^m = r_b^m(t) + R_b^m(t)\ r_c^b + S_i\ R_b^m(t)\ R_c^b\ r_i^c$$



Direct Georeferencing – ISO

Photogrammetric Point Positioning

Multi-Camera Photogrammetric Systems:



Multi-Camera Systems

A rigid-relationship among the cameras

Airborne Mobile Mapping System

Photogrammetric Point Positioning

Multi-Camera Photogrammetric Systems:

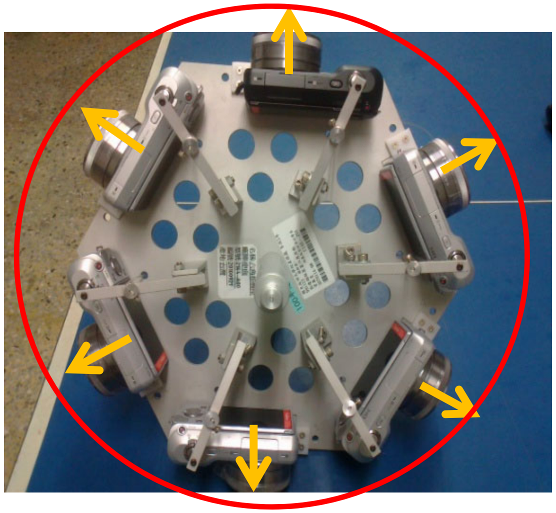
Multi-Camera Systems

A rigid-relationship among the cameras

Terrestrial Mobile Mapping System

Photogrammetric Point Positioning

Multi-Camera Photogrammetric Systems:



Multi-Camera Systems

A rigid-relationship among the cameras

Portable Panoramic Image Mapping System

Photogrammetric Point Positioning

GNSS/INS-Assisted Multi-Camera Photogrammetric System:

$$r_I^m = r_b^m(t) + R_b^m(t)\, r_{c_r}^b + R_b^m(t)\, R_{c_r}^b\, r_{c_j}^{c_r} + S_i^{c_j}\, R_b^m(t)\, R_{c_r}^b\, R_{c_j}^{c_r}\, r_i^{c_j}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_b^m (t)_{GNSS/INS} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_b^m (t) + \begin{bmatrix} e_X \\ e_Y \\ e_Z \end{bmatrix}_b^m (t)$$

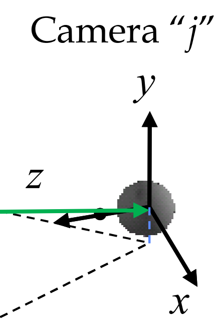
$$\begin{bmatrix} \omega \\ \phi \\ \kappa \end{bmatrix}_b^m (t)_{GNSS/INS} = \begin{bmatrix} \omega \\ \phi \\ \kappa \end{bmatrix}_b^m (t) + \begin{bmatrix} e_\omega \\ e_\phi \\ e_\kappa \end{bmatrix}_b^m (t)$$

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{cj}^{cr} (\,prior\,) = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{cj}^{cr} + \begin{bmatrix} e_{\Delta X} \\ e_{\Delta Y} \\ e_{\Delta Z} \end{bmatrix}_{cj}^{cr}$$

$$\begin{bmatrix} \Delta\omega \\ \Delta\varphi \\ \Delta\kappa \end{bmatrix}_{cj}^{cr} (\,prior\,) = \begin{bmatrix} \Delta\omega \\ \Delta\varphi \\ \Delta\kappa \end{bmatrix}_{cj}^{cr} + \begin{bmatrix} e_{\Delta\omega} \\ e_{\Delta\varphi} \\ e_{\Delta\kappa} \end{bmatrix}_{cj}^{cr}$$

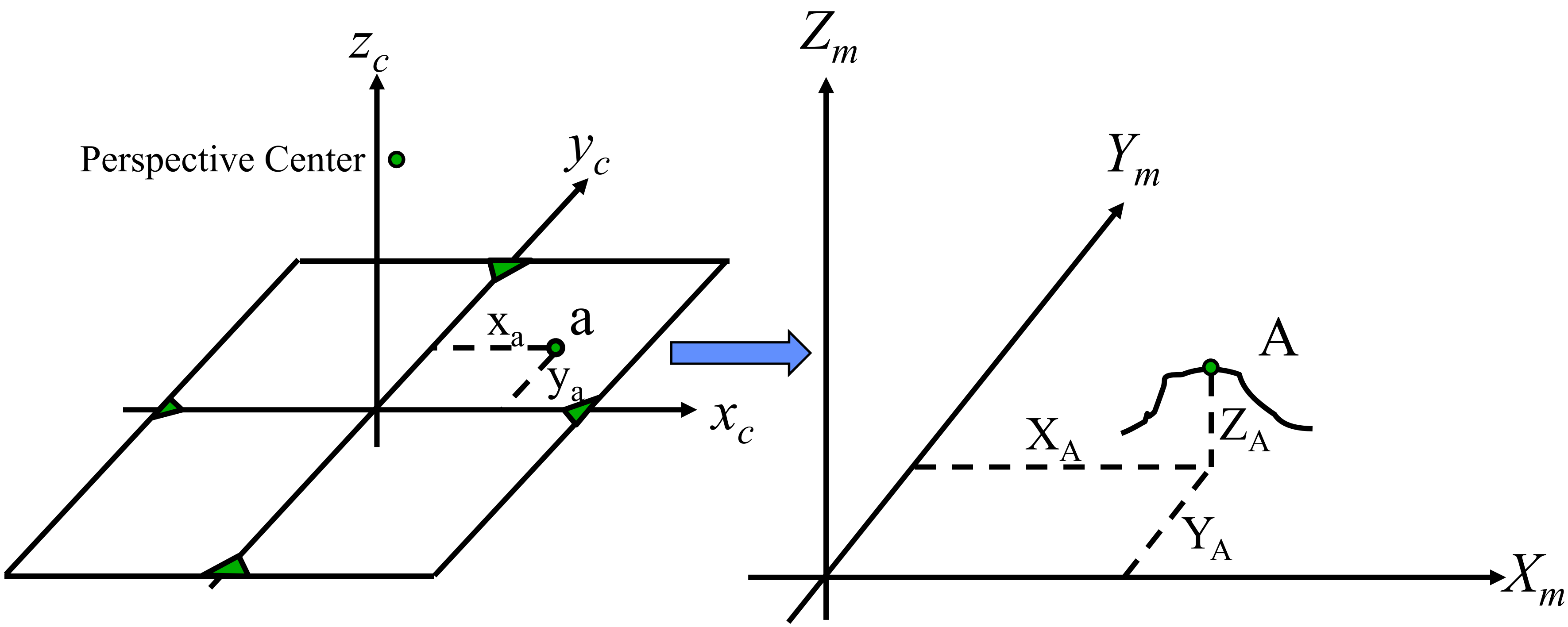
$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{cr}^{b} (\,prior\,) = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{cr}^{b} + \begin{bmatrix} e_{\Delta X} \\ e_{\Delta Y} \\ e_{\Delta Z} \end{bmatrix}_{cr}^{b}$$

$$\begin{bmatrix} \Delta\omega \\ \Delta\varphi \\ \Delta\kappa \end{bmatrix}_{cr}^{b} (\,prior\,) = \begin{bmatrix} \Delta\omega \\ \Delta\varphi \\ \Delta\kappa \end{bmatrix}_{cr}^{b} + \begin{bmatrix} e_{\Delta\omega} \\ e_{\Delta\varphi} \\ e_{\Delta\kappa} \end{bmatrix}_{cr}^{b}$$



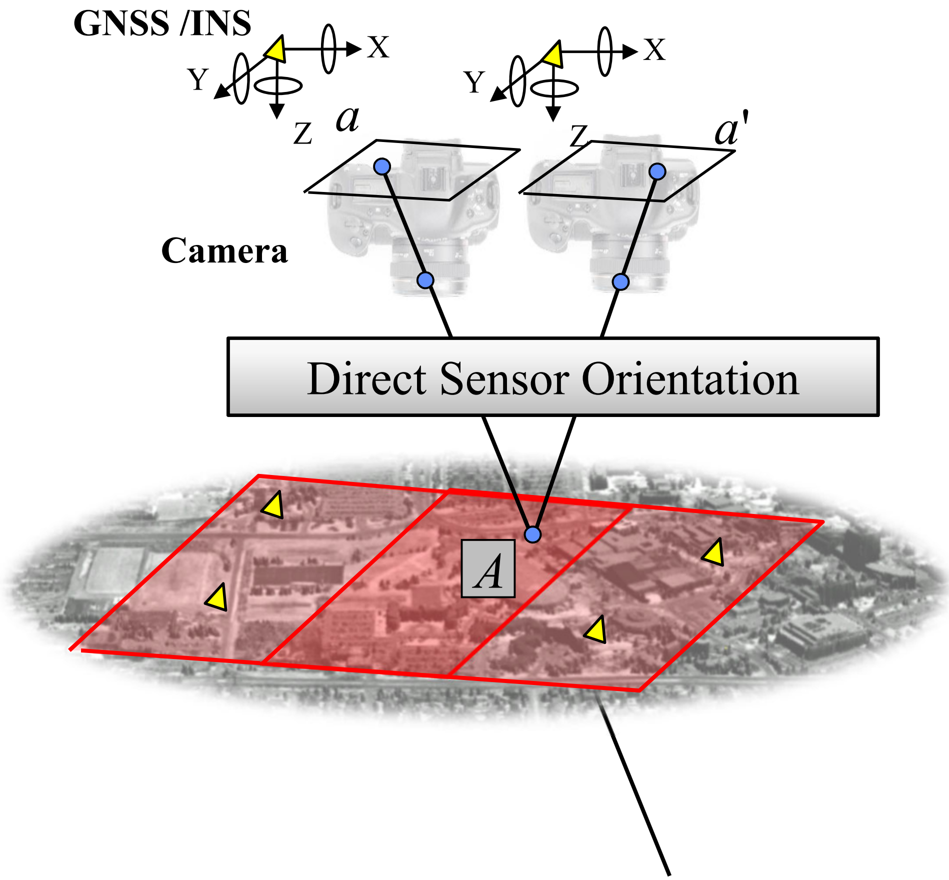
Y_G X_G

Photogrammetric Point Positioning



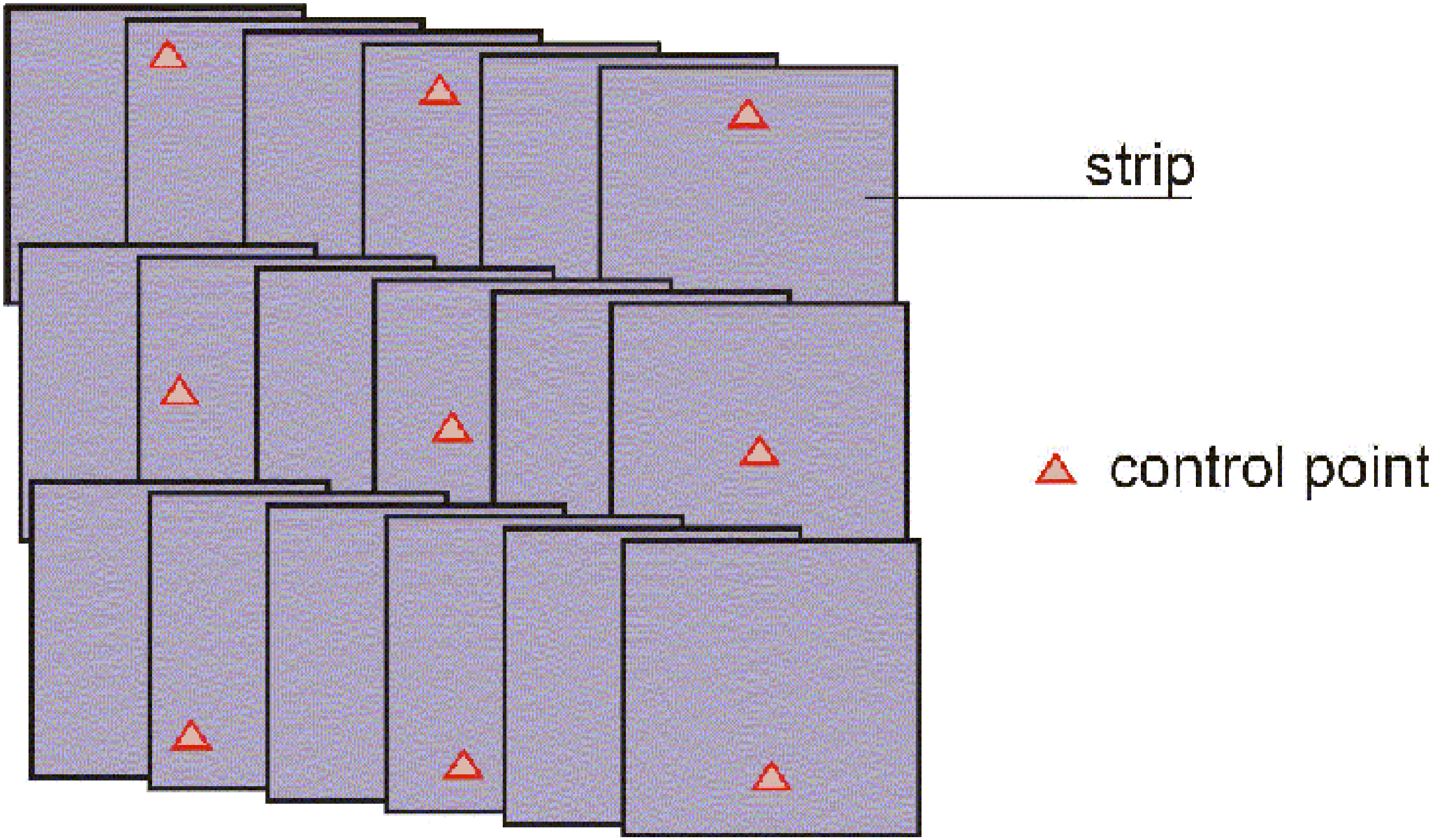
$$x_a = f_x(X_A, Y_A, Z_A, IOPs, EOPs)$$
$$y_a = f_y(X_A, Y_A, Z_A, IOPs, EOPs)$$

Photogrammetric Point Positioning



Bundle Block Adjustment

Bundle Block Adjustment



60% Overlap and 20% side lap

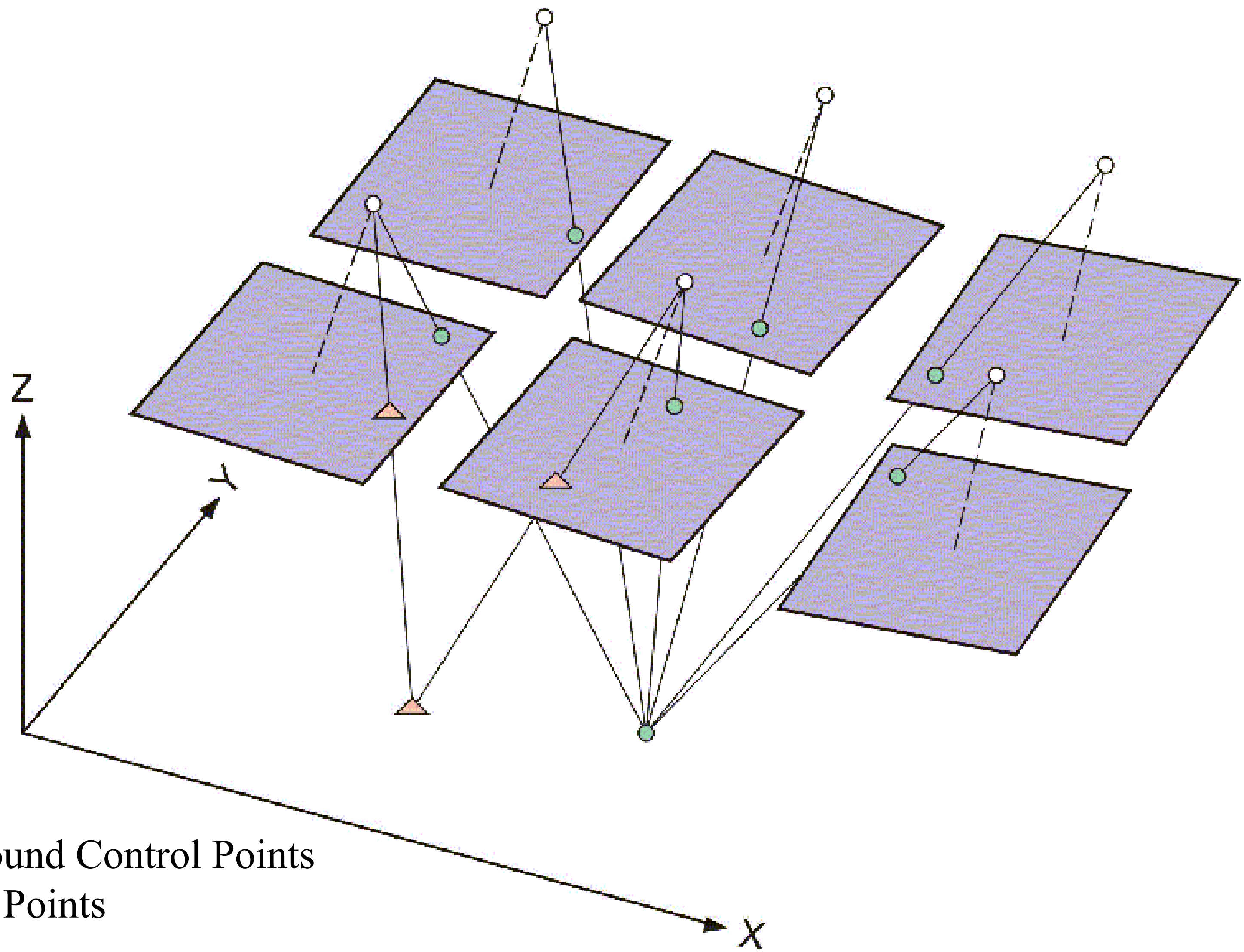
Bundle Block Adjustment

- Direct relationship between image and ground coordinates
- We measure the image coordinates in the images of the block.
- Using the collinearity equations, we can relate the image coordinates, corresponding ground coordinates, IOPs, and EOPs.
- Using a simultaneous least squares adjustment, we can solve for the:
  - Ground coordinates of tie points,
  - EOPs, and
  - IOPs (Camera Calibration Procedure).

Bundle Block Adjustment: Concept

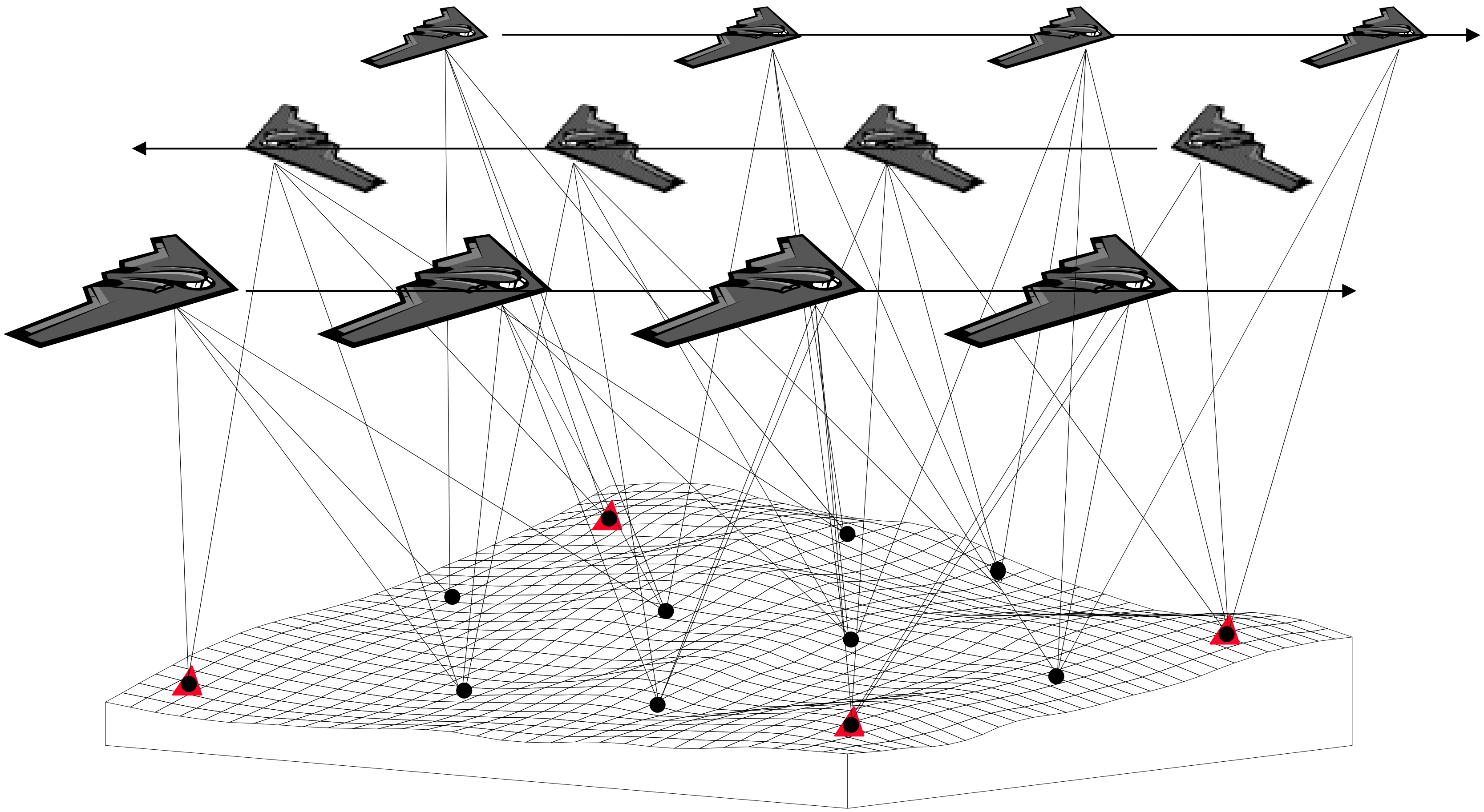
- The image coordinate measurements and IOPs define a bundle of light rays.
- The EOPs define the position and attitude of the bundles in space.
- During the adjustment: The bundles are rotated (ω, ϕ, κ) and shifted (X_o, Y_o, Z_o) until:
  - Conjugate light rays intersect as well as possible at the locations of object space tie points.
  - Light rays corresponding to ground control points pass through the object points as close as possible.

Bundle Block Adjustment: Concept



▲ Ground Control Points
● Tie Points

Bundle Block Adjustment: Concept



▲ Ground Control Points

● Tie Points

Least Squares Adjustment

- Prior to the adjustment, we need to identify:
  - The unknown parameters
  - Observable quantities
  - The mathematical relationship between the unknown parameters and the observable quantities
- Linearize the mathematical relationship (if it is not linear)
- Apply least squares adjustment formulas

Unknown Parameters

- Unknown parameters might include:
  - Ground coordinates of tie points (points that appear in more than one image)
  - Exterior orientation parameters of the involved imagery
  - Interior orientation parameters of the involved cameras (for camera calibration purposes)

Observable Quantities

- Observable quantities might include:
  - The ground coordinates of control points
  - Image coordinates of tie as well as control points
  - Interior orientation parameters of the involved cameras
  - Exterior orientation parameters of the involved imagery (from a GNSS/INS unit onboard)

Mathematical Model

$$x_a = x_p - c\frac{r_{11}(X_A - X_O) + r_{21}(Y_A - Y_O) + r_{31}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + \Delta x + e_x$$

$$y_a = y_p - c\frac{r_{12}(X_A - X_O) + r_{22}(Y_A - Y_O) + r_{32}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + \Delta y + e_y$$

$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} \sim (0, \sigma_o^2\ P^{-1})$$

Mathematical Model

- $\Delta x = \Delta x_{\text{Radial Lens Distortion}} + \Delta x_{\text{Decentric Lens Distortion}} + \Delta x_{\text{Atmospheric Refraction}} + \Delta x_{\text{Affine Deformation}} + \text{etc....}$

- $\Delta y = \Delta y_{\text{Radial Lens Distortion}} + \Delta y_{\text{Decentric Lens Distortion}} + \Delta y_{\text{Atmospheric Refraction}} + \Delta y_{\text{Affine Deformations}} + \text{etc....}$

Distortion Parameters

$$\Delta x_{\text{Radial Lens Distortion}} = \bar{x}\ (k_1 r^2 + k_2 r^4 + k_3 r^6 + \ldots.)$$

$$\Delta y_{\text{Radial Lens Distortion}} = \bar{y}\ (k_1 r^2 + k_2 r^4 + k_3 r^6 + \ldots.)$$

$$\Delta x_{\text{Decentric Lens Distortion}} = (1 + p_3^2 r^2)\{p_1(r^2 + 2\bar{x}^2) + 2 p_2 \bar{x}\ \bar{y}\ \}$$

$$\Delta y_{\text{Decentric Lens Distortion}} = (1 + p_3^2 r^2)\{2 p_1 \bar{x}\ \bar{y}\ + p_2(r^2 + 2\bar{y}^2)\}$$

where: $r = \{(x - x_p)^2 + (y - y_p)^2\}^{0.5}$

$$\bar{x} = x - x_p$$

$$\bar{y} = y - y_p$$

Least Squares Adjustment

- Gauss Markov Model
  Observation Equations

$$y = A\,x + e \qquad\qquad e \sim (0, \sigma_o^2 P^{-1})$$

$$y \qquad n \times 1 \;\; \textit{observation vector}$$

$$A \qquad n \times m \;\; \textit{design matrix}$$

$$x \qquad m \times 1 \;\; \textit{vector of unknowns}$$

$$e \qquad n \times 1 \;\; \textit{noise contaminating the observation vector}$$

$$\sigma_o^2 P^{-1} \qquad n \times n \;\; \textit{variance covariance matrix of the noise vector}$$

Least Squares Adjustment

$$\hat{x} = (A^T P A)^{-1} A^T P y$$

$$D\{\hat{x}\} = \sigma_o^2 \, (A^T P A)^{-1}$$

$$\tilde{e} = y - A\hat{x}$$

$$\hat{\sigma}_o^2 = (\tilde{e}^T P \tilde{e}) \, / \, (n - m)$$

Non-Linear System

$$Y = a(X) + e$$

$a(X)$ is the non − linear function

We use Taylor Series Expansion

$$Y \approx a(X_o) + \left.\frac{\partial a}{\partial X}\right|_{X_o} (X - X_o) + e \qquad \text{(We ignore higher order terms)}$$

Where :

X_o is approximate values for the unknown parameters

$$Y - a(X_o) = \left.\frac{\partial a}{\partial X}\right|_{X_o} (X - X_o) + e$$

$$y = A\,x + e$$

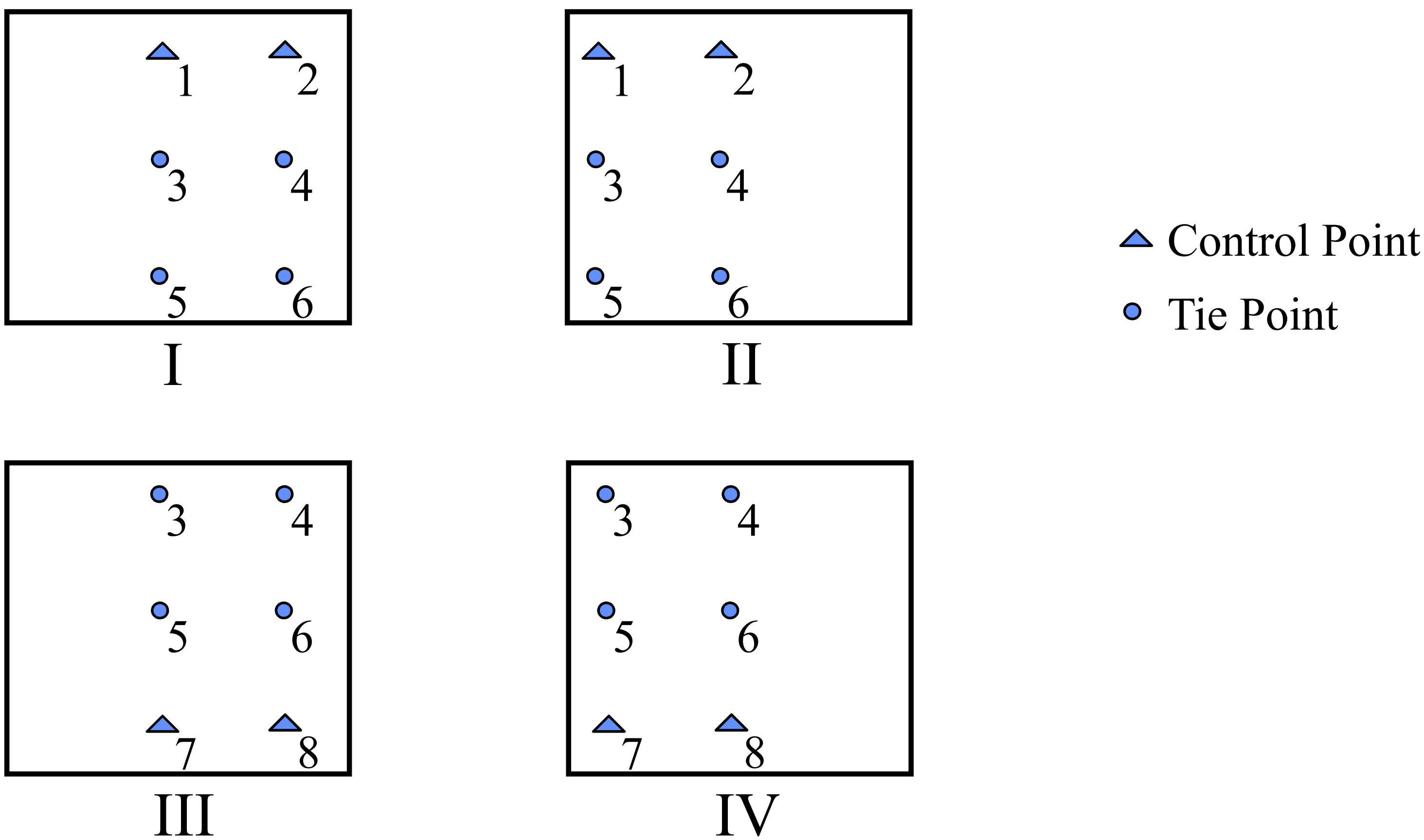
Where :

$$y = Y - a(X_o)$$

$$A = \left.\frac{\partial a}{\partial X}\right|_{X_o}$$

- Iterative solution for the unknown parameters
- When should we stop the iterations?

Example (4 Images in Two Strips)



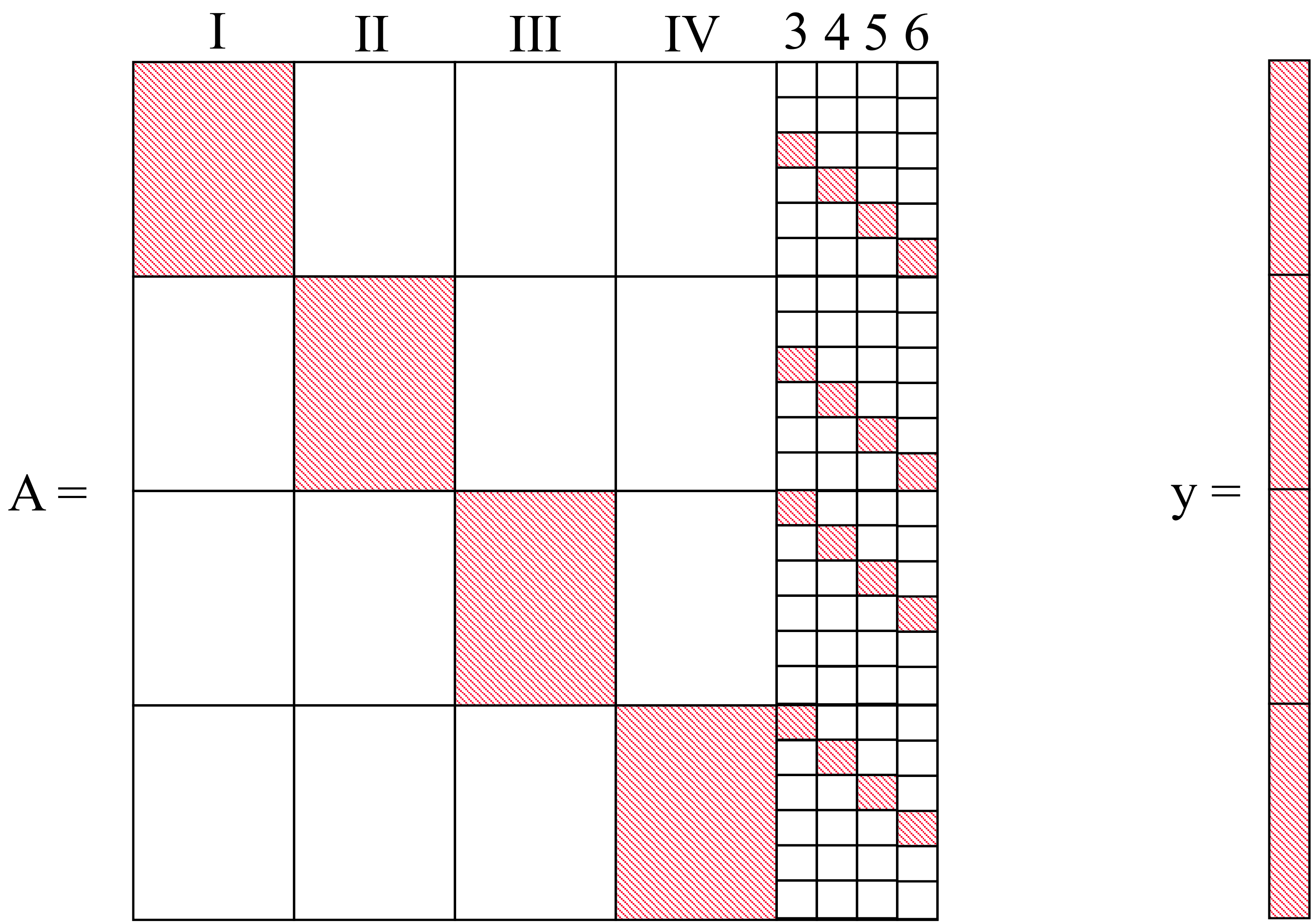
Balance Between Observations & Unknowns

- Number of observations:
  - 4 x 6 x 2 = 48 observations (collinearity equations)
- Number of unknowns:
  - 4 x 6 + 3 x 4 = 36 unknowns
- Redundancy:
  - 12
- Assumptions:
  - IOPs are assumed to be known and errorless.
  - Ground coordinates of the control points are errorless.

Structure of the Design Matrix (BA)

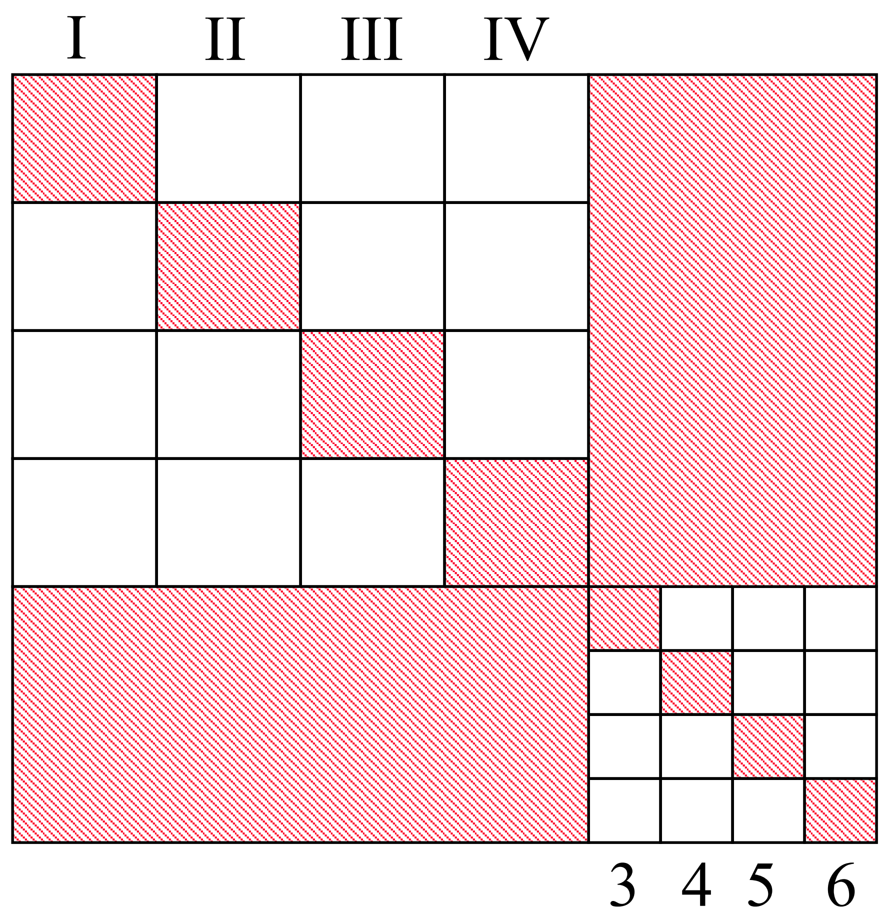
- $Y = a(X) + e \qquad e \sim (0, \sigma^2 P^{-1})$
- Using approximate values for the unknown parameters (X^o) and partial derivatives, the above equations can be linearized leading to the following equations:
- $y_{48x1} = A_{48x36}\, x_{36x1} + e_{48x1} \qquad e \sim (0, \sigma^2 P^{-1})$

Structure of the Design Matrix



Structure of the Normal Matrix

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix} =$$

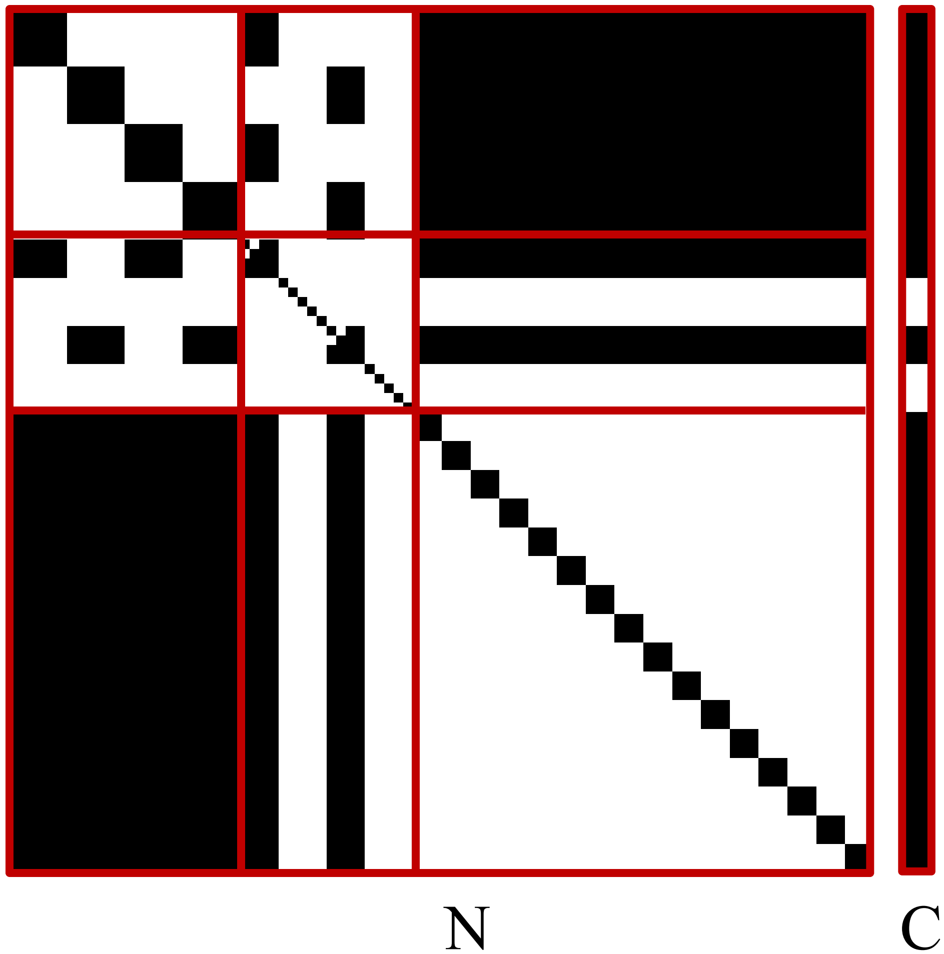


Sample Data

- 2 cameras.
- 4 images.
- 16 points.

- All the points appear in all the images
- Two images were captured by each camera

Structure of the Normal Matrix: Example



Observation Equations

$$y_{n\times 1} = A_{n\times m}\ x_{m\times 1} + e_{n\times 1} \qquad e \sim (0, \sigma_o^2\ P^{-1})$$

$$y_{n\times 1} = A_{1_{n\times 6m_1}}\ x_{1_{6m_1\times 1}} + A_{2_{n\times 3m_2}}\ x_{2_{3m_2\times 1}} + e_{n\times 1}$$

$$y_{n\times 1} = \begin{bmatrix} A_{1_{n\times 6m_1}} & A_{2_{n\times 3m_2}} \end{bmatrix} \begin{bmatrix} x_{1_{6m_1\times 1}} \\ x_{2_{3m_2\times 1}} \end{bmatrix} + e_{n\times 1}$$

- **n ≡ Number of observations (image coordinate measurements)**
- **m ≡ Number of unknowns:**
  - **m_1 ≡ Number of images ⇒ 6 m_1 (EOPs of the images)**
  - **m_2 ≡ Number of tie points ⇒ 3 m_2 (ground coordinates of tie points)**
  - **$m = 6\ m_1 + 3\ m_2$**

Normal Equation Matrix

$$N_{(6m_1+3m_2)\times(6m_1+3m_2)} = \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} P \begin{bmatrix} A_1 & A_2 \end{bmatrix}$$

$$N = \begin{bmatrix} N_{11_{6m_1\times 6m_1}} & N_{12_{6m_1\times 3m_2}} \\ N_{12_{3m_2\times 6m_1}}^T & N_{22_{3m_2\times 3m_2}} \end{bmatrix}$$

$$C_{(6m_1+3m_2)\times 1} = \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} P\ y = \begin{bmatrix} A_1^T P y \\ A_2^T P y \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1\times 1}} \\ C_{2_{3m_2\times 1}} \end{bmatrix}$$

Normal Equation Matrix

- N_{11} is a block diagonal matrix with 6x6 sub-blocks along the diagonal.
- N_{22} is a block diagonal matrix with 3x3 sub-blocks along the diagonal.

$$\begin{bmatrix} N_{11_{6m_1 \times 6m_1}} & N_{12_{6m_1 \times 3m_2}} \\ N^T_{12_{3m_2 \times 6m_1}} & N_{22_{3m_2 \times 3m_2}} \end{bmatrix} \begin{bmatrix} \hat{x}_{1_{6m_1 \times 1}} \\ \hat{x}_{2_{3m_2 \times 1}} \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1 \times 1}} \\ C_{2_{3m_2 \times 1}} \end{bmatrix}$$

- Question: Under which circumstances will we deviate from this structure?

Reduction of the Normal Equation Matrix

$$N_{11_{6m_1\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} + N_{12_{6m_1\times 3m_2}} \hat{x}_{2_{3m_2\times 1}} = C_{1_{6m_1\times 1}}$$

$$N^T_{12_{3m_2\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} + N_{22_{3m_2\times 3m_2}} \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

- Solving for x_2 first:

$$N^T_{12_{3m_2\times 6m_1}} \left(N^{-1}_{11_{6m_1\times 6m_1}} C_{1_{6m_1\times 1}} - N^{-1}_{11_{6m_1\times 6m_1}} N_{12_{6m_1\times 3m_2}} \hat{x}_{2_{3m_2\times 1}}\right) + N_{22_{3m_2\times 3m_2}} \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

$$\hat{x}_{2_{3m_2\times 1}} = \left(N_{22_{3m_2\times 3m_2}} - N^T_{12_{3m_2\times 6m_1}} N^{-1}_{11_{6m_1\times 6m_1}} N_{12_{6m_1\times 3m_2}}\right)^{-1} \left(C_{2_{3m_2\times 1}} - N^T_{12_{3m_2\times 6m_1}} N^{-1}_{11_{6m_1\times 6m_1}} C_{1_{6m_1\times 1}}\right)$$

$$\hat{x}_{1_{6m_1\times 1}} = \left(N^{-1}_{11_{6m_1\times 6m_1}} C_{1_{6m_1\times 1}} - N^{-1}_{11_{6m_1\times 6m_1}} N_{12_{6m_1\times 3m_2}} \hat{x}_{2_{3m_2\times 1}}\right)$$

- $3m_2 < 6m_1$
- Remember: N_{11} is a block diagonal matrix with 6x6 sub-blocks along the diagonal.

Reduction of the Normal Equation Matrix

$$N_{11_{6m_1\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} + N_{12_{6m_1\times 3m_2}} \hat{x}_{2_{3m_2\times 1}} = C_{1_{6m_1\times 1}}$$

$$N^T_{12_{3m_2\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} + N_{22_{3m_2\times 3m_2}} \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

- Solving for x_1 first:

$$N_{11_{6m_1\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} + N_{12_{3m_2\times 3m_2}} \left(N^{-1}_{22_{3m_2\times 3m_2}} C_{2_{3m_2\times 1}} - N^{-1}_{22_{3m_2\times 3m_2}} N^T_{12_{3m_2\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} \right) = C_{1_{6m_1\times 1}}$$

$$\hat{x}_{1_{6m_1\times 1}} = \left(N_{11_{6m1\times 6m1}} - N_{12_{6m_1\times 3m_2}} N^{-1}_{22_{3m_2\times 3m_2}} N^T_{12_{3m_2\times 6m_1}} \right)^{-1} \left(C_{1_{6m_1\times 1}} - N_{12_{6m_1\times 3m_2}} N^{-1}_{22_{3m_2\times 2m_2}} C_{2_{3m_2\times 1}} \right)$$

$$\hat{x}_{2_{3m_2\times 1}} = \left(N^{-1}_{22_{3m_2\times 3m_2}} C_{2_{3m_2\times 1}} - N^{-1}_{22_{3m_2\times 3m_2}} N^T_{12_{3m_2\times 6m_1}} \hat{x}_{1_{6m_1\times 1}} \right)$$

- $6m_1 < 3m_2$
- Remember: N_{22} is a block diagonal matrix with 3x3 sub-blocks along the diagonal.

Reduction of the Normal Equation Matrix

- Variance covariance matrix of the estimated parameters:

$$D\{\hat{x}_{1_{6m_1\times 1}}\} = \sigma_o^2 \left(N_{11_{6m1\times 6m1}} - N_{12_{6m_1\times 3m_2}} N^{-1}_{22_{3m_2\times 3m_2}} N^{T}_{12_{3m_2\times 6m_1}}\right)^{-1}$$

$$D\{\hat{x}_{2_{3m_2\times 1}}\} = \sigma_o^2 \left(N_{22_{3m_2\times 3m_2}} - N^{T}_{12_{3m_2\times 6m_1}} N^{-1}_{11_{6m_1\times 6m_1}} N_{12_{6m_1\times 3m_2}}\right)^{-1}$$

Building the Normal Equation Matrix

- We would like to investigate the possibility of sequentially building up the normal equation matrix without fully building the design matrix.
- (x_{ij}, y_{ij}) image coordinates of the i^{th} point in the j^{th} image

$$y_{2\times1_{ij}} = A_{1_{2\times6_{ij}}} \; x_{1_{6\times1_j}} + A_{2_{2\times3_{ij}}} \; x_{2_{3\times1_i}} + e_{2\times1_{ij}}$$

$$y_{2\times1_{ij}} = \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} + e_{2\times1_{ij}}$$

Normal Equation Matrix

$$y_{2\times1_{ij}} = \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} + e_{2\times1_{ij}}$$

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} \\ A^T_{2_{3\times2_{ij}}} \end{bmatrix} P_{ij} \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} \\ A^T_{2_{3\times2_{ij}}} \end{bmatrix} P_{ij} \; y_{2\times1_{ij}}$$

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{1_{6\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{2_{3\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \end{bmatrix}$$

Normal Equation Matrix

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{1_{6\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{2_{3\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \end{bmatrix}$$

$$\begin{bmatrix} N_{11_{ij}} & N_{12_{ij}} \\ N^T_{12_{ij}} & N_{22_{ij}} \end{bmatrix}_{9\times9} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix}_{9\times1} = \begin{bmatrix} C_{1_{ij}} \\ C_{2_{ij}} \end{bmatrix}_{9\times1}$$

- Note: We cannot solve this matrix for the:
  - The Exterior Orientation Parameters of the jth image, and
  - The ground coordinates of the ith point.

Normal Equation Matrix

$$\begin{bmatrix} N_{11_{6m_1 \times 6m_1}} & N_{12_{6m_1 \times 3m_2}} \\ N^T_{12_{3m_2 \times 6m_1}} & N_{22_{3m_2 \times 3m_2}} \end{bmatrix} \begin{bmatrix} \hat{x}_{1_{6m_1 \times 1}} \\ \hat{x}_{2_{3m_2 \times 1}} \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1 \times 1}} \\ C_{2_{3m_2 \times 1}} \end{bmatrix}$$

- Question: How can we sequentially build the above matrices?
- Assumption: All the points are common to all the images.

N_{11} - Matrix

$$N_{11_{(6m_1 \times 6m_1)}} = \begin{bmatrix} \sum_{i=1}^{m_2} N_{11_{i1}} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \sum_{i=1}^{m_2} N_{11_{i2}} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \sum_{i=1}^{m_2} N_{11_{i3}} & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \sum_{i=1}^{m_2} N_{11_{im_1}} \end{bmatrix}$$

- If all the points are not common to all the images:
    - The summation should be carried over all the points that appear in the image under consideration.

N_{22} - Matrix

$$N_{22_{(3m_2 \times 3m_2)}} = \begin{bmatrix} \sum_{j=1}^{m_1} N_{22_{1j}} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \sum_{j=1}^{m_1} N_{22_{2j}} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \sum_{j=1}^{m_1} N_{22_{3j}} & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \sum_{j=1}^{m_1} N_{22_{m_2 j}} \end{bmatrix}$$

- If all the points are not common to all the images:
  - The summation should be carried over all the images within which the point under consideration appears.

N_{12} - Matrix

$$N_{12_{(6m_1\times 3m_2)}} = \begin{bmatrix} N_{12_{11}} & N_{12_{21}} & N_{12_{31}} & \cdots & \cdots & N_{12_{m_2 1}} \\ N_{12_{12}} & N_{12_{22}} & N_{12_{32}} & \cdots & \cdots & N_{12_{m_2 2}} \\ N_{12_{13}} & N_{12_{23}} & N_{12_{23}} & \cdots & \cdots & N_{12_{m_2 3}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N_{12_{1m_1}} & N_{12_{2m_1}} & N_{12_{3m_1}} & \cdots & \cdots & N_{12_{m_2 m_1}} \end{bmatrix}$$

- If point "i" does not appear in image "j":
  - $(N_{12})_{ij} = 0$

C - Matrix

$$C_{1_{6m_1\times 1}} = \begin{bmatrix} \sum_{i=1}^{m_2} C_{1_{i1}} \\ \sum_{i=1}^{m_2} C_{1_{i2}} \\ \sum_{i=1}^{m_2} C_{1_{i3}} \\ \vdots \\ \vdots \\ \sum_{i=1}^{m_2} C_{1_{im_1}} \end{bmatrix} \qquad C_{2_{3m_2\times 1}} = \begin{bmatrix} \sum_{j=1}^{m_1} C_{2_{1j}} \\ \sum_{j=1}^{m_1} C_{2_{2j}} \\ \sum_{j=1}^{m_1} C_{2_{3j}} \\ \vdots \\ \vdots \\ \sum_{j=1}^{m_1} C_{2_{m_2 j}} \end{bmatrix}$$

- Once again, we assumed that all the points are common to all the images.

Precision of Bundle Block Adjustment

- The precision of the estimated EOPs as well as the ground coordinates of tie points can be obtained by the product of:
  – The estimated variance component, and
  – The inverse of the normal equation matrix (cofactor matrix).
- The precision depends on the following factors:
  – Geometric configuration of the image block
    - Base-Height ratio
  – Image scale
  – Image coordinate measurement precision

Precision of Bundle Block Adjustment

- Precision of a single model: If we have
  - Bundle block adjustment with additional parameters that compensate for various distortions
  - Regular blocks with 60%overlap and 20% side lap
  - Signalized targets

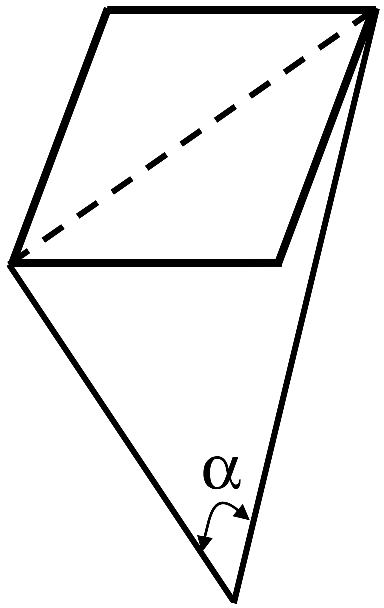
$\sigma_{XY} = \pm 3 \mu m$

$\sigma_Z = \pm 0.003\%$ of the camera principal distance (NA and WA cameras)

$\sigma_Z = \pm 0.004\%$ of the camera principal distance (SWA cameras)

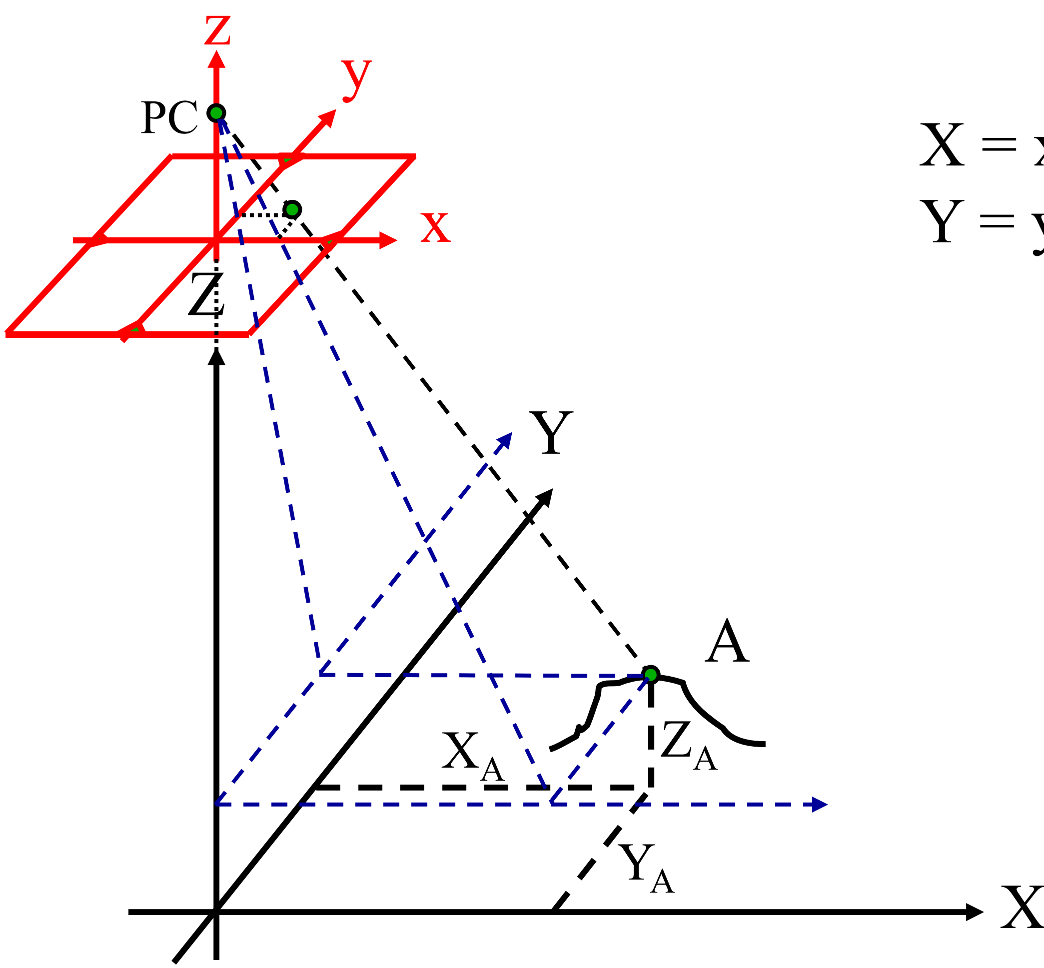
These precision values are given in the image space

Camera Classification



- α < 75° Normal angle camera (NA)
- 100° > α > 75° Wide angle camera (WA)
- α > 100° Super wide angle camera (SWA)

Precision of Bundle Block Adjustment



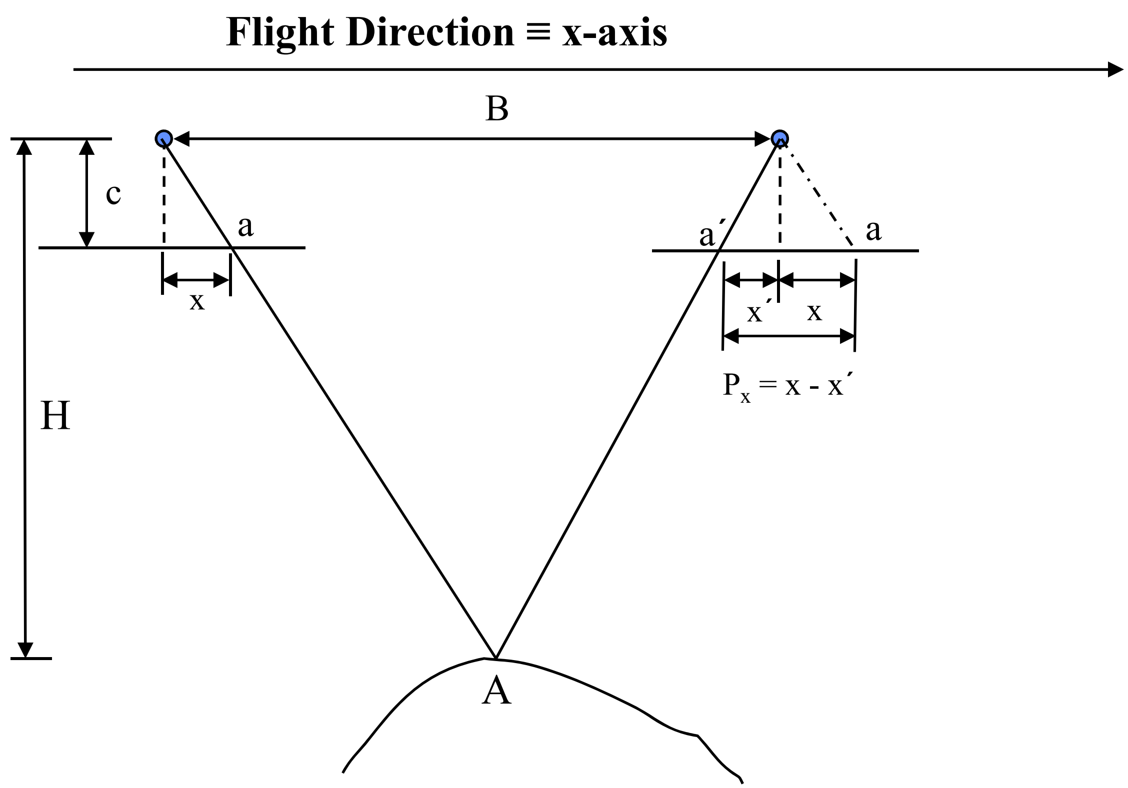
X = x * Z / c
Y = y * Z / c

$$\sigma_X = \frac{Z}{c}\sigma_x$$

$$\sigma_Y = \frac{Z}{c}\sigma_y$$

Precision of Bundle Block Adjustment

Vertical Precision

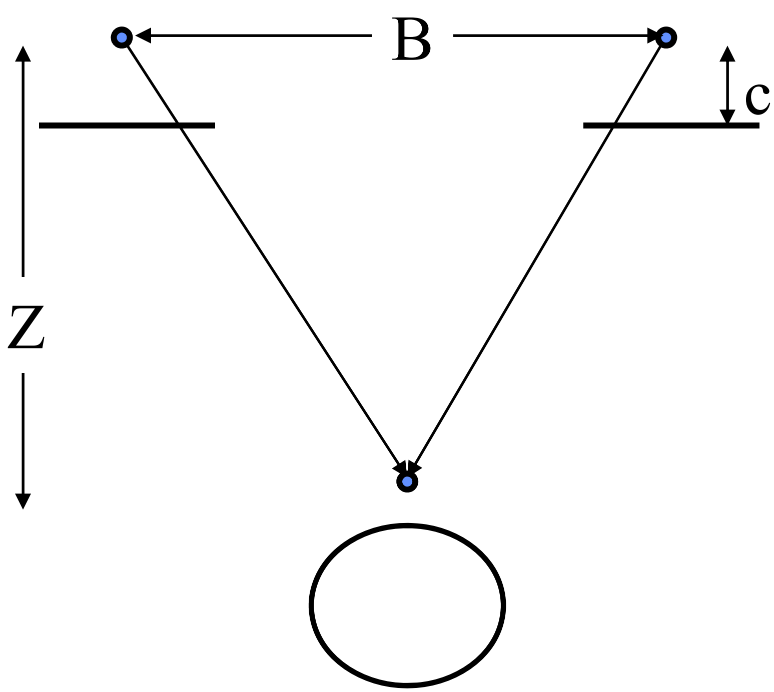


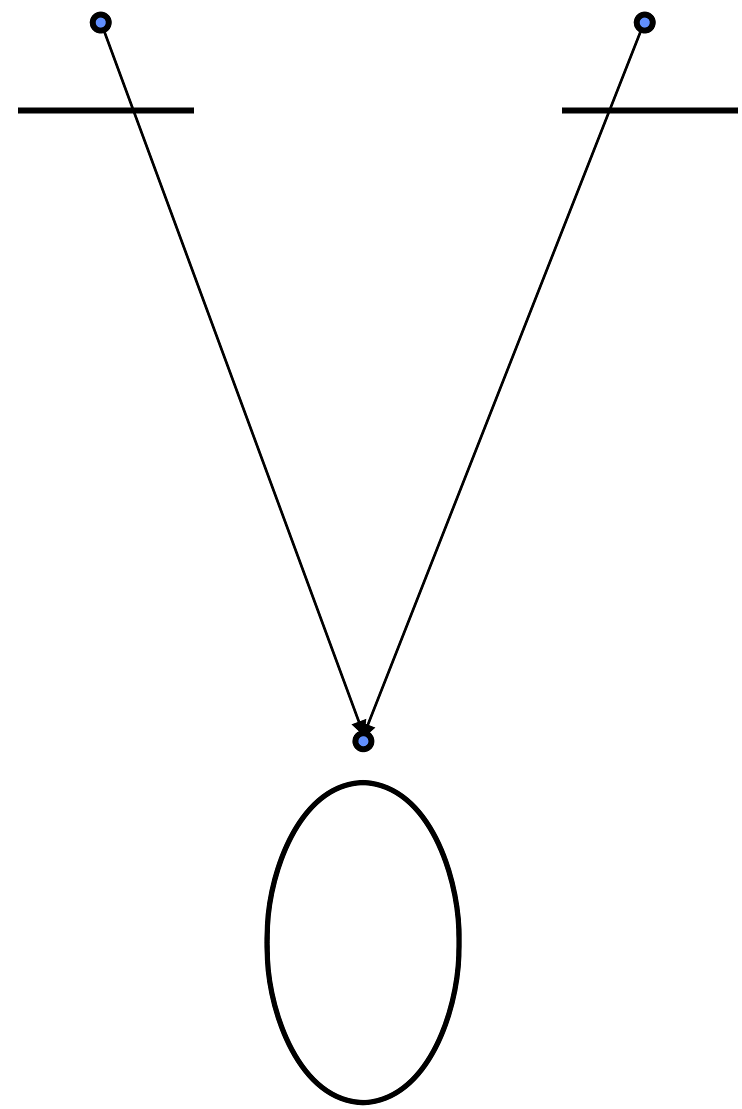
$P_x / B = c / H$
$H = B c / P_x$

Precision of Bundle Block Adjustment

Vertical Precision

$$\sigma_Z = \frac{Z}{c}\frac{Z}{B}\sigma_{p_x}$$

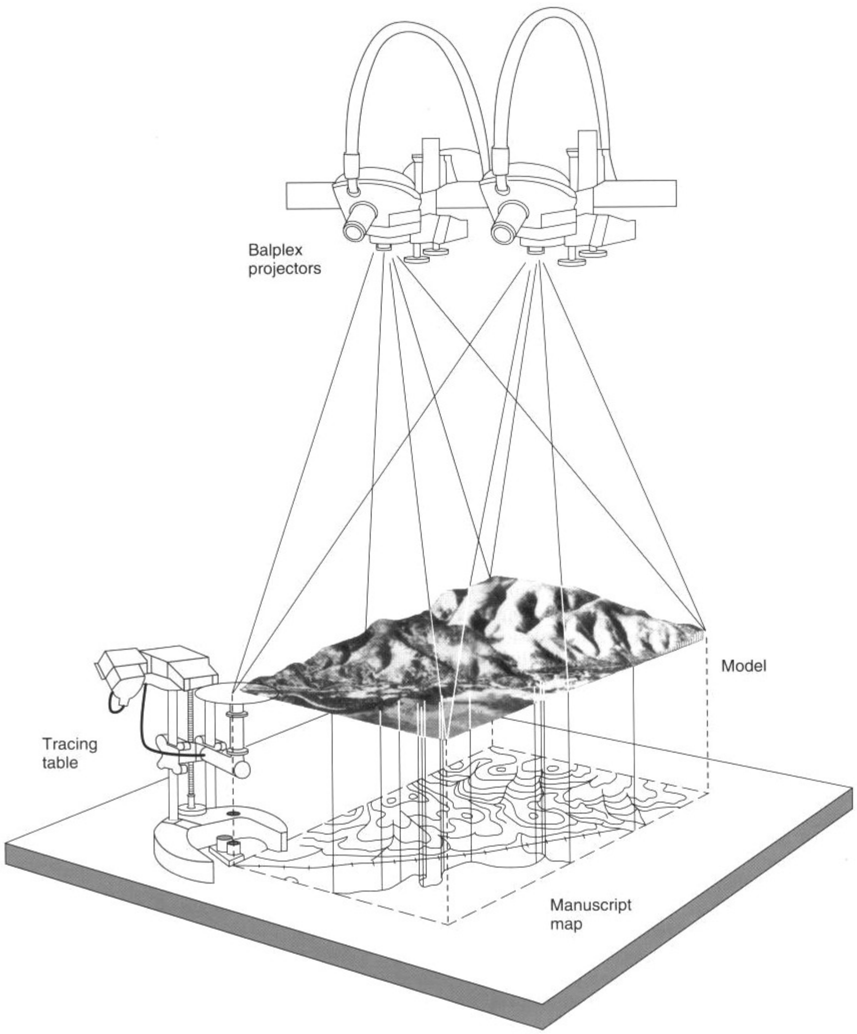




Advantages of Bundle Block Adjustment

- Most accurate triangulation technique since we have direct transformation between image and ground coordinates.
- Straight forward to include parameters that compensate for various deviations from the collinearity model.
- Straight forward to include additional observations:
  - GNSS/INS observations at the exposure stations
  - Object space distances
- Can be used for normal, convergent, aerial, and close range imagery
- After the adjustment, the EOPs can be set on analogue and analytical plotters for compilation purposes.

Photogrammetric Compilation



Disadvantages of Bundle Block Adjustment

- Model is non linear: approximations as well as partial derivatives are needed.
- Requires computer intensive computations.
- Analogue instruments cannot be used (they cannot measure image coordinate measurements).
- The adjustment cannot be separated into planimetric and vertical adjustment.

Bundle Adjustment: Final Remarks

- Elementary Unit: Images
- Measurements: Image coordinates
- Mathematical model: Collinearity equations
- Instruments: Comparators, analytical plotters, and Digital Photogrammetric Workstations (DPW)
- Required computer power: Very large
- Expected accuracy: High

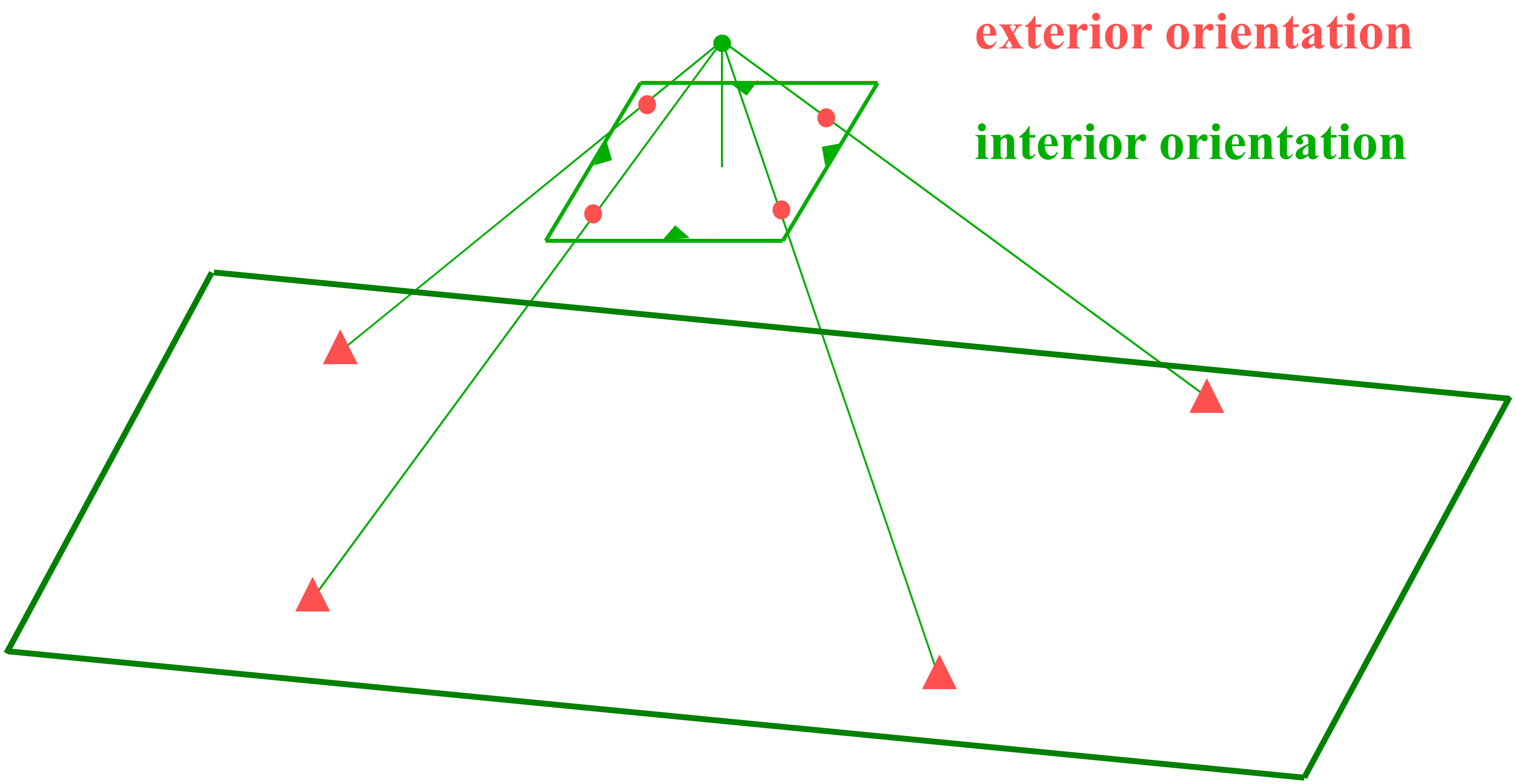
Special Cases

- Resection
- Intersection
- Stereo-pair orientation
- Relative orientation

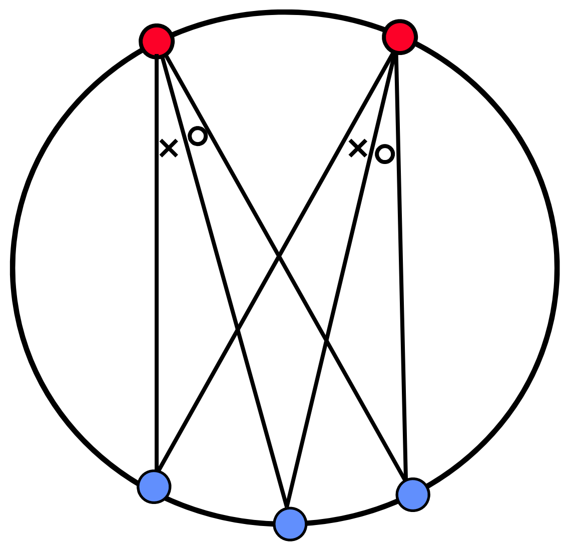
Resection

- We are dealing with one image.
- We would like to determine the EOPs of this image using GCPs.
- Q: What is the minimum GCPs requirements?
  - At least 3 non-collinear GCPs are required to estimate the 6 EOPs.
  - At least 5 non-collinear (well distributed in 3-D) GCPs are required to estimate the 6 EOPs and the 3 IOPs (x_p, y_p, c).
- Critical surface:
  - The GCPs and the perspective center lie on a common cylinder.

Resection



Resection - Critical Surface

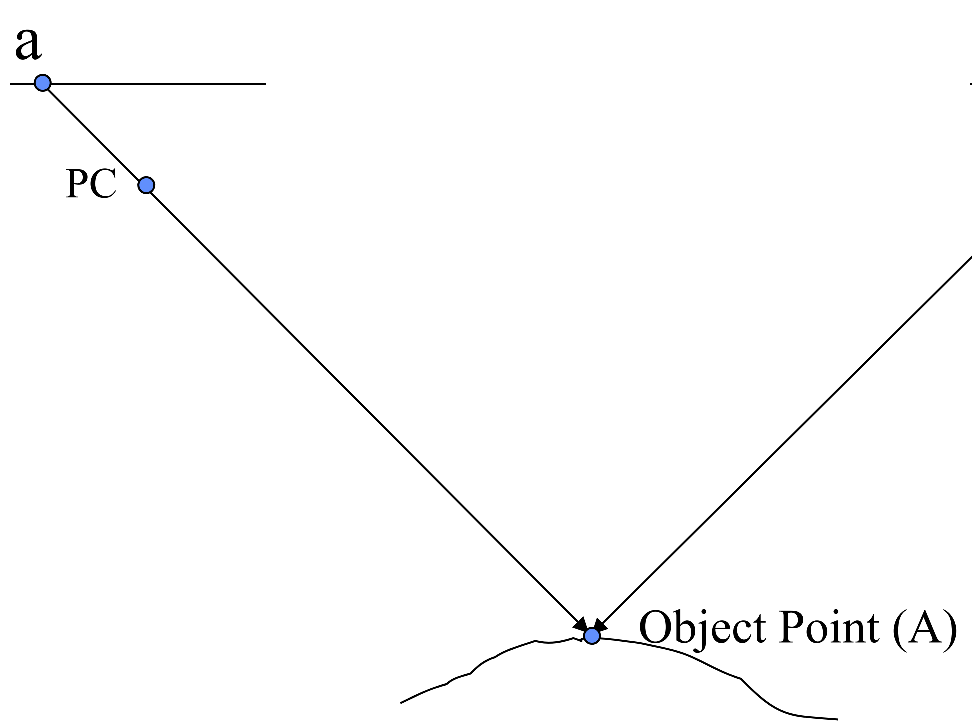


- Question: Which one of the EOPs cannot be determined?

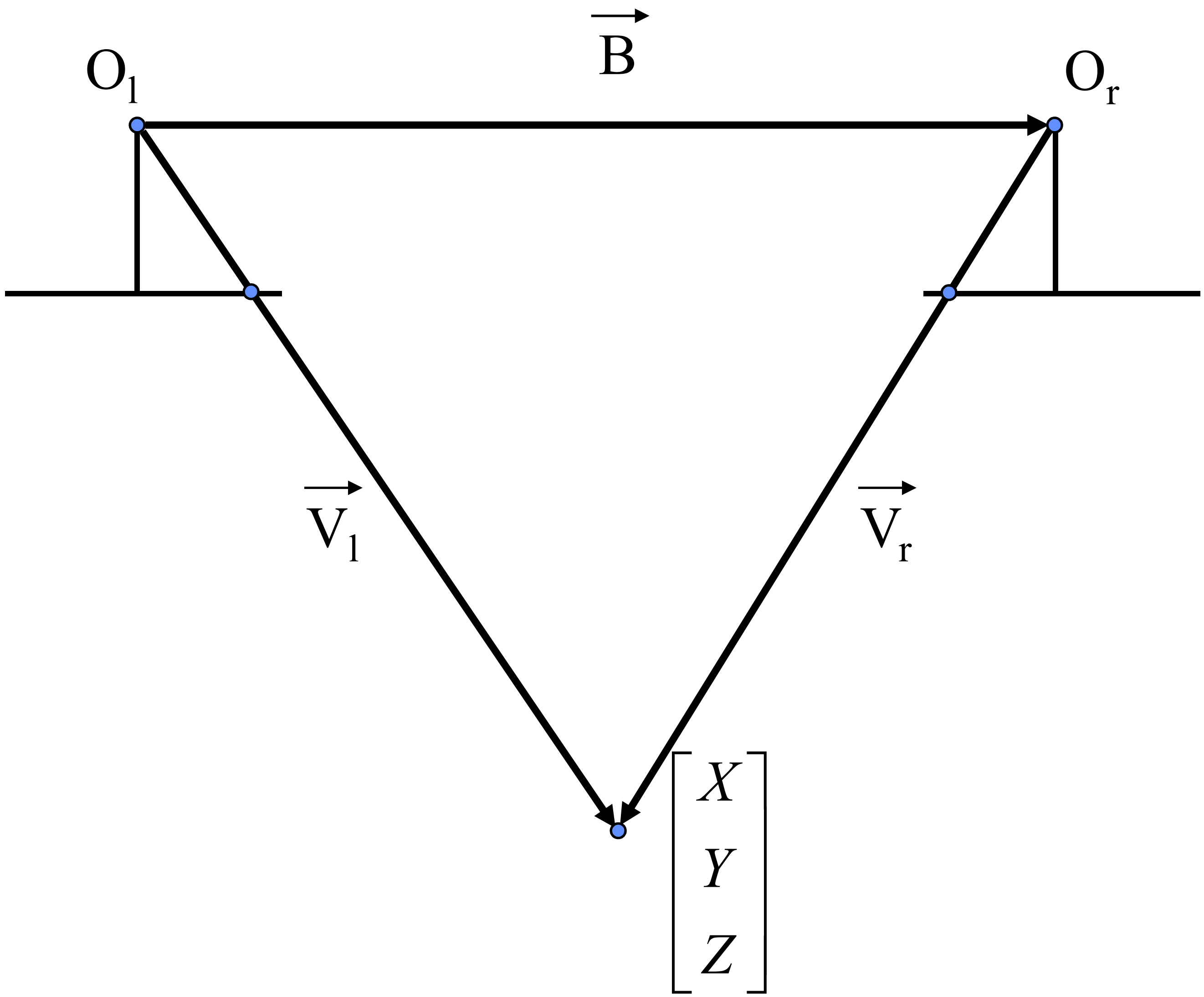
Intersection

- We are dealing with two images.
- The EOPs of these images are available.
- The IOPs of the involved camera(s) are also available.
- We want to estimate the ground coordinates of points in the overlap area.
- For each tie point, we have:
  - 4 Observation equations
  - 3 Unknowns
  - Redundancy = 1
- Non-linear model: approximations are needed

Intersection



Intersection: Linear Model



Intersection: Linear Model

$$\vec{B} = \begin{bmatrix} X_{O_r} - X_{O_l} \\ Y_{O_r} - Y_{O_l} \\ Z_{O_r} - Z_{O_l} \end{bmatrix}$$

- These vectors are given w.r.t. the ground coordinate system.

$$\vec{V}_l = \lambda \, R_{(\omega_l,\, \phi_l,\, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix}$$

$$\vec{V}_r = \mu \, R_{(\omega_r,\, \phi_r,\, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

Intersection: Linear Model

$$\vec{V}_l = \vec{B} + \vec{V}_r$$

$$\begin{bmatrix} X_{o_r} - X_{o_l} \\ Y_{o_r} - Y_{o_l} \\ Z_{o_r} - Z_{o_l} \end{bmatrix} = \lambda \, R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix} - \mu \, R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

- Three equations in two unknowns (λ, μ).
- They are linear equations.

Intersection: Linear Model

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O_l} \\ Y_{O_l} \\ Z_{O_l} \end{bmatrix} + \lambda \, R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix}$$

Or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O_r} \\ Y_{O_r} \\ Z_{O_r} \end{bmatrix} + \mu \, R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

Stereo-pair Orientation

- Given:
  - Stereo-pair: two images with at least 50% overlap
  - Image coordinates of some tie points
  - Image and ground coordinates of control points
- Required:
  - The ground coordinates of the tie points
  - The EOPs of the involved images
- Mini-Bundle Adjustment Procedure

Stereo-pair Orientation

- Example:
  - Given:
    - 1 Stereo-pair
    - 20 tie points
    - No ground control points
  - Question:
    - Can we estimate the ground coordinates of the points as well as the exterior orientation parameters of that stereo-pair?
  - Answer:
    - NO

Summary

- Photogrammetry: Definition and applications
- Photogrammetric tools:
  - Rotation matrices
  - Photogrammetric orientation: interior and exterior orientation
  - Collinearity equations/conditions
- Photogrammetric bundle adjustment
  - Structure of the design and normal matrices
- Special cases:
  - Resection, intersection, and stereo-pair orientation