Chapters 1 – 8: Overview

- Chapter 1: Introduction
- Chapters 2 – 4: Data acquisition
- Chapters 5 – 8: Data manipulation
  - Chapter 5: Vertical imagery
  - Chapter 6: Image coordinate measurements and refinement
  - Chapter 7: Mathematical model and bundle block adjustment
  - Chapter 8: Theory of orientation & photogrammetric triangulation

- This chapter will cover more details about the bundle adjustment procedure

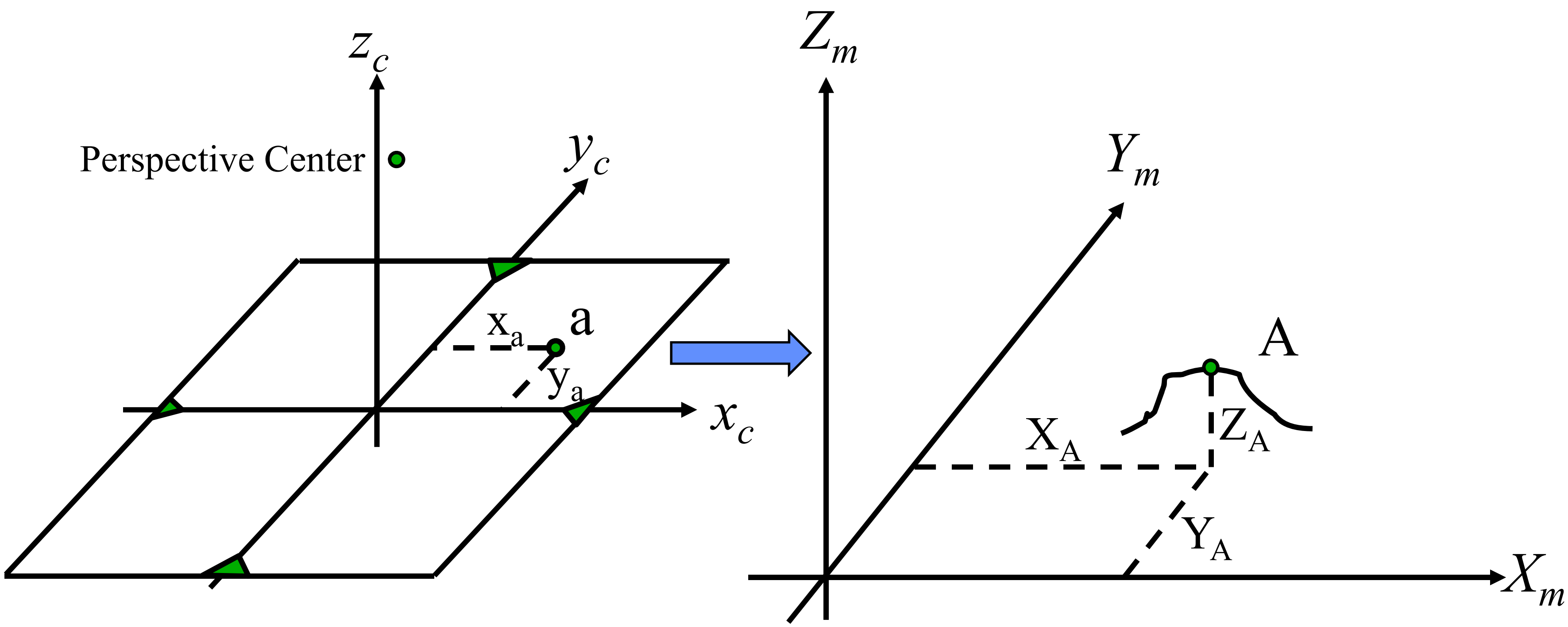
CE 59700: Chapter 9

Photogrammetric Bundle Adjustment

Overview

- Photogrammetric point positioning
- Photogrammetric bundle adjustment
  - Structure of the design and normal matrices
  - Sequential estimation of the unknown parameters
  - Sequential building of the normal equation matrix
  - Rules of thumb for the expected precision from a bundle adjustment procedure
- Special cases:
  - Resection, intersection, and stereo-pair orientation

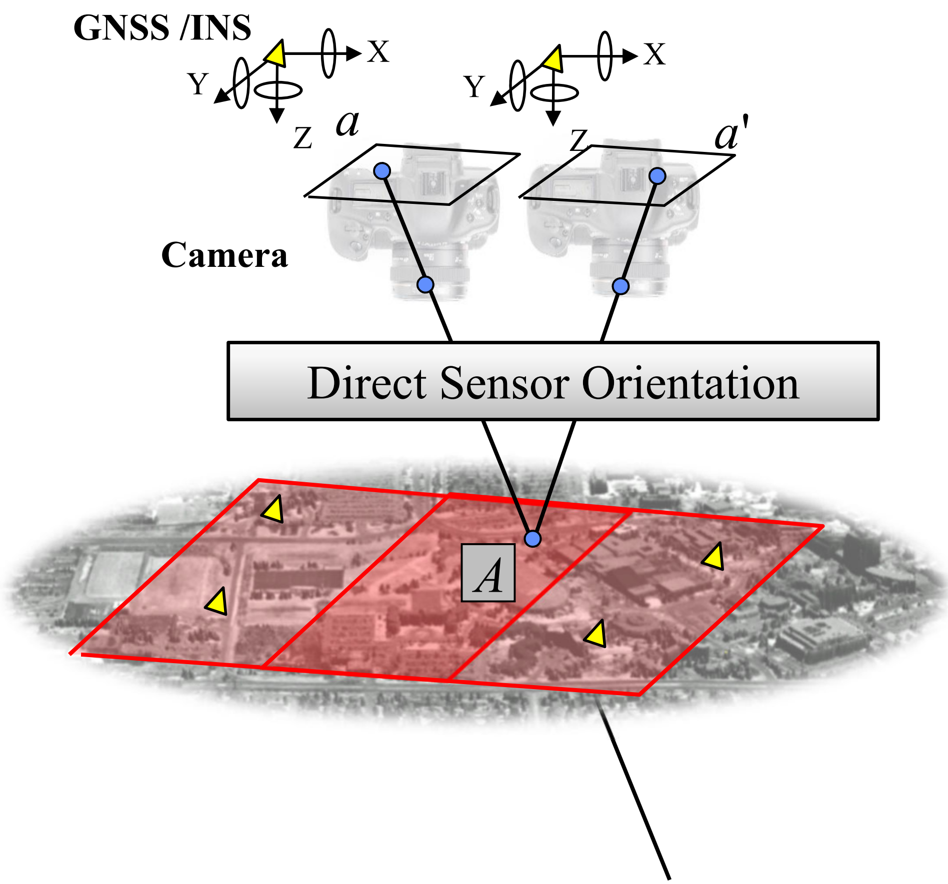
Photogrammetric Point Positioning



$$x_a = f_x(X_A, Y_A, Z_A, IOPs, EOPs)$$

$$y_a = f_y(X_A, Y_A, Z_A, IOPs, EOPs)$$

Photogrammetric Point Positioning



Collinearity Equations: Stochastic Model

$$x_a = x_p - c \frac{r_{11}(X_A - X_O) + r_{21}(Y_A - Y_O) + r_{31}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + dist_x + e_x$$

$$y_a = y_p - c \frac{r_{12}(X_A - X_O) + r_{22}(Y_A - Y_O) + r_{32}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + dist_y + e_y$$

$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} \sim (0, \sigma_o^2 P^{-1})$$

Distortion Model

- $dist_x = \Delta x_{\text{Radial Lens Distortion}} + \Delta x_{\text{Decentric Lens Distortion}} + \Delta x_{\text{Atmospheric Refraction}} + \Delta x_{\text{Affine Deformation}} + \text{etc….}$

- $dist_y = \Delta y_{\text{Radial Lens Distortion}} + \Delta y_{\text{Decentric Lens Distortion}} + \Delta y_{\text{Atmospheric Refraction}} + \Delta y_{\text{Affine Deformations}} + \text{etc….}$

Distortion Parameters

$$\Delta x_{\text{Radial Lens Distortion}} = \bar{x}\ (k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

$$\Delta y_{\text{Radial Lens Distortion}} = \bar{y}\ (k_1 r^2 + k_2 r^4 + k_3 r^6 + ...)$$

$$\Delta x_{\text{Decentering Lens Distortion}} = (1 + p_3^2\ r^2)\ \{p_1\ (r^2 + 2\bar{x}^2) + 2p_2\ \bar{x}\ \bar{y}\}$$

$$\Delta y_{\text{Decentering Lens Distortion}} = (1 + p_3^2\ r^2)\ \{2p_1\ \bar{x}\ \bar{y} + p_2\ (r^2 + 2\bar{y}^2)\}$$

where: $r = \{(x - x_p)^2 + (y - y_p)^2\}^{0.5}$

$$\bar{x} = x - x_p$$

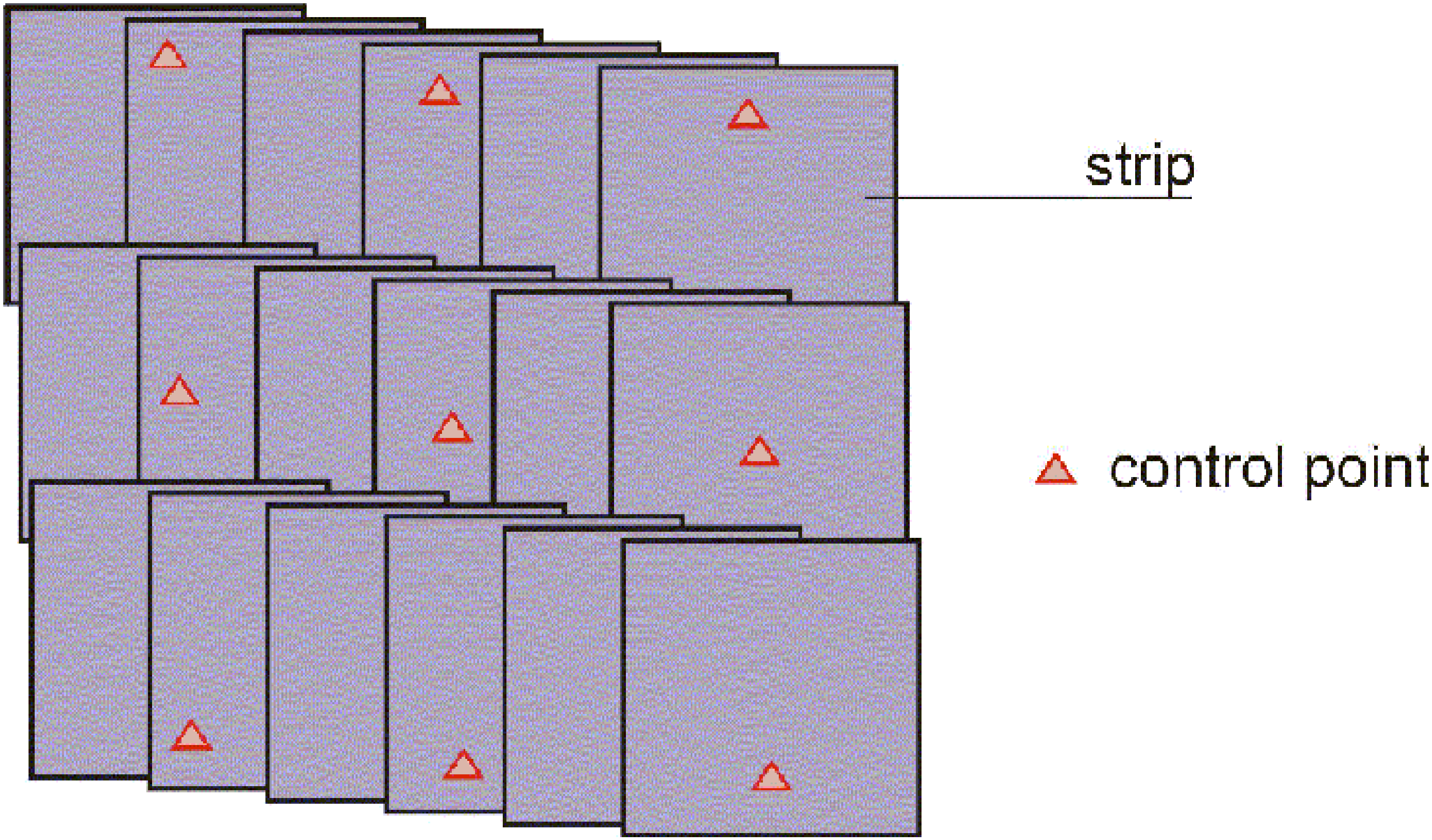
$$\bar{y} = y - y_p$$

Collinearity Equations: Involved Parameters

- Image coordinates (x_a, y_a)
- Ground coordinates (X_A, Y_A, Z_A)
- Exterior Orientation Parameters $(X_O, Y_O, Z_O, \omega, \phi, \kappa)$
- Interior Orientation Parameters:
  - x_p, y_p, c
  - Camera-related distortion parameters that compensate for deviations from the assumed perspective geometry
    - Radial lens distortion (RLD), Decentering lens distortion (DLD), Affine deformation (AD)

Bundle Block Adjustment

Bundle Block Adjustment



60% Overlap and 20% side lap

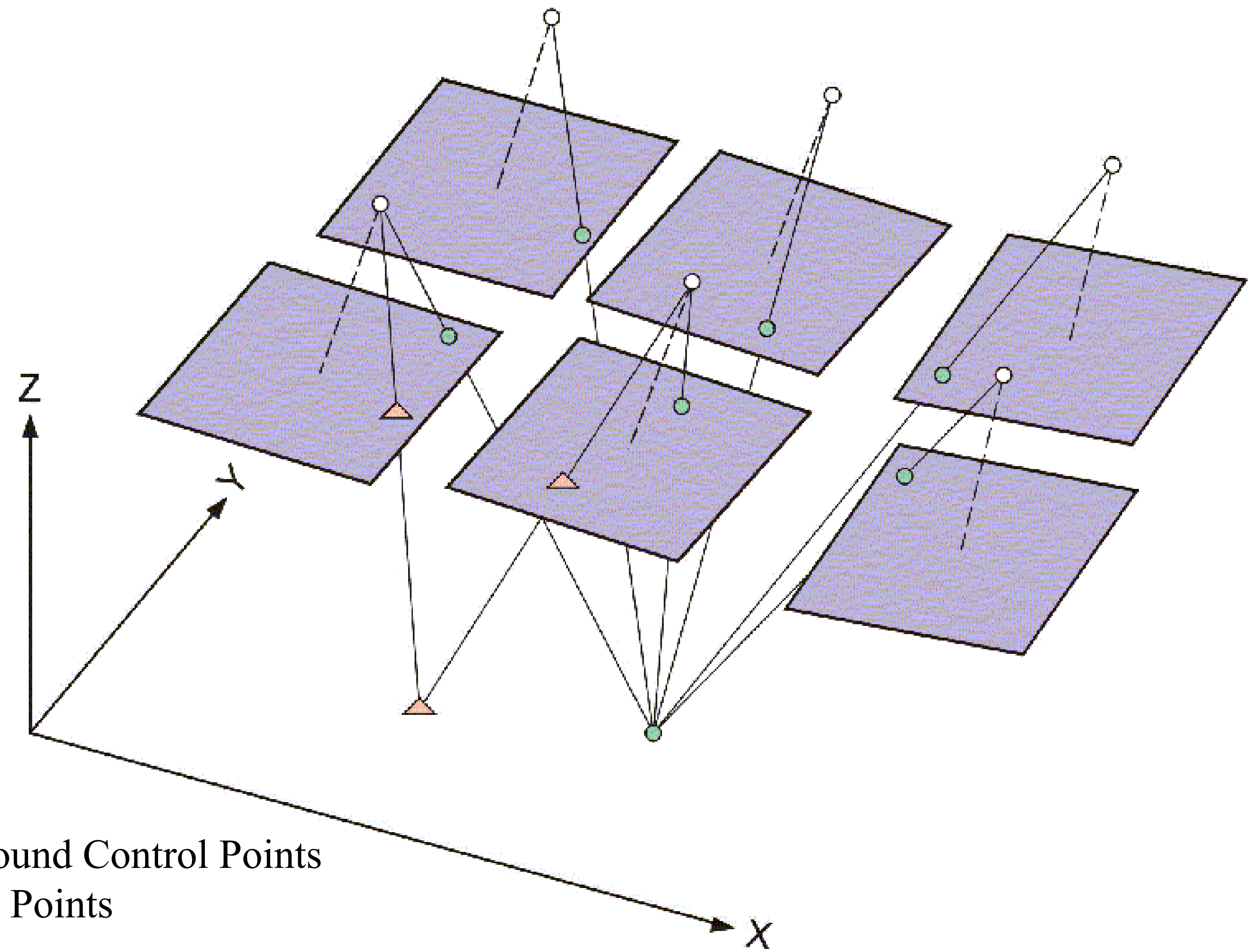
Bundle Block Adjustment

- Direct relationship between image and ground coordinates
- We measure the image coordinates in the images of the block.
- Using the collinearity equations, we can relate the image coordinates, the corresponding ground coordinates, the IOPs, and the EOPs.
- Using a simultaneous least squares adjustment, we can solve for the:
  - The ground coordinates of tie points,
  - The EOPs, and
  - The IOPs (Camera Calibration Procedure).

Bundle Block Adjustment: Concept

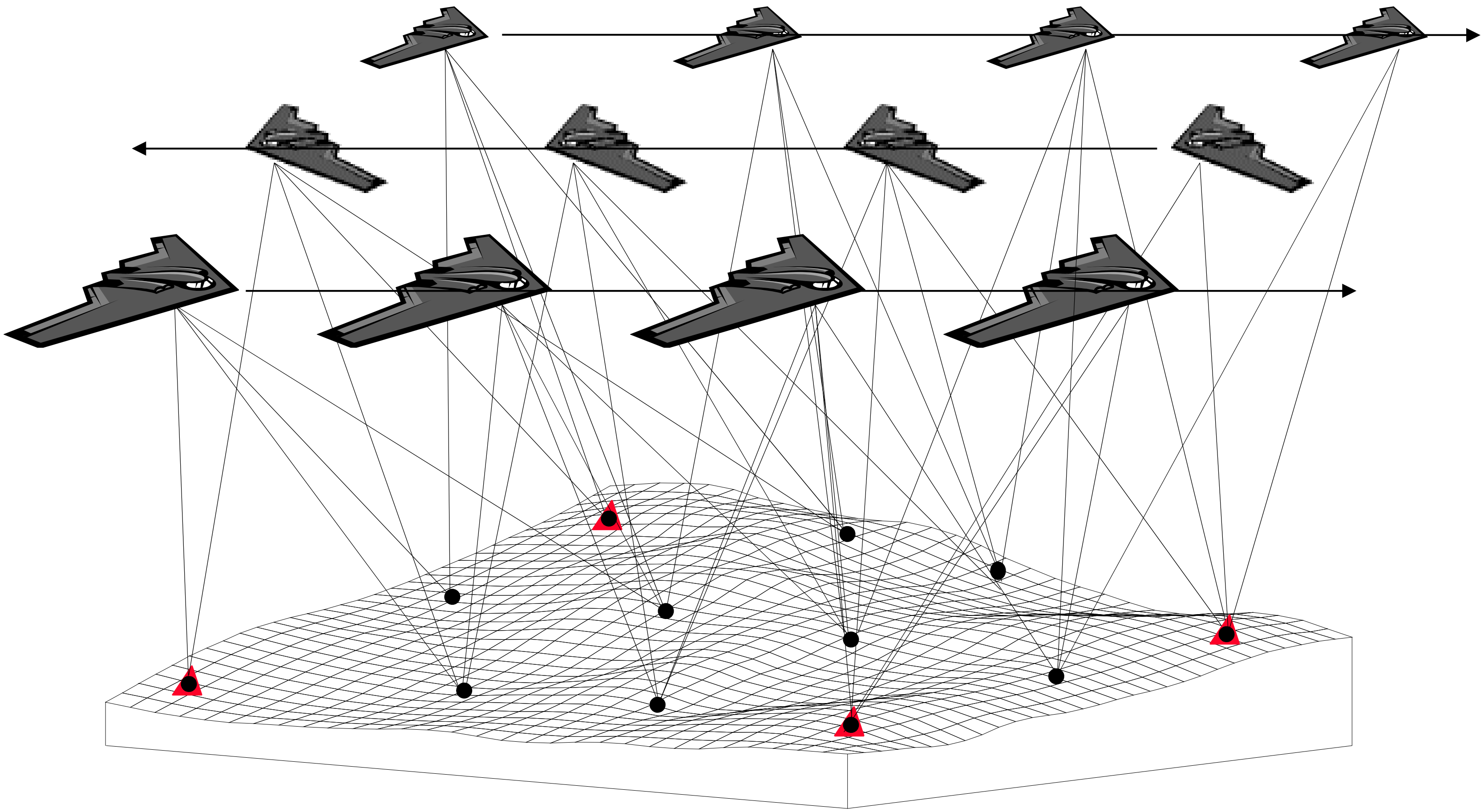
- The image coordinate measurements and the IOPs define a bundle of light rays.
- The EOPs define the position and the attitude of the bundles in space.
- During the adjustment: The bundles are rotated (ω, ϕ, κ) and shifted (X_o, Y_o, Z_o) until:
  - Conjugate light rays intersect as well as possible at the locations of object space tie points.
  - Light rays corresponding to ground control points pass through the object points as close as possible.

Bundle Block Adjustment: Concept



- ▲ Ground Control Points
- ● Tie Points

Bundle Block Adjustment: Concept



▲ Ground Control Points
● Tie Points

Least Squares Adjustment

Gauss Markov Model
Observation Equations

$$y = A\,x + e \qquad\qquad e \sim (0, \sigma_o^2 P^{-1})$$

y $\quad n \times 1$ ***observation vector***

A $\quad n \times m$ ***design matrix***

x $\quad m \times 1$ ***vector of unknowns***

e $\quad n \times 1$ ***noise contaminating the observation vector***

$\sigma_o^2 P^{-1}$ $\quad n \times n$ ***variance covariance matrix of the noise vector***

Least Squares Adjustment

$$\hat{x} = (A^T P A)^{-1} A^T P y$$

$$D\{\hat{x}\} = \sigma_o^2 \, (A^T P A)^{-1}$$

$$\tilde{e} = y - A\hat{x}$$

$$\hat{\sigma}_o^2 = (\tilde{e}^T P \tilde{e}) \, / \, (n - m)$$

Non Linear System

$Y = a(X) + e$

$a(X)$ is the non − linear function

We use Taylor Series Expansion

$$Y \approx a(X_o) + \left.\frac{\partial a}{\partial X}\right|_{X_o} (X - X_o) + e \qquad \text{(We ignore higher order terms)}$$

Where :

X_o is approximate values for the unknown parameters

$$Y - a(X_o) = \left.\frac{\partial a}{\partial X}\right|_{X_o} (X - X_o) + e$$

$y = A\,x + e$

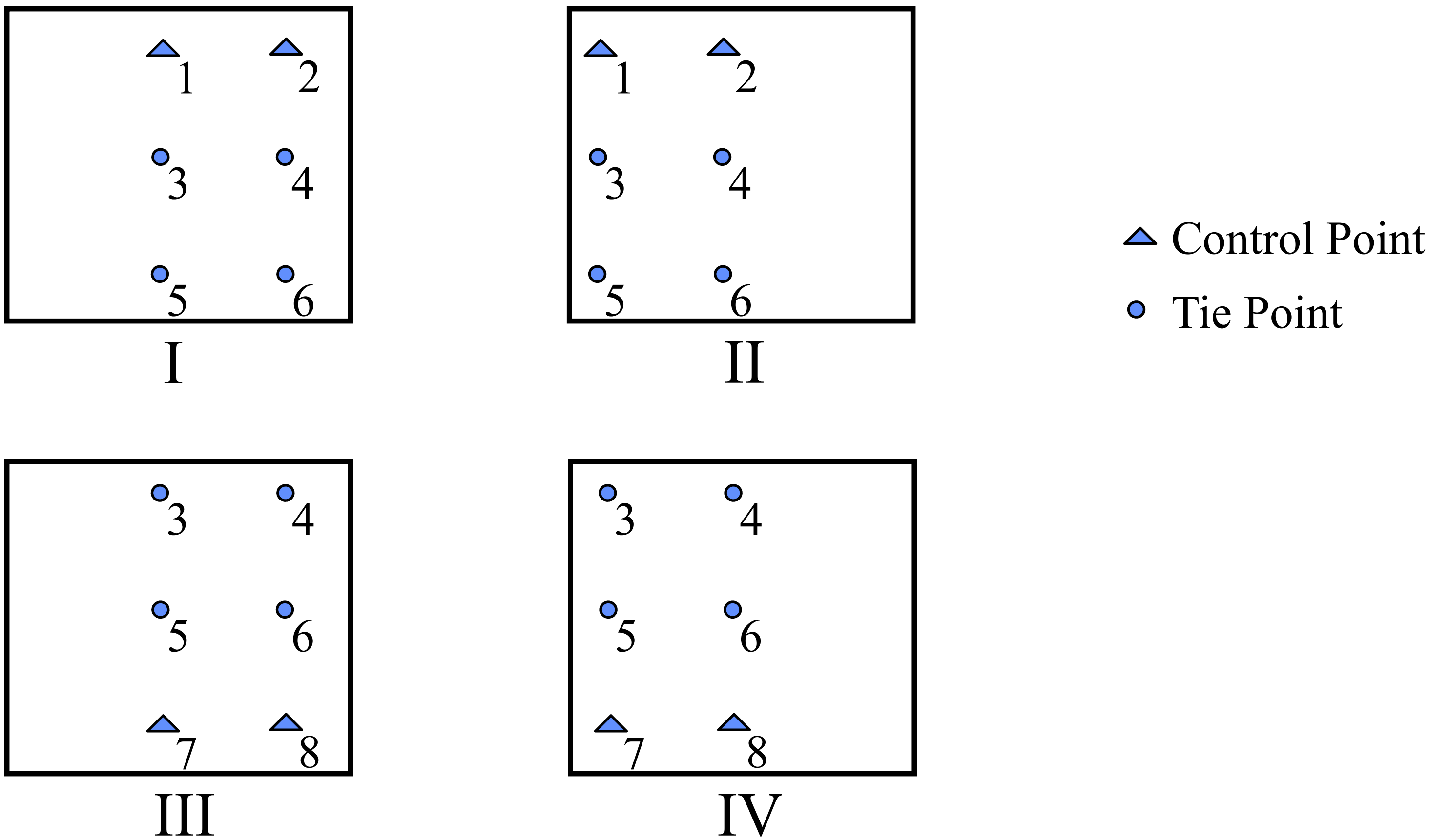
Where :

$y = Y - a(X_o)$

$$A = \left.\frac{\partial a}{\partial X}\right|_{X_o}$$

- Iterative solution for the unknown parameters
- When should we stop the iterations?

Example (4 Images in Two Strips)



Balance Between Observations & Unknowns

- Number of observations:
  - 4 x 6 x 2 = 48 observations (collinearity equations)
- Number of unknowns:
  - 4 x 6 + 3 x 4 = 36 unknowns
- Redundancy:
  - 12
- Assumptions:
  - IOPs are assumed to be known and errorless.
  - Ground coordinates of the control points are errorless.

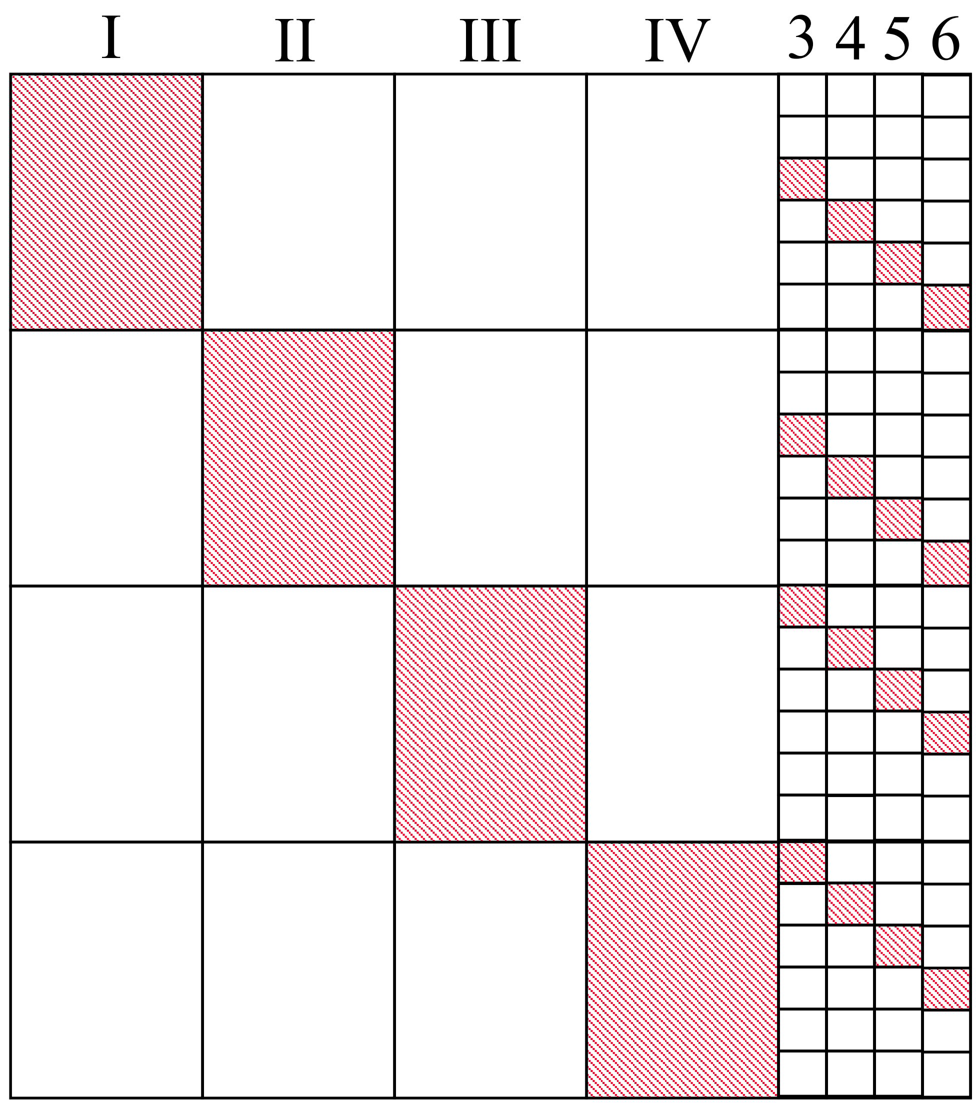
Structure of the Design Matrix (BA)

- $Y = a(X) + e \qquad e \sim (0, \sigma^2 P^{-1})$

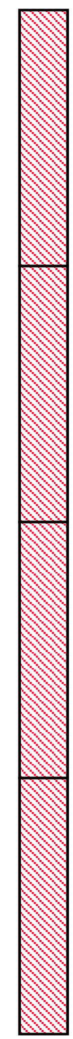
- Using approximate values for the unknown parameters (X^o) and partial derivatives, the above equations can be linearized leading to the following equations:
- $y_{48x1} = A_{48x36}\, x_{36x1} + e_{48x1} \qquad e \sim (0, \sigma^2 P^{-1})$

Structure of the Design Matrix

A =

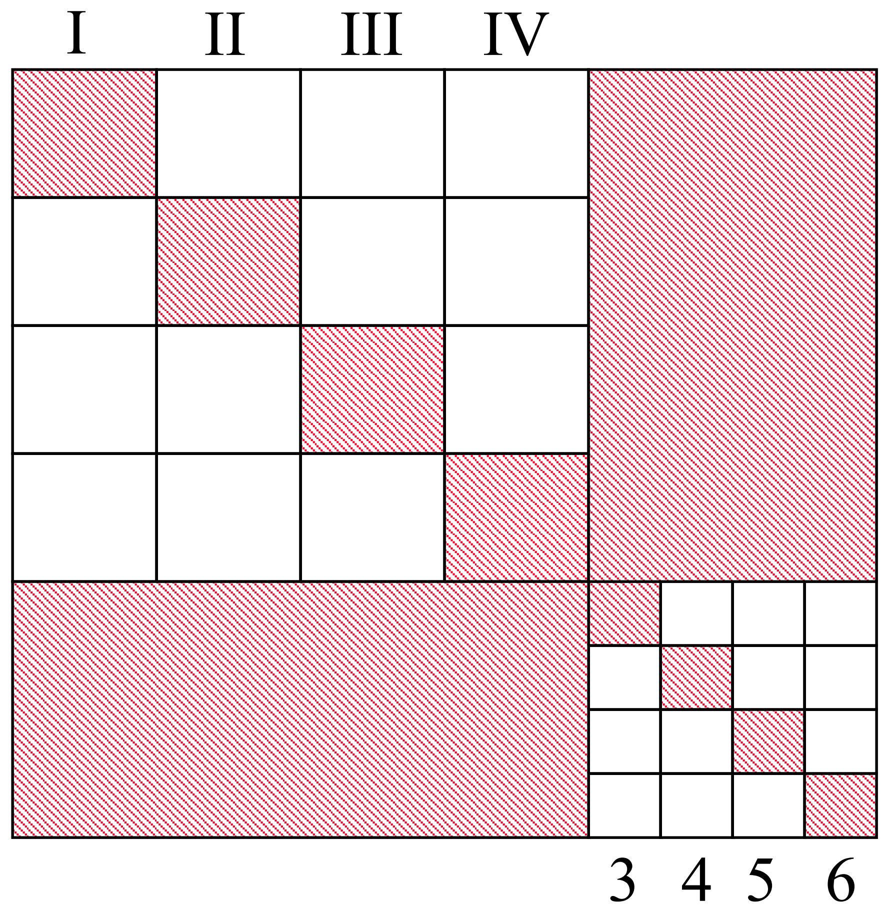


y =



Structure of the Normal Matrix

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{12}^{T} & N_{22} \end{bmatrix} =$$

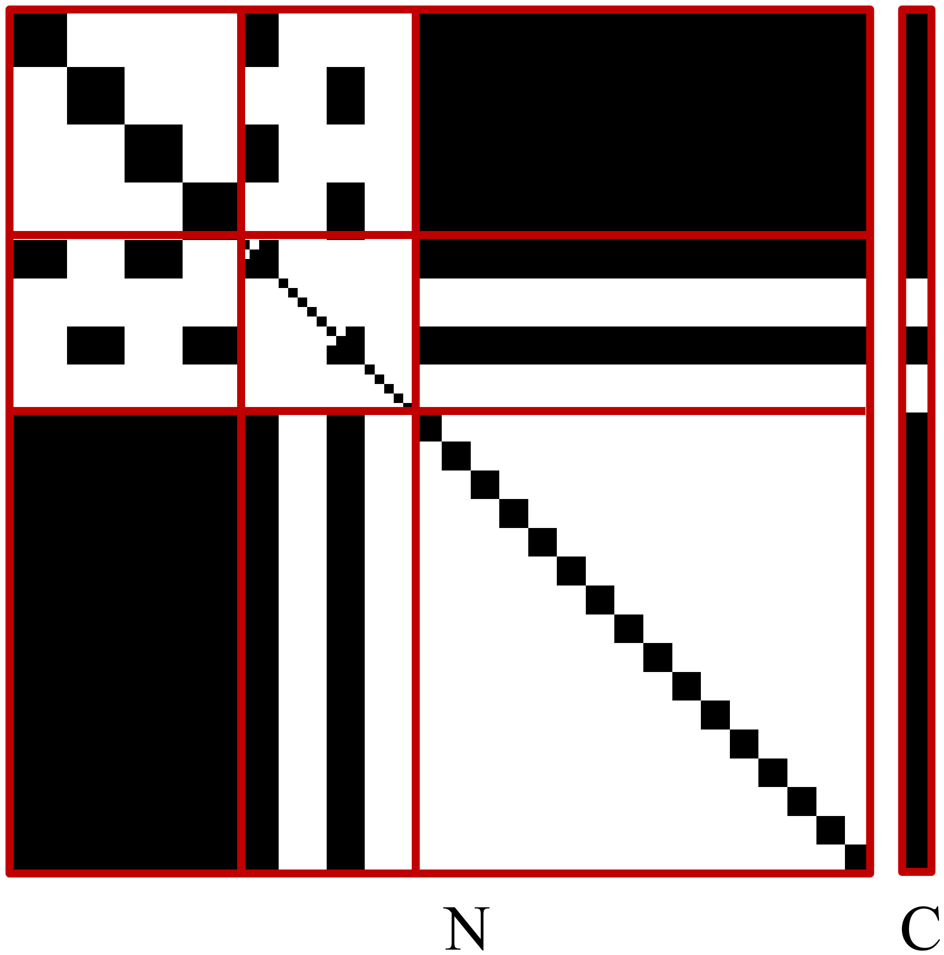


Sample Data

- 2 cameras.
- 4 images.
- 16 points.

- All the points appear in all the images
- Two images were captured by each camera

Structure of the Normal Matrix: Example



Observation Equations

$$y_{n\times 1} = A_{n\times m}\ x_{m\times 1} + e_{n\times 1} \qquad e \sim (0, \sigma_o^2\ P^{-1})$$

$$y_{n\times 1} = A_{1_{n\times 6m_1}}\ x_{1_{6m_1\times 1}} + A_{2_{n\times 3m_2}}\ x_{2_{3m_2\times 1}} + e_{n\times 1}$$

$$y_{n\times 1} = \begin{bmatrix} A_{1_{n\times 6m_1}} & A_{2_{n\times 3m_2}} \end{bmatrix} \begin{bmatrix} x_{1_{6m_1\times 1}} \\ x_{2_{3m_2\times 1}} \end{bmatrix} + e_{n\times 1}$$

- **n ≡ Number of observations (image coordinate measurements)**
- **m ≡ Number of unknowns:**
  - **m_1 ≡ Number of images ⇒ 6 m_1 (EOPs of the images)**
  - **m_2 ≡ Number of tie points ⇒ 3 m_2 (ground coordinates of tie points)**
  - **$m = 6\ m_1 + 3\ m_2$**

Normal Equation Matrix

$$N_{(6m_1+3m_2)\times(6m_1+3m_2)} = \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} P \begin{bmatrix} A_1 & A_2 \end{bmatrix}$$

$$N = \begin{bmatrix} N_{11_{6m_1\times 6m_1}} & N_{12_{6m_1\times 3m_2}} \\ N^T_{12_{3m_2\times 6m_1}} & N_{22_{3m_2\times 3m_2}} \end{bmatrix}$$

$$C_{(6m_1+3m_2)\times 1} = \begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} P\ y = \begin{bmatrix} A_1^T Py \\ A_2^T Py \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1\times 1}} \\ C_{2_{3m_2\times 1}} \end{bmatrix}$$

Normal Equation Matrix

- N_{11} is a block diagonal matrix with 6x6 sub-blocks along the diagonal.
- N_{22} is a block diagonal matrix with 3x3 sub-blocks along the diagonal.

$$\begin{bmatrix} N_{11_{6m_1\times 6m_1}} & N_{12_{6m_1\times 3m_2}} \\ N^T_{12_{3m_2\times 6m_1}} & N_{22_{3m_2\times 3m_2}} \end{bmatrix} \begin{bmatrix} \hat{x}_{1_{6m_1\times 1}} \\ \hat{x}_{2_{3m_2\times 1}} \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1\times 1}} \\ C_{2_{3m_2\times 1}} \end{bmatrix}$$

- Question: Under which circumstances will we deviate from this structure?

Reduction of the Normal Equation Matrix

$$N_{11_{6m_1\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}} + N_{12_{6m_1\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}} = C_{1_{6m_1\times 1}}$$

$$N^T_{12_{3m_2\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}} + N_{22_{3m_2\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

- Solving for x_2 first:

$$N^T_{12_{3m_2\times 6m_1}}\left(N^{-1}_{11_{6m_1\times 6m_1}}\ C_{1_{6m_1\times 1}} - N^{-1}_{11_{6m_1\times 6m_1}}\ N_{12_{6m_1\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}}\right) + N_{22_{3m_2\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

$$\hat{x}_{2_{3m_2\times 1}} = \left(N_{22_{3m_2\times 3m_2}} - N^T_{12_{3m_2\times 6m_1}}\ N^{-1}_{11_{6m_1\times 6m_1}}\ N_{12_{6m_1\times 3m_2}}\right)^{-1}\left(C_{2_{3m_2\times 1}} - N^T_{12_{3m_2\times 6m_1}}\ N^{-1}_{11_{6m_1\times 6m_1}}\ C_{1_{6m_1\times 1}}\right)$$

$$\hat{x}_{1_{6m_1\times 1}} = \left(N^{-1}_{11_{6m_1\times 6m_1}}\ C_{1_{6m_1\times 1}} - N^{-1}_{11_{6m_1\times 6m_1}}\ N_{12_{6m_1\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}}\right)$$

- $3m_2 < 6m_1$
- Remember: N_{11} is a block diagonal matrix with 6x6 sub-blocks along the diagonal.

Reduction of the Normal Equation Matrix

$$N_{11_{6m_1\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}} + N_{12_{6m_1\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}} = C_{1_{6m_1\times 1}}$$

$$N^T_{12_{3m_2\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}} + N_{22_{3m_2\times 3m_2}}\ \hat{x}_{2_{3m_2\times 1}} = C_{2_{3m_2\times 1}}$$

- Solving for x_1 first:

$$N_{11_{6m_1\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}} + N_{12_{3m_2\times 3m_2}}\ \left(N^{-1}_{22_{3m_2\times 3m_2}}\ C_{2_{3m_2\times 1}} - N^{-1}_{22_{3m_2\times 3m_2}}\ N^T_{12_{3m_2\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}}\right) = C_{1_{6m_1\times 1}}$$

$$\hat{x}_{1_{6m_1\times 1}} = \left(N_{11_{6m1\times 6m1}} - N_{12_{6m_1\times 3m_2}}\ N^{-1}_{22_{3m_2\times 3m_2}}\ N^T_{12_{3m_2\times 6m_1}}\right)^{-1} \left(C_{1_{6m_1\times 1}} - N_{12_{6m_1\times 3m_2}}\ N^{-1}_{22_{3m_2\times 2m_2}}\ C_{2_{3m_2\times 1}}\right)$$

$$\hat{x}_{2_{3m_2\times 1}} = \left(N^{-1}_{22_{3m_2\times 3m_2}}\ C_{2_{3m_2\times 1}} - N^{-1}_{22_{3m_2\times 3m_2}}\ N^T_{12_{3m_2\times 6m_1}}\ \hat{x}_{1_{6m_1\times 1}}\right)$$

- $6m_1 < 3m_2$
- Remember: N_{22} is a block diagonal matrix with 3x3 sub-blocks along the diagonal.

Reduction of the Normal Equation Matrix

- Variance covariance matrix of the estimated parameters:

$$D\{\hat{x}_{1_{6m_1\times 1}}\} = \sigma_o^2 \left(N_{11_{6m_1\times 6m_1}} - N_{12_{6m_1\times 3m_2}} \; N^{-1}_{22_{3m_2\times 3m_2}} \; N^{T}_{12_{3m_2\times 6m_1}}\right)^{-1}$$

$$D\{\hat{x}_{2_{3m_2\times 1}}\} = \sigma_o^2 \left(N_{22_{3m_2\times 3m_2}} - N^{T}_{12_{3m_2\times 6m_1}} \; N^{-1}_{11_{6m_1\times 6m_1}} \; N_{12_{6m_1\times 3m_2}}\right)^{-1}$$

Building the Normal Equation Matrix

- We would like to investigate the possibility of sequentially building up the normal equation matrix without fully building the design matrix.
- (x_{ij}, y_{ij}) image coordinates of the i^{th} point in the j^{th} image

$$y_{2\times1_{ij}} = A_{1_{2\times6_{ij}}}\ x_{1_{6\times1_j}} + A_{2_{2\times3_{ij}}}\ x_{2_{3\times1_i}} + e_{2\times1_{ij}}$$

$$y_{2\times1_{ij}} = \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} + e_{2\times1_{ij}}$$

Normal Equation Matrix

$$y_{2\times1_{ij}} = \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} + e_{2\times1_{ij}}$$

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} \\ A^T_{2_{3\times2_{ij}}} \end{bmatrix} P_{ij} \begin{bmatrix} A_{1_{2\times6_{ij}}} & A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} \\ A^T_{2_{3\times2_{ij}}} \end{bmatrix} P_{ij} \; y_{2\times1_{ij}}$$

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{1_{6\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{2_{3\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \end{bmatrix}$$

Normal Equation Matrix

$$\begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{1_{6\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} A_{1_{2\times6_{ij}}} & A^T_{2_{3\times2_{ij}}} P_{ij} A_{2_{2\times3_{ij}}} \end{bmatrix} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix} = \begin{bmatrix} A^T_{1_{6\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \\ A^T_{2_{3\times2_{ij}}} P_{ij} \; y_{2\times1_{ij}} \end{bmatrix}$$

$$\begin{bmatrix} N_{11_{ij}} & N_{12_{ij}} \\ N^T_{12_{ij}} & N_{22_{ij}} \end{bmatrix}_{9\times9} \begin{bmatrix} x_{1_{6\times1_j}} \\ x_{2_{3\times1_i}} \end{bmatrix}_{9\times1} = \begin{bmatrix} C_{1_{ij}} \\ C_{2_{ij}} \end{bmatrix}_{9\times1}$$

- Note: We cannot solve this matrix for the:
  - The Exterior Orientation Parameters of the j^{th} image, and
  - The ground coordinates of the i^{th} point.

Normal Equation Matrix

$$\begin{bmatrix} N_{11_{6m_1 \times 6m_1}} & N_{12_{6m_1 \times 3m_2}} \\ N_{12_{3m_2 \times 6m_1}}^T & N_{22_{3m_2 \times 3m_2}} \end{bmatrix} \begin{bmatrix} \hat{x}_{1_{6m_1 \times 1}} \\ \hat{x}_{2_{3m_2 \times 1}} \end{bmatrix} = \begin{bmatrix} C_{1_{6m_1 \times 1}} \\ C_{2_{3m_2 \times 1}} \end{bmatrix}$$

- Question: How can we sequentially build the above matrices?
- Assumption: All the points are common to all the images.

N_{11} - Matrix

$$N_{11_{(6m_1 \times 6m_1)}} = \begin{bmatrix} \sum_{i=1}^{m_2} N_{11_{i1}} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \sum_{i=1}^{m_2} N_{11_{i2}} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \sum_{i=1}^{m_2} N_{11_{i3}} & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \sum_{i=1}^{m_2} N_{11_{im_1}} \end{bmatrix}$$

- If all the points are not common to all the images:
    - The summation should be carried over all the points that appear in the image under consideration.

N_{22} - Matrix

$$N_{22_{(3m_2 \times 3m_2)}} = \begin{bmatrix} \sum_{j=1}^{m_1} N_{22_{1j}} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \sum_{j=1}^{m_1} N_{22_{2j}} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \sum_{j=1}^{m_1} N_{22_{3j}} & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \sum_{j=1}^{m_1} N_{22_{m_2 j}} \end{bmatrix}$$

- If all the points are not common to all the images:
  - The summation should be carried over all the images within which the point under consideration appears.

N_{12} - Matrix

$$N_{12_{(6m_1 \times 3m_2)}} = \begin{bmatrix} N_{12_{11}} & N_{12_{21}} & N_{12_{31}} & \cdots & \cdots & N_{12_{m_2 1}} \\ N_{12_{12}} & N_{12_{22}} & N_{12_{32}} & \cdots & \cdots & N_{12_{m_2 2}} \\ N_{12_{13}} & N_{12_{23}} & N_{12_{23}} & \cdots & \cdots & N_{12_{m_2 3}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ N_{12_{1m_1}} & N_{12_{2m_1}} & N_{12_{3m_1}} & \cdots & \cdots & N_{12_{m_2 m_1}} \end{bmatrix}$$

- If point "i" does not appear in image "j":
  - $(N_{12})_{ij} = 0$

C - Matrix

$$C_{1_{6m_1\times 1}} = \begin{bmatrix} \sum_{i=1}^{m_2} C_{1_{i1}} \\ \sum_{i=1}^{m_2} C_{1_{i2}} \\ \sum_{i=1}^{m_2} C_{1_{i3}} \\ \vdots \\ \vdots \\ \sum_{i=1}^{m_2} C_{1_{im_1}} \end{bmatrix} \qquad C_{2_{3m_2\times 1}} = \begin{bmatrix} \sum_{j=1}^{m_1} C_{2_{1j}} \\ \sum_{j=1}^{m_1} C_{2_{2j}} \\ \sum_{j=1}^{m_1} C_{2_{3j}} \\ \vdots \\ \vdots \\ \sum_{j=1}^{m_1} C_{2_{m_2 j}} \end{bmatrix}$$

- Once again, we assumed that all the points are common to all the images.

Precision of Bundle Block Adjustment

- The precision of the estimated EOPs as well as the ground coordinates of tie points can be obtained by the product of:
  - The estimated variance component, and
  - The inverse of the normal equation matrix (cofactor matrix).
- The precision depends on the following factors:
  - Geometric configuration of the image block
    - Base-Height ratio
  - Image scale
  - Image coordinate measurement precision

Precision of Bundle Block Adjustment

- Precision of a single model: If we have
  - Bundle block adjustment with additional parameters that compensate for various distortions
  - Regular blocks with 60%overlap and 20% side lap
  - Signalized targets

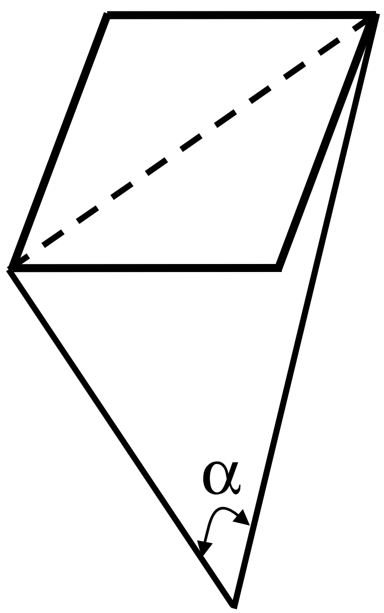
$\sigma_{XY} = \pm 3 \mu m$

$\sigma_Z = \pm 0.003\%$ of the camera principal distance (NA and WA cameras)

$\sigma_Z = \pm 0.004\%$ of the camera principal distance (SWA cameras)

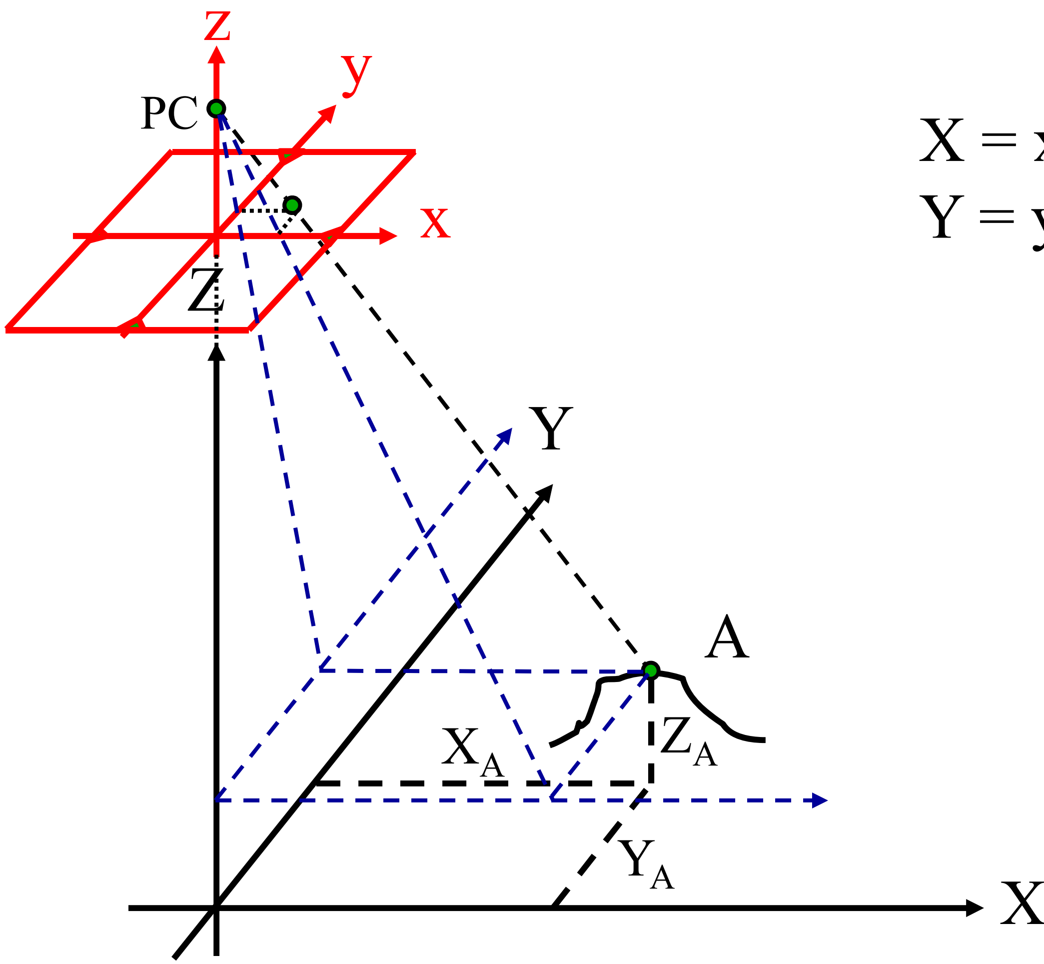
These precision values are given in the image space

Camera Classification



- α < 75° Normal angle camera (NA)
- 100° > α > 75° Wide angle camera (WA)
- α > 100° Super wide angle camera (SWA)

Precision of Bundle Block Adjustment



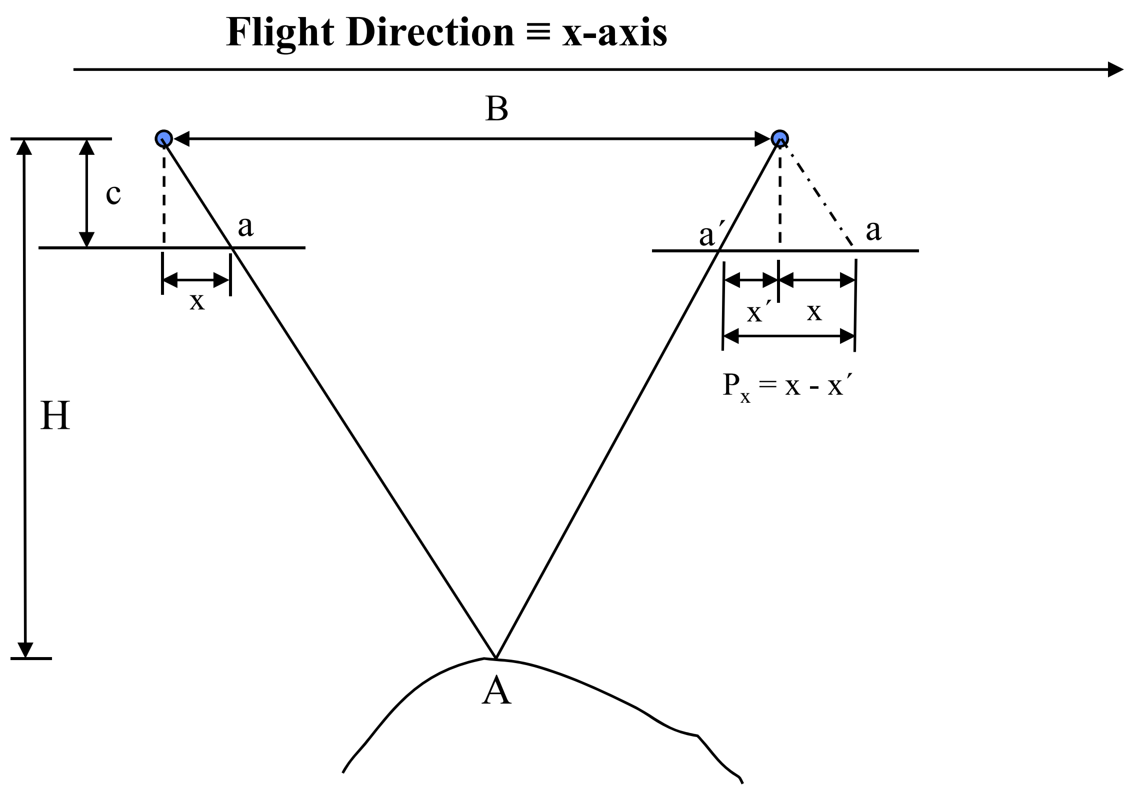
$X = x * Z / c$
$Y = y * Z / c$

$$\sigma_X = \frac{Z}{c}\,\sigma_x$$

$$\sigma_Y = \frac{Z}{c}\,\sigma_y$$

Precision of Bundle Block Adjustment

Vertical Precision

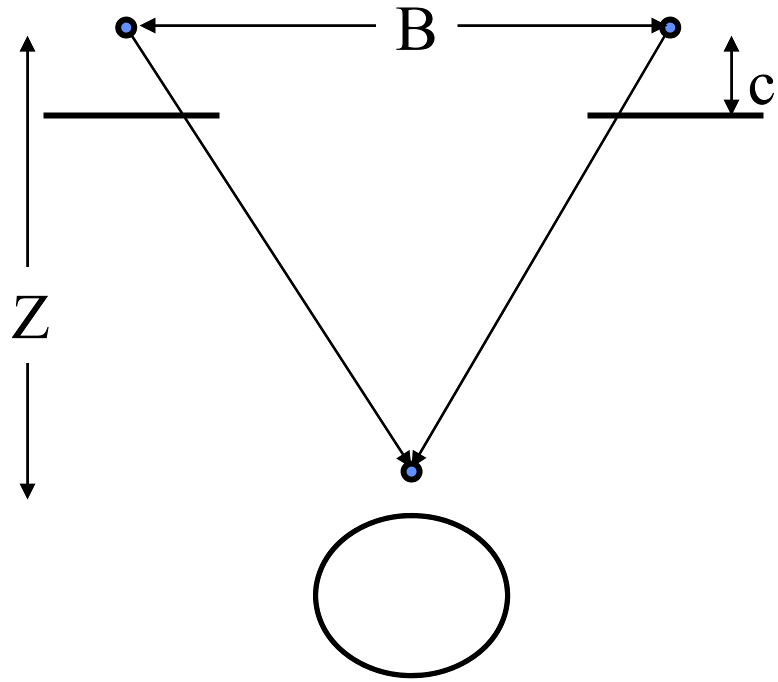


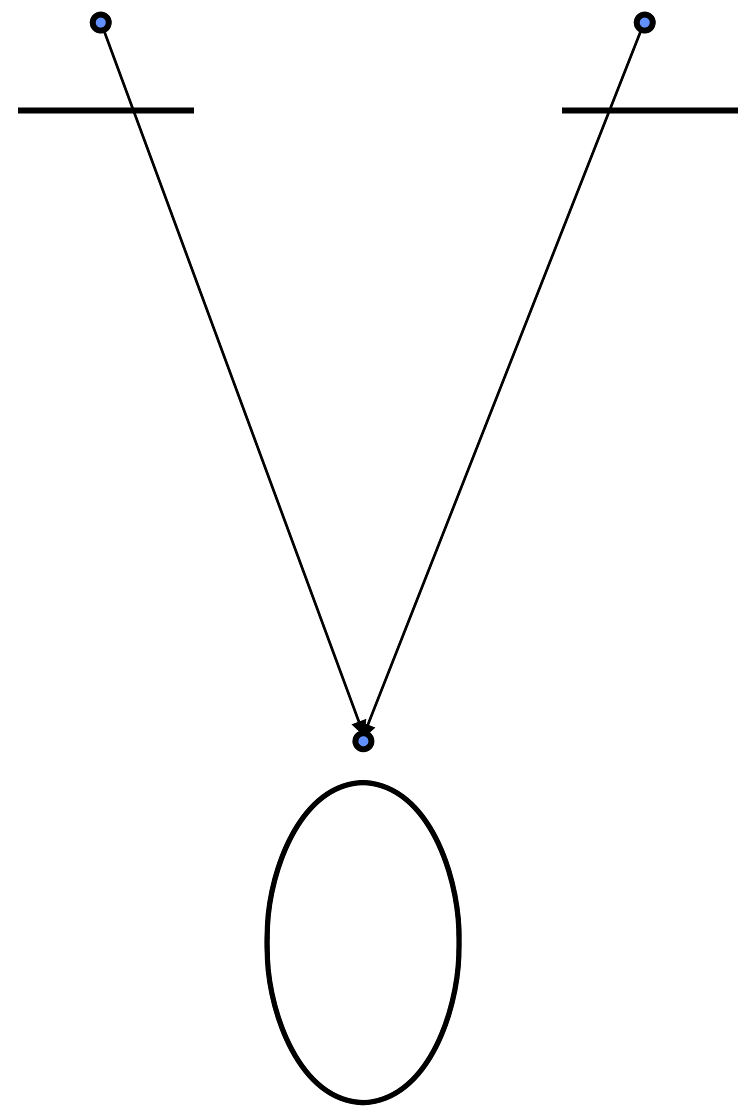
$P_x / B = c / H$
$H = B\, c / P_x$

Precision of Bundle Block Adjustment

Vertical Precision

$$\sigma_Z = \frac{Z}{c}\,\frac{Z}{B}\,\sigma_{p_x}$$

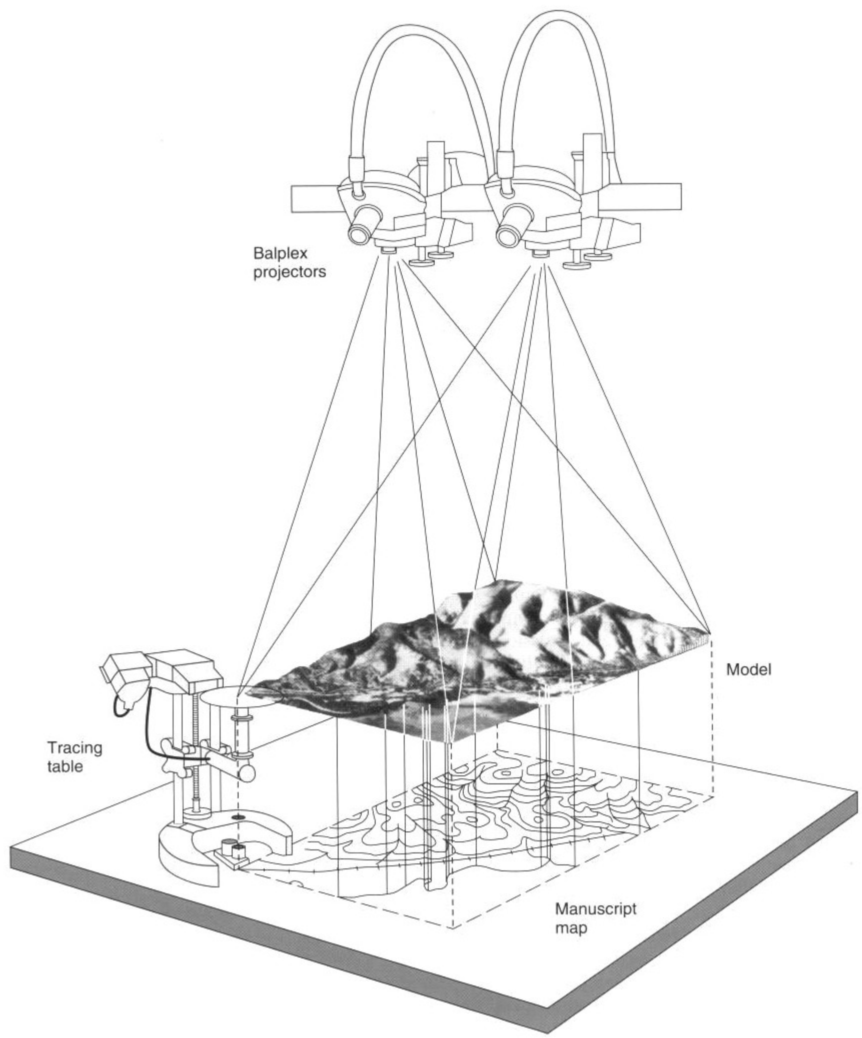




Advantages of Bundle Block Adjustment

- Most accurate triangulation technique since we have direct transformation between image and ground coordinates.
- Straight forward to include parameters that compensate for various deviations from the collinearity model.
- Straight forward to include additional observations:
  - GNSS/INS observations at the exposure stations
  - Object space distances
- Can be used for normal, convergent, aerial, and close range imagery
- After the adjustment, the EOPs can be set on analog and analytical plotters as well as digital photogrammetric workstations for compilation purposes.

Photogrammetric Compilation



Disadvantages of Bundle Block Adjustment

- Model is non linear: approximations as well as partial derivatives are needed.
- Requires computer intensive computations
- Analog instruments cannot be used (they cannot measure image coordinate measurements).
- The adjustment cannot be separated into planimetric and vertical adjustment.

Bundle Adjustment: Final Remarks

- Elementary Unit: Images
- Measurements: Image coordinates
- Mathematical model: Collinearity equations
- Instruments: Comparators, analytical plotters, and Digital Photogrammetric Workstations (DPW)
- Required computer power: Very large
- Expected accuracy: High

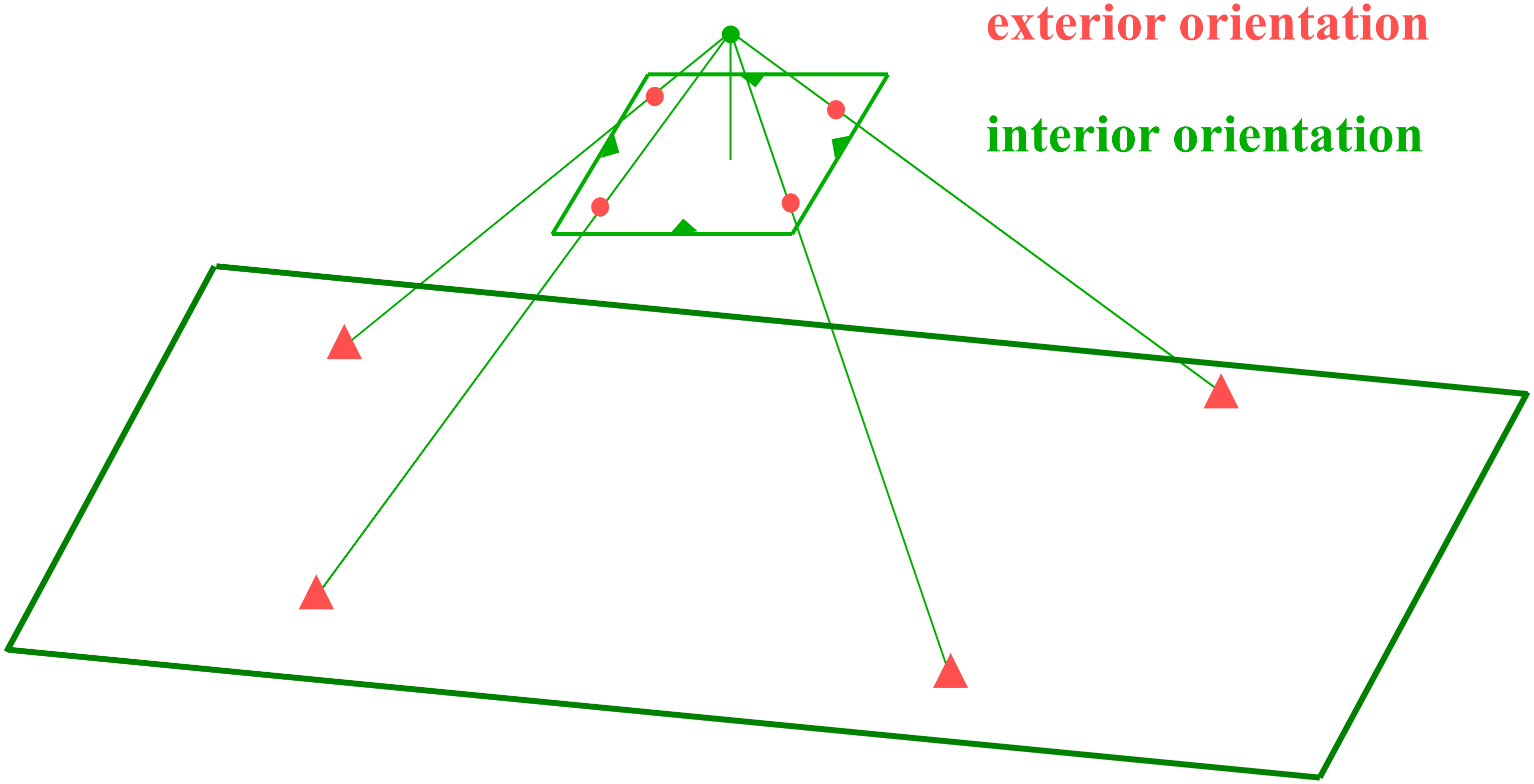
Special Cases

- Resection
- Intersection
- Stereo-pair orientation
- Relative orientation (Discussed in Chapter 8)
  - Dependent Relative Orientation (DRO), and
  - Independent Relative Orientation

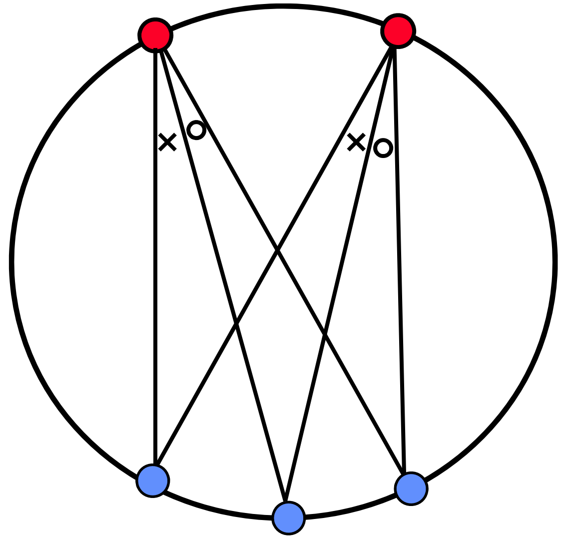
Resection

- We are dealing with one image.
- We would like to determine the EOPs of this image using GCPs.
- Q: What is the minimum GCP requirements?
  - At least 3 non-collinear GCPs are required to estimate the 6 EOPs.
  - At least 5 non-collinear (well distributed in 3-D) GCPs are required to estimate the 6 EOPs and the 3 IOPs (x_p, y_p, c).
- Critical surface:
  - The GCPs and the perspective center lie on a common cylinder.

Resection



Resection - Critical Surface

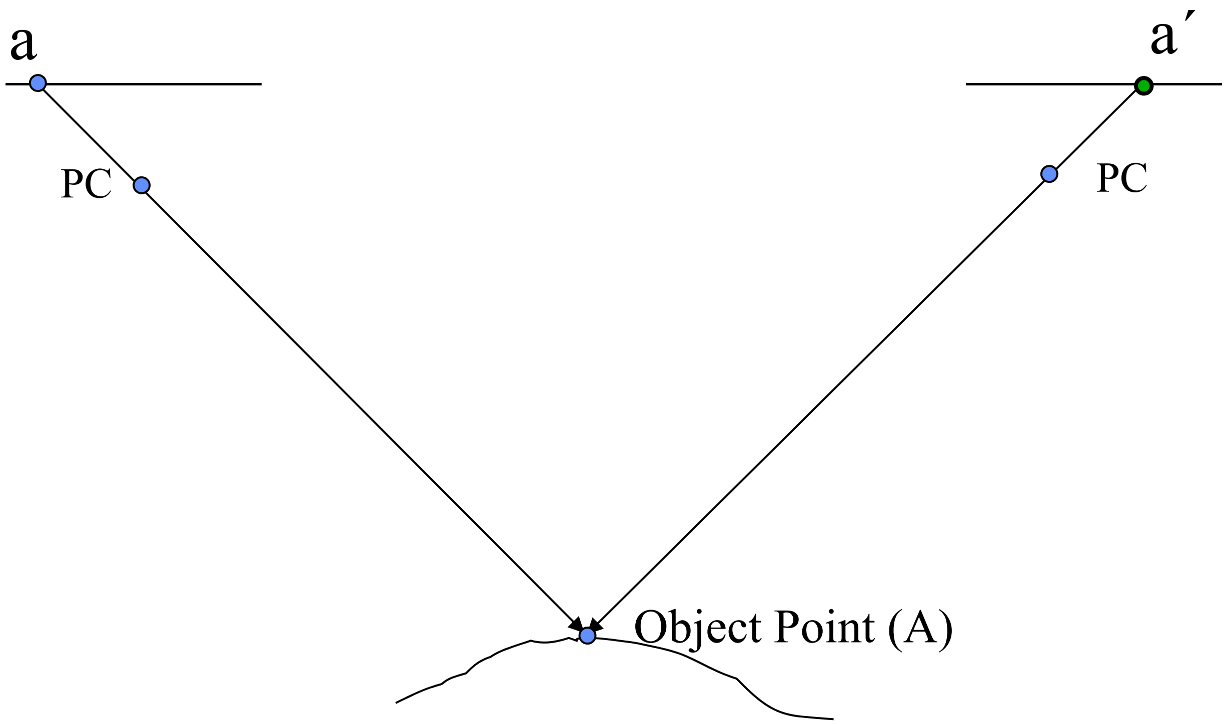


- Question: Which parameter of the EOPs cannot be determined?

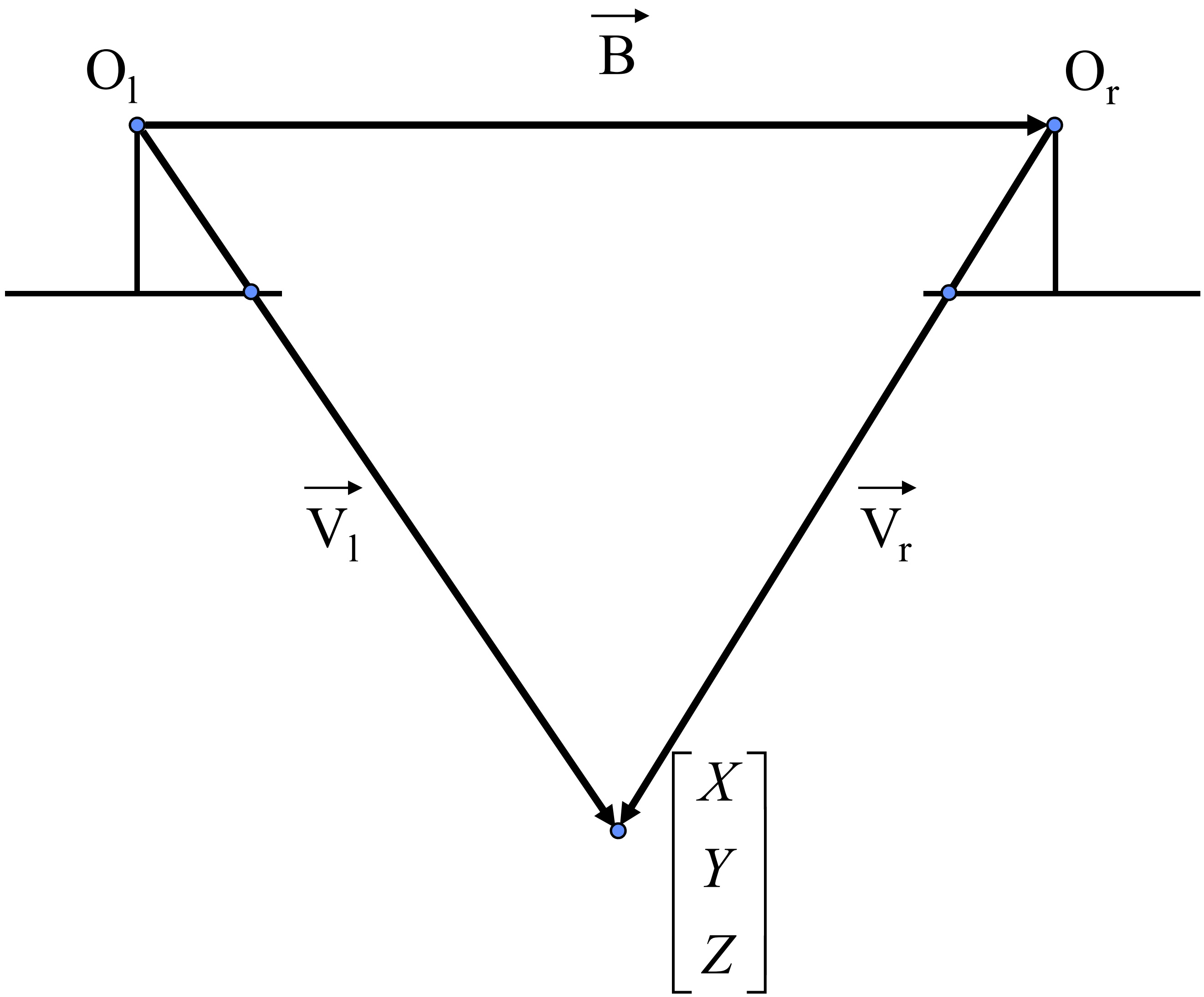
Intersection

- We are dealing with two images.
- The EOPs of these images are available.
- The IOPs of the involved camera(s) are also available.
- We want to estimate the ground coordinates of tie points in the overlap area.
- For each tie point, we have:
  - 4 Observation equations
  - 3 Unknowns
  - Redundancy = 1
- Non-linear model: approximations are needed

Intersection



Intersection: Linear Model



Intersection: Linear Model

$$\vec{B} = \begin{bmatrix} X_{O_r} - X_{O_l} \\ Y_{O_r} - Y_{O_l} \\ Z_{O_r} - Z_{O_l} \end{bmatrix}$$

- These vectors are given w.r.t. the ground coordinate system.

$$\vec{V}_l = \lambda \; R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix}$$

$$\vec{V}_r = \mu \; R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

Intersection: Linear Model

$$\vec{V}_l = \vec{B} + \vec{V}_r$$

$$\begin{bmatrix} X_{o_r} - X_{o_l} \\ Y_{o_r} - Y_{o_l} \\ Z_{o_r} - Z_{o_l} \end{bmatrix} = \lambda \, R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix} - \mu \, R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

- Three equations in two unknowns (λ, μ).
- They are linear equations.

Intersection: Linear Model

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O_l} \\ Y_{O_l} \\ Z_{O_l} \end{bmatrix} + \lambda \, R_{(\omega_l, \phi_l, \kappa_l)} \begin{bmatrix} x_l - x_p \\ y_l - y_p \\ -c \end{bmatrix}$$

Or:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_{O_r} \\ Y_{O_r} \\ Z_{O_r} \end{bmatrix} + \mu \, R_{(\omega_r, \phi_r, \kappa_r)} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ -c \end{bmatrix}$$

Stereo-pair Orientation

- Given:
  - Stereo-pair: two images with at least 50% overlap
  - Image coordinates of some tie points
  - Image and ground coordinates of control points
- Required:
  - The ground coordinates of the tie points
  - The EOPs of the involved images
- Mini-Bundle Adjustment Procedure

Stereo-pair Orientation

- Example:
  - Given:
    - 1 Stereo-pair
    - 20 tie points
    - No ground control points
  - Question:
    - Can we estimate the ground coordinates of the tie points as well as the exterior orientation parameters of that stereo-pair?
  - Answer:
    - NO

Summary

- Photogrammetry: Definition and applications
- Photogrammetric tools:
  - Rotation matrices
  - Photogrammetric orientation: interior and exterior orientation
  - Collinearity equations/conditions
- Photogrammetric bundle adjustment
  - Structure of the design and normal matrices
- Special cases:
  - Resection, intersection, and stereo-pair orientation