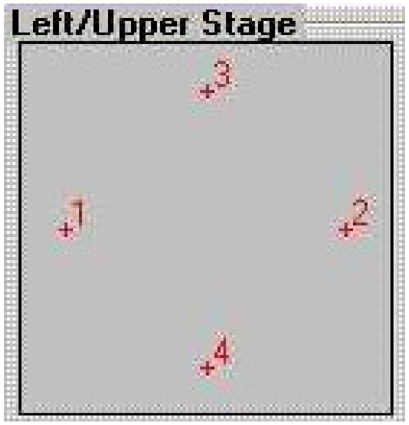
Numerical Example

Comparator-to-Image Coordinate Transformation

Affine Transformation

Reduction of Comparator Coordinates



Given:
Comparator
Aerial images (left & right)
Image coordinates of Fiducial Marks (CCC)

Fiducial marks' measurements using the comparator for the given stereo pair left & right:

Left

Point	Given Image Coordinates, mm (CCC)		Measured Comparator Coordinates, mm	
	x	y	x′	y′
1	-113.026	-0.02	7.256	120.694
2	113.002	-0.028	233.322	119.212
3	-0.006	112.993	121.043	233.005
4	-0.009	-113.045	119.542	6.897

Right

Point	Given Image Coordinates, mm (CCC)		Measured Comparator Coordinates, mm	
	x	y	x′	y′
1	-113.026	-0.02	10.964	119.440
2	113.002	-0.028	237.032	120.11
3	-0.006	112.993	123.676	232.826
4	-0.009	-113.045	124.328	6.721

Reduction of Comparator Coordinates

Affine transformation parameters:

The affine model:

$x = a_0 + a_1 x' + a_2 y'$

$y = b_0 + b_1 x' + b_2 y'$

Measure two sets of coordinates for each of the four Fiducial marks.

For each of these measurements, write two equations according to the above model.

Since we have eight equations in six unknowns, least squares adjustment should be performed.

Observation Equation Model:

$y = A x + e$

where,

y: vector of observations, A: the design matrix, x : vector of parameters, e: vector of errors

Balance of observations and parameters:

Number of observations, n = 4 points x 2 coordinates/point = 8

Number of parameters, m = 6 (a_0, a_1, a_2, b_0, b_1, b_2)

Redundancy = n – m = 8 – 6 = 2

Reduction of Comparator Coordinates

Left image

Point	Image Coordinates, mm		Comparator Coordinates, mm	
	x	y	x`	y`
1	-113.026	-0.02	7.256	120.694
2	113.002	-0.028	233.322	119.212
3	-0.006	112.993	121.043	233.005
4	-0.009	-113.045	119.542	6.897

$$x = a_0 + a_1\,x' + a_2\,y' + b_0(0) + b_1\,(0) + b_2\,(0)$$

$$y = a_0(0) + a_1\,(0) + a_2\,(0) + b_0 + b_1\,x' + b_2\,y'$$

Applying the equations for each point:

$$y = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}_{8\times1} = \begin{bmatrix} -113.026 \\ -0.02 \\ 113.002 \\ -0.028 \\ -0.006 \\ 112.993 \\ -0.009 \\ -113.045 \end{bmatrix}_{8\times1} = \begin{bmatrix} 1 & 7.256 & 120.694 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 7.256 & 120.694 \\ 1 & 233.322 & 119.212 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 233.322 & 119.212 \\ 1 & 121.043 & 233.005 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 121.043 & 233.005 \\ 1 & 119.542 & 6.897 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 119.542 & 6.897 \end{bmatrix}_{8\times6} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{6\times1} + \begin{bmatrix} e_{x_1} \\ e_{y_1} \\ e_{x_2} \\ e_{y_2} \\ e_{x_3} \\ e_{y_3} \\ e_{x_4} \\ e_{y_4} \end{bmatrix}_{8\times1}$$

Same is also done for the right image.

Reduction of Comparator Coordinates

Left

$$y = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}_{8\times1} = \begin{bmatrix} -113.026 \\ -0.02 \\ 113.002 \\ -0.028 \\ -0.006 \\ 112.993 \\ -0.009 \\ -113.045 \end{bmatrix}_{8\times1} = \begin{bmatrix} 1 & 7.256 & 120.694 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 7.256 & 120.694 \\ 1 & 233.322 & 119.212 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 233.322 & 119.212 \\ 1 & 121.043 & 233.005 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 121.043 & 233.005 \\ 1 & 119.542 & 6.897 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 119.542 & 6.897 \end{bmatrix}_{8\times6} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{6\times1} + \begin{bmatrix} e_{x_1} \\ e_{y_1} \\ e_{x_2} \\ e_{y_2} \\ e_{x_3} \\ e_{y_3} \\ e_{x_4} \\ e_{y_4} \end{bmatrix}_{8\times1}$$

Right

$$y = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}_{8\times1} = \begin{bmatrix} -113.026 \\ -0.02 \\ 113.002 \\ -0.028 \\ -0.006 \\ 112.993 \\ -0.009 \\ -113.045 \end{bmatrix}_{8\times1} = \begin{bmatrix} 1 & 10.964 & 119.440 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 10.964 & 119.440 \\ 1 & 237.032 & 120.111 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 237.032 & 120.111 \\ 1 & 123.676 & 232.826 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 123.676 & 232.826 \\ 1 & 124.328 & 6.721 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 124.328 & 6.721 \end{bmatrix}_{8\times6} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{6\times1} + \begin{bmatrix} e_{x_1} \\ e_{y_1} \\ e_{x_2} \\ e_{y_2} \\ e_{x_3} \\ e_{y_3} \\ e_{x_4} \\ e_{y_4} \end{bmatrix}_{8\times1}$$

Reduction of Comparator Coordinates

The normal equations system:

N x = C

where, $N = A^T PA$, $C = A^T P y$,

with $P = [I]_8$, the above reduces to:

$N = A^T A$, $C = A^T y$, and

$\mathbf{x} = N^{-1} C = (A^T A)^{-1} A^T y = [a_0, a_1, a_2, b_0, b_1, b_2]^T$

Solving this system for the parameters vector using MATLAB.

Left

$$x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{6\times1} = \begin{bmatrix} -119.4805 \\ 0.9998 \\ -0.0066 \\ -120.7187 \\ 0.0065 \\ 0.9996 \end{bmatrix}_{6\times1} \qquad e = 1.0\times10^{-3} \begin{bmatrix} -0.4937 \\ 0.0118 \\ -0.4938 \\ 0.0118 \\ 0.4938 \\ -0.0118 \\ 0.4937 \\ -0.0118 \end{bmatrix}_{8\times1}$$

Right

$$x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_{6\times1} = \begin{bmatrix} -124.3337 \\ 0.9998 \\ 0.0029 \\ -119.3906 \\ -0.0030 \\ 0.9997 \end{bmatrix}_{6\times1} \qquad e = 1.0\times10^{-3} \begin{bmatrix} -0.2533 \\ -0.0057 \\ -0.2533 \\ -0.0057 \\ 0.2533 \\ 0.0057 \\ 0.2533 \\ 0.0057 \end{bmatrix}_{8\times1}$$

Reduction of Comparator Coordinates

In its final format, the affine transformation model looks like:

Left	**Right**
$x = -119.4805 + 0.9998\, x' - 0.0066\, y'$ $y = -120.7187 + 0.0065\, x' + 0.9996\, y'$	$x = -124.3337 + 0.9998\, x' + 0.0029\, y'$ $y = -119.3906 - 0.0030\, x' + 0.9997\, y'$

$\hat{\sigma}_o = 6.985$ e-004 $\qquad\qquad$ $\hat{\sigma}_o = 3.583$ e-004