Nonlinear pull-in with surface traction in MPM

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Verification of pull-in with static nonlinear Euler-Bernoulli numerical solution

The solution for lateral deflection, y, of a fixed-fixed beam due to a point load is linear and is a function of the point load P and the point x where it is applied:

$$y_{single} = y(P, x)$$

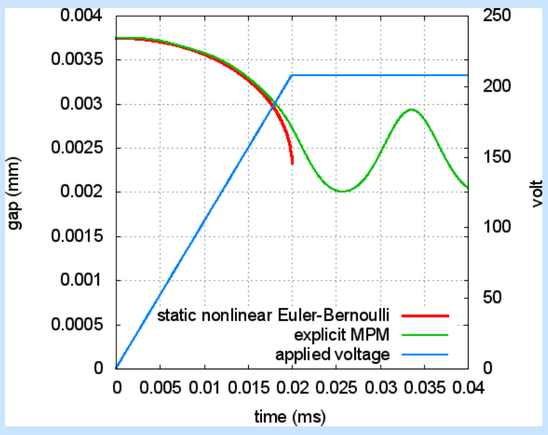
A 1D beam is divided into segments, each of area A, and multiple point loads are superposed to find the deflection at the midpoint of the beam:

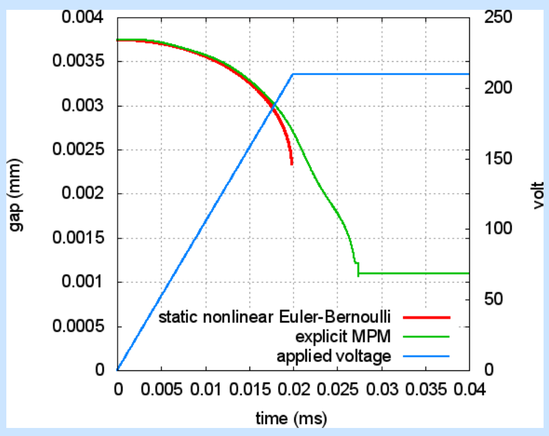
$$\delta_{gap} = \sum_i^n y(f(\delta_i) A, x_i)$$

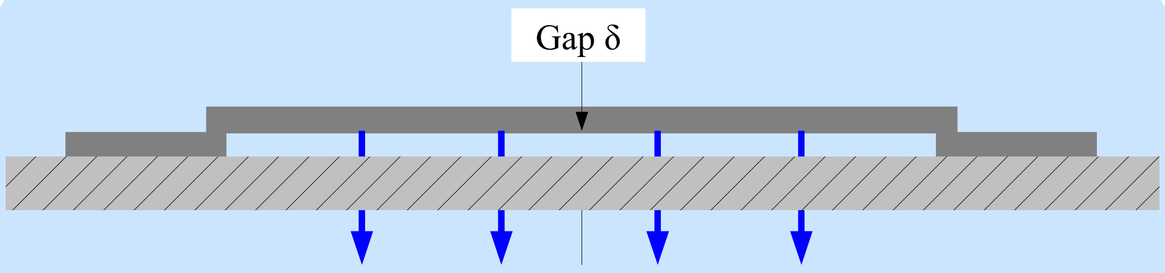
The deflection at each point is updated, and the electro-static force for the new point position is re-calculated. Iteration proceeds until the change in deflection falls below a tolerance. Numerical convergence fails when pull-in occurs.

This solution is static and relies on assumptions of linear elasticity, infinitesimal deformation and infinitesimal beam thickness. It may be used to verify quasi-static large scale simulations of beam shape and pull-in voltage when the assumptions hold.

The following plots demonstrate that a small change of voltage makes a big difference in behavior, as expected for pull-in:
- at 210V the membrane deforms and oscillates about the deformed position, but does not pull in
- at 212V the membrane pulls in, contacting the electrode across the bottom and staying there







$$\sigma \nabla + f = \rho a$$ 3D Equation of Motion

$$0 = \sigma(\boldsymbol{x}, 0) = \boldsymbol{v}(\boldsymbol{x}, 0)$$ Initial conditions

$$0 = \boldsymbol{v}(0, t)$$ Boundary condition – membrane fixed to base

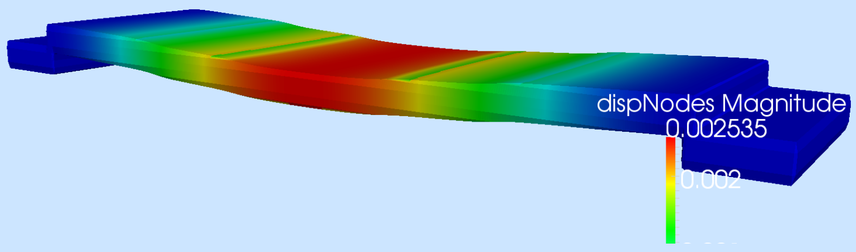
$$f(\delta) = \frac{A\epsilon}{2}\left(\frac{V}{\delta}\right)^2$$ Non-linear electro-static forcing function

Where:
- σ = stress
- f = electro-static forcing
- ρ = density
- a = acceleration
- x = position
- v = velocity
- A = area
- ε = permittivity of air
- δ = gap
- V = voltage

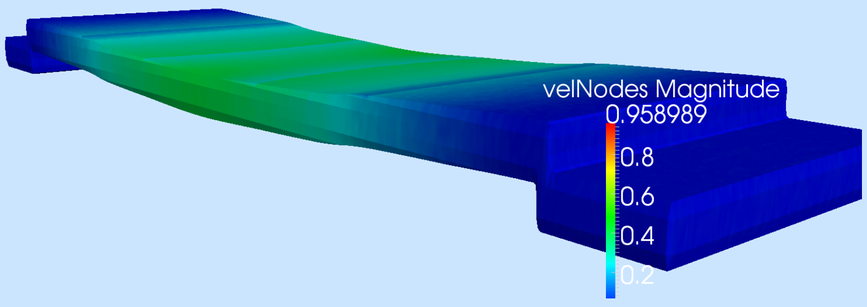
Beam dimensions:
- Length = 0.4 (mm)
- Width = 0.124 (mm)
- Thickness = 0.0045 (mm)

MPM discretized with:
- 4 elements thickness
- 353 elements length
- 112 elements width

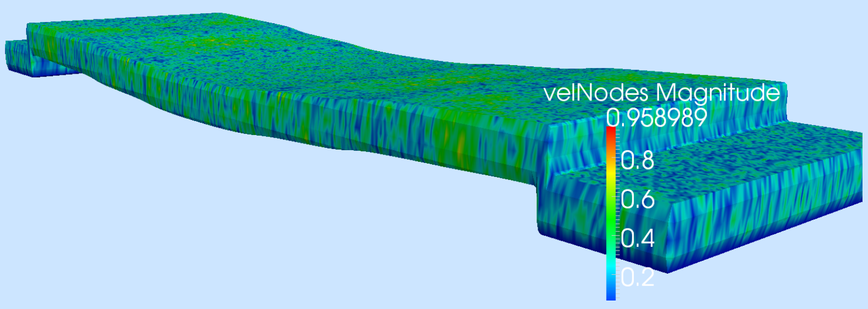
Displacement of the pulled-in membrane:



Velocity of the membrane before pull-in:



Velocity of the membrane after pull-in; note the complicated high frequency vibrations induced by contact:



Surface particles for MPM

Force on each particle due to traction and/or pressure:

$$f_I^{surf} = a(p\boldsymbol{n} + \boldsymbol{t})$$

Integrate particle forces to the grid:

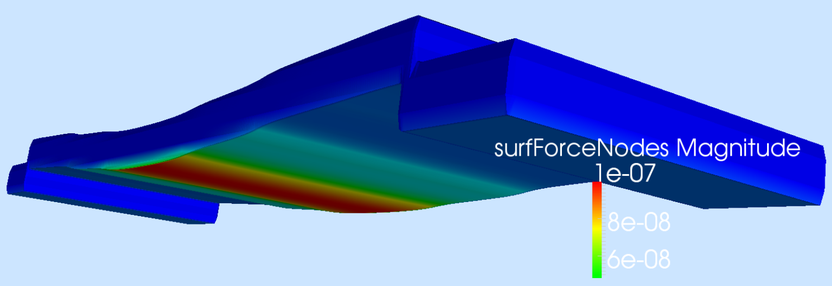
$$f_i^{ext} = \sum_I S_{iI} f_I^{surf}$$

The area and normal of each surface particle must be determined in the current configuration via Nanson's relation:

$$a\boldsymbol{n} = det(\boldsymbol{F}) A \boldsymbol{N} \cdot \boldsymbol{F}^{-1}$$

Where:
- A = reference area
- a = current area
- N = reference surface normal
- n = current surface normal
- F = deformation gradient
- p = pressure
- t = traction
- S = shape function

A view of the magnitude of the electro-static surface force:



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