Verification of Visco-Plastic and Creep Models in MEMOSA

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Objective: To Implement Visco-Plastic and Creep Models in FVM-Based Structural Solver in Memosa

- Implement visco-plastic and creep models in the FVM-based structural solver
- Use implicit discretization for elastic part of stress-strain relationship and explicit discretization for the plastic part
- Implement a general non-linear stress-strain rate constitutive law in the FVM structural model to characterize any creep mechanism
- Verify visco-plastic model by studying steady-state deformation in a visco-plastic perforated tensile strip
- Verify creep model by studying long-term creep behavior of a fixedfixed beam subjected to uniform distributed load

Governing Equations

$$\frac{\partial^2(\rho \mathbf{w})}{\partial t^2} - \nabla \cdot \boldsymbol{\sigma} = \rho \mathbf{f}$$

The strain tensor is assumed to be the sum of elastic and plastic components

$$\varepsilon = \frac{1}{2} \left[\nabla \mathbf{w} + (\nabla \mathbf{w})^{\mathrm{T}} \right] = \varepsilon^{e} + \varepsilon^{\mu}$$

The stress tensor is related to the elastic part of the strain tensor:

$$\sigma = 2\mu(\varepsilon - \varepsilon^p) + \lambda \operatorname{tr}(\varepsilon - \varepsilon^p)\mathbf{I}$$

$$W = displacement vector$$

- σ = stress tensor
- \mathcal{E} = strain tensor

$$\mathcal{E}_e$$
 = elastic strain

$$\mathcal{E}_p$$
 = plastic strain

$$\mu = \frac{E}{2(1+\nu)}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$\frac{\partial^2 (\rho \mathbf{w})}{\partial t^2} - \nabla \cdot \left[\mu \nabla \mathbf{w} + \mu (\nabla \mathbf{w})^{\mathrm{T}} + \lambda \mathbf{I} \mathrm{tr} (\nabla \mathbf{w}) \right] = \rho \mathbf{f} \cdot \nabla \cdot \left[2 \mu \varepsilon_p + \lambda \mathbf{I} \mathrm{tr} (\varepsilon_p) \right]$$



Visco-Plastic Model



Viscous properties appear only after material has yielded

Creep Model



- Stage I characterized by decreasing creep rate
- Stage II characterized by steady-state creep rate, and lasts longest
- Stage III characterized by rapidly increasing creep rate

Numerical Procedure

• Step 1: Assume known values of σ^N , \mathcal{E}^N , \mathcal{E}^N_p , and \mathbf{w}^N at time level N

• Step 2: Calculate non-linear strain rate $\dot{\varepsilon}_p^N = F(\sigma^N)$. Calculate change in non-linear strain as:

$$\Delta \varepsilon_p^N = \dot{\varepsilon}_p^N \Delta t$$

• Step 3: Advance time as:

$$^{N+1} = t^N + \Delta t$$

• Step 4: Update non-linear strain at the current time level using explicit Euler procedure:

$$\varepsilon_p^{N+1} = \varepsilon_p^N + \Delta \varepsilon_p^N$$

- Step 5: Explicity calculate the contribution due to plasticity on the force balance equations
- Step 6: Obtain deformation at the new time level \mathbf{w}^{N+1}
- Step 7: Calculate stress at new time level *N*+1. Go to step 1



Creep Deformation in a Fixed-Fixed Beam



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Von-M	lises	Stress	(Pa)	1
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 $s = \sigma - \frac{1}{3} \operatorname{Itr}(\sigma)$

 $\sigma_{eff} = \sqrt{\frac{3}{2}s_{ij}s_{ij}}$

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Yield	Displacement at E (mm)						
Stress (Pa)	Present Work	CV_FVM	FE	MFVP			
5.5x10 ⁹	0.33	0.3239	0.3258	0.3398			
6.5x10 ⁹	0.313	0.3111	0.3123	0.3186			
7.5x10 ⁹	0.3061	0.3047	0.3058	0.3099			
Infinity	0.3015	0.299	0.2999	0.3019			

Conclusions

- FVM solver for non-linear stress-strain modeling in MEMOSA has been developed
- The solver uses implicit formulation for elastic part of the strain tensor and explicit formulation for the plastic part
- Visco-plastic and creep models have been implemented in the solver
- Any creep mechanism in RF MEMS device can be simulated using the solver by using proper value of the creep stress exponent m
- The solver has been verified for two test cases: (1) a viscoplastic perforated tensile strip, and (2) creep deformation in a fixed-fixed beam



Future Work

- Implement alternate creep models in the solver
- Compare numerical result with experimental data
- Predict creep in PRISM device for range of voltages

References

• Taylor, G. A., Bailey, C., and Cross, M., 1995, "Solution of the Elastic/Visco-Plastic Constitutive Equations: A Finite Volume Approach," Applied Mathematical Modelling, **19**(12), pp. 746-760.