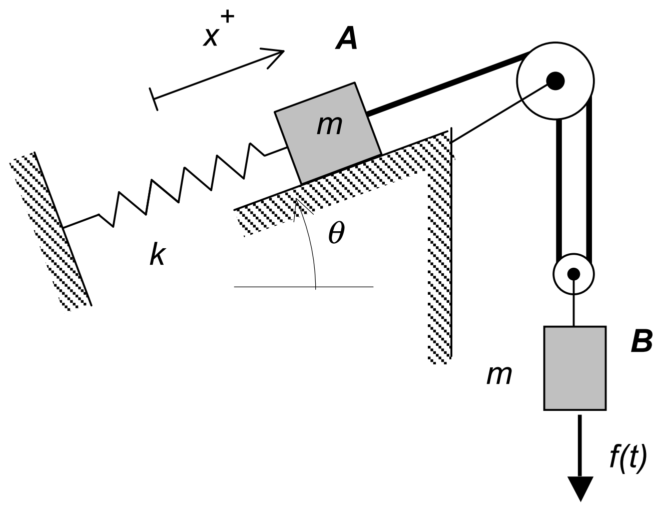
Extra examples for SDOF free response

Example 1

Given: The system shown below.

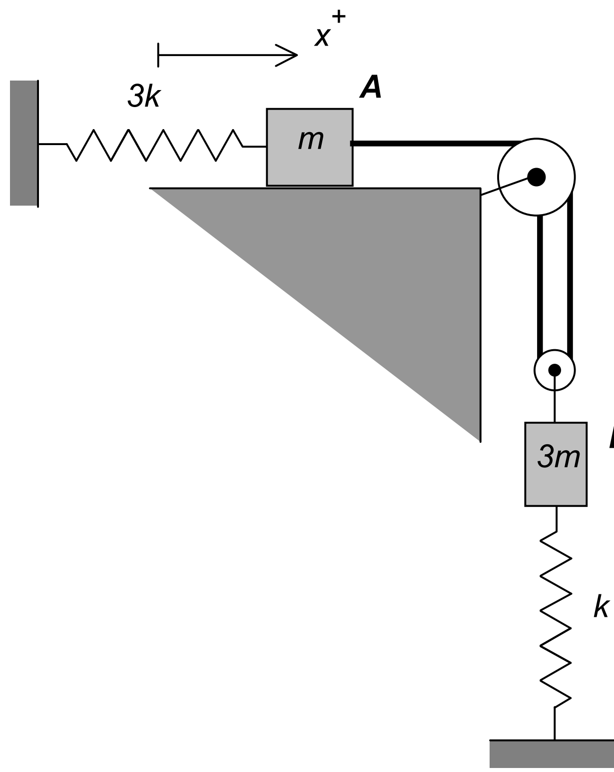
Find: The EOM of this system in terms of the coordinate x and the natural frequency of the system.



Example 2

Given: Blocks A and B (having masses of m and $3m$, respectively) are connected by a cable-pulley system as shown below. A spring of stiffness $3k$ is attached between A and ground. A second spring of stiffness k is attached between block B and ground. Let x describe the position of A, where the springs are unstretched when $x = 0$.

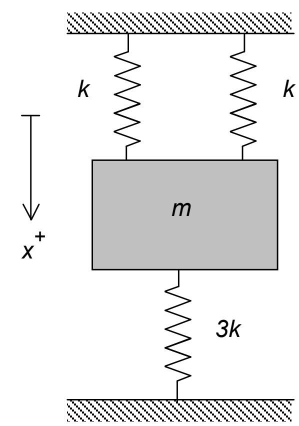
Find: The EOM for the system in terms of the coordinate x and the natural frequency of free oscillations for this system.



Example 3

Given: The block has a downward speed of 2 m/s as it passes through its equilibrium position.

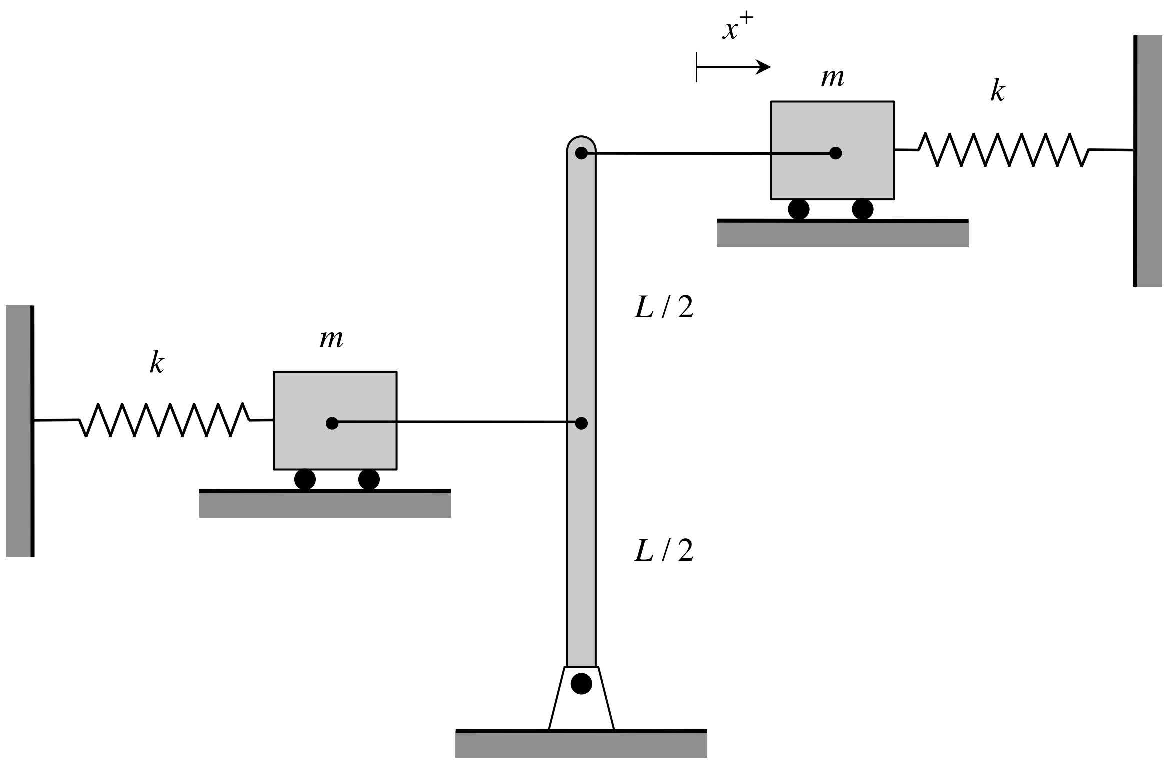
Find: The maximum acceleration of the block over one cycle of oscillation. Use $k = 1000$ N/m and $m = 50$ kg.



Example 4

Given: The system shown below.

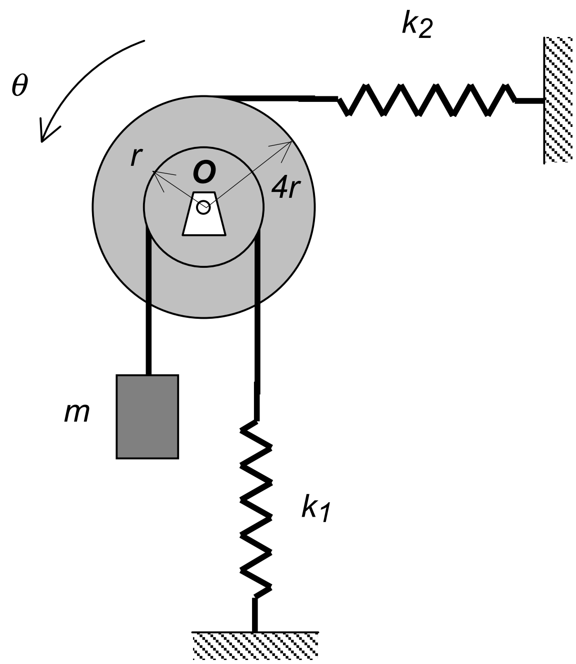
Find: The differential equation of motion for the system using the coordinate x and the natural frequency of the system. Assume small oscillations of the system.



Example 5

Given: The pulley has a mass moment of inertia of I_O.

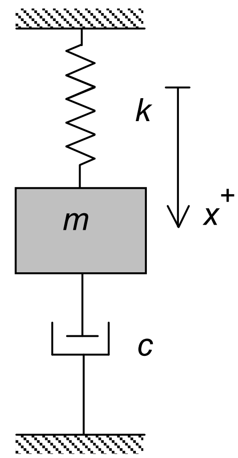
Find: The differential equation of motion for the system in terms of θ and the natural frequency for the system.



Example 6

Given: The system shown below.

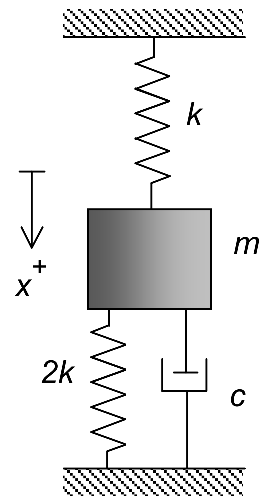
Find: The value of c corresponding to critical damping. Use $k = 30$ kN/m and $m = 35$ kg.



Example 7

Given: The system shown below.

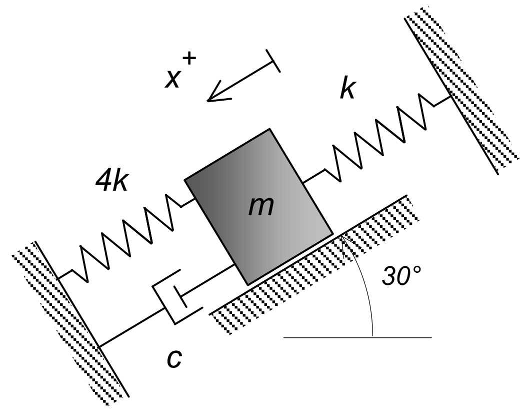
Find: The value of the damping constant c, such that the system has 50 percent of critical damping. Use $k = 3000$ N/m and $m = 10$ kg.



Example 8

Given: The system shown below.

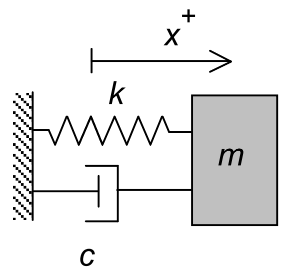
Find: The value of the damping constant c, such that the system has 50 percent of critical damping. Use $k = 2000$ N/m and $m = 10$ kg.



Example 9

Given: The system shown below.

Find: The free response of the system corresponding to $x(0) = x_0$ and $\dot{x}(0) = 0$. Use $c = 2.5$ lb-s/ft, $k = 36$ lb/ft and $mg = 8$ lb.



Example 10

Given: The addition of damping to an undamped system causes the period to increase by 25 percent.

Find: The value of the damping ratio after the addition of damping.

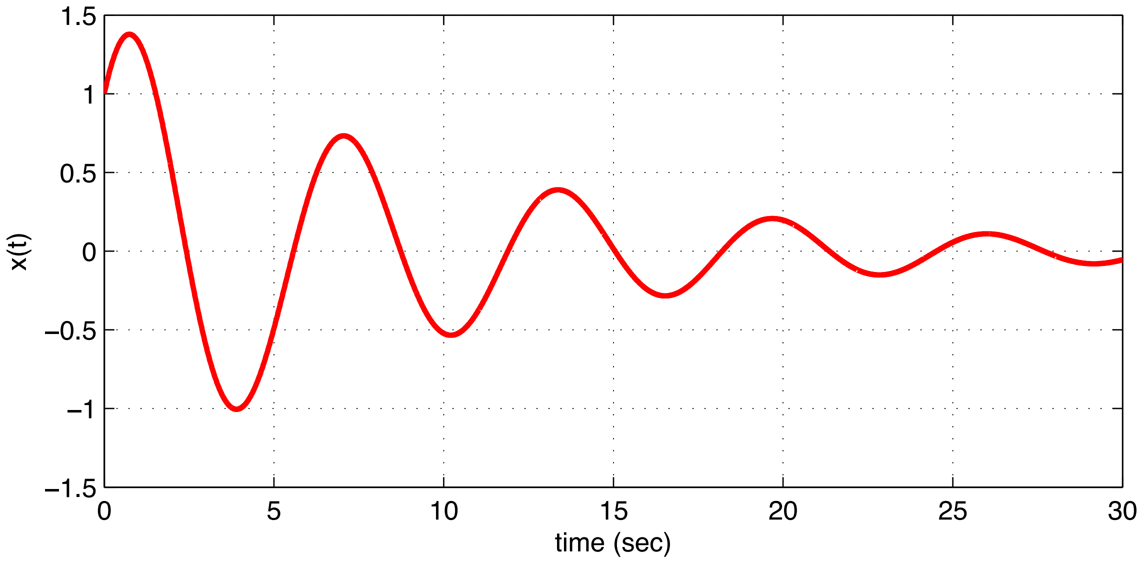
Example 11

Given: The free response of the single degree of freedom system:

$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

shown below. It is known that $M = 2$ kg.

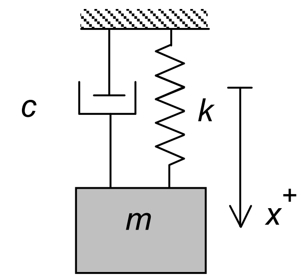
Find: The damping coefficient C from this free response plot.



Example 12

Given: The system shown below is released from rest under the action of gravity.

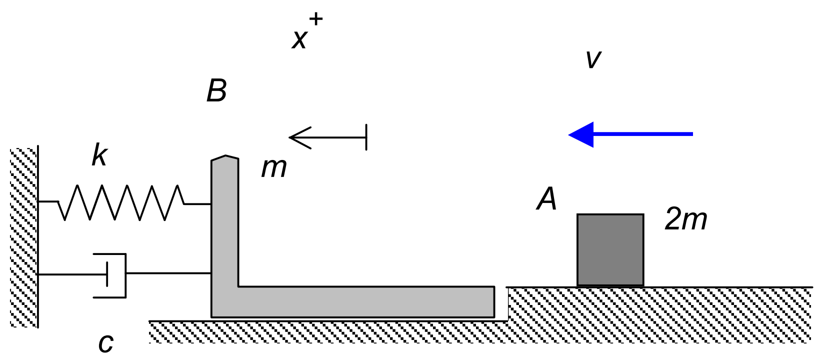
Find: The initial overshoot past the static equilibrium state of the system. Use $m = 3$ kg, $c = 18$ N-s/m and $k = 108$ N/m.



Example 13

Given: Block A strikes stationary block B with a speed of v. Upon impact, A sticks to B. Assume all surfaces to be smooth.

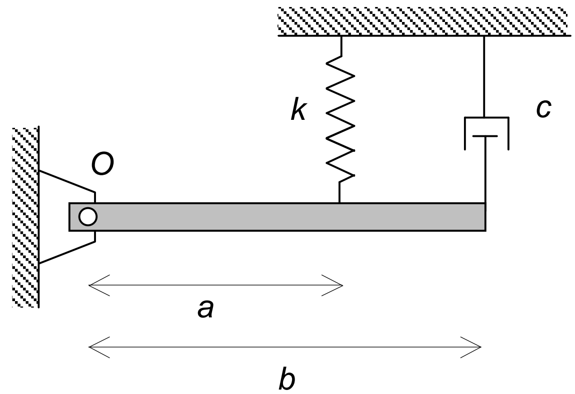
Find: The time history of motion $x(t)$ of the system after A sticks to B. Use $v = 30$ m/s, $k = 3000$ N/m, $c = 30$ N-s/m and $m = 10$ kg.



Example 14

Given: The system moves in a horizontal plane.

Find: The value of c for critical damping. Assume small oscillations.



Example 15

The location center of percussion P for a rigid body is given by: $h = I_O/md$, where h is the distance from the support point O to P, d is the distance from the support point O to the center of mass G and I_O is the mass moment of inertia of the body about the support point O. In this example, we will explore using the free vibration response of a baseball bat suspended from support point O to determine the location of the bat's center of percussion.

1. Draw an FBD of the bat.
2. Develop the equation of motion (EOM) of the bat in terms of the angle θ. Linearize this EOM for small θ (recall that for small θ we have $\sin\theta \approx \theta$).
3. Based on your linearized EOM, what is the natural frequency of free response of the bat in terms of the parameters of the problem?
4. Determine the relationship between the distance h to the center of percussion and the natural frequency of free oscillations for the bat.
5. Discuss how you could set up a simple experiment to determine the location of the center of percussion of the bat.