ME 200 – Thermodynamics 1
Chapter 6 In-Class Notes
for Spring 2023

Entropy

- Definition of Entropy
- Entropy Property Evaluations
  - real fluids, ideal gases, incompressible liquids
- T-s Diagrams
- Entropy Generation and Balances
- Isentropic Processes and Efficiencies
- ….

Lecture 24
Introduction to Entropy

- Definition of Entropy
- Entropy Evaluation for Real Fluids
- T-s Diagrams

Entropy

Clausius defined a property called entropy whose incremental change for a closed system undergoing an internally reversible process is

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

T is absolute temperature at the boundary inside the system where heat transfer occurs

For a finite process

$$\Delta S = S_2 - S_1 = \int_1^2 dS = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

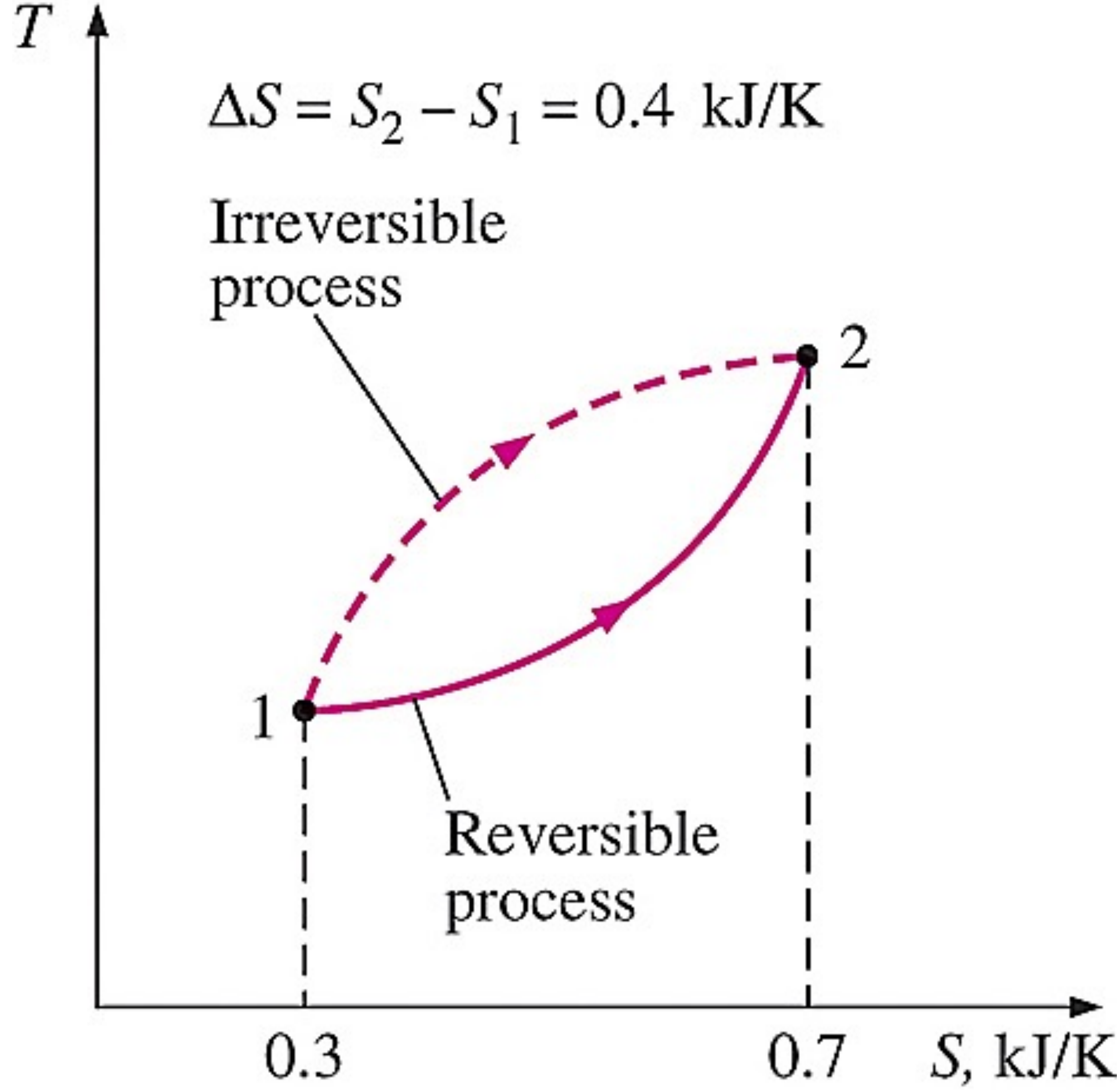
Important Notes on Entropy

1. For an internally reversible process, entropy can only change if there is a heat transfer (entropy increases for heat in and decreases for heat out).
2. Irreversibilities can also cause entropy changes.
3. Even though the definition for ΔS is based on an internally reversible process, ΔS only depends on the beginning and end states and not the process (i.e., it's a property)
4. Two ways to evaluate ΔS

   1. Internally Rev. Closed Process $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$

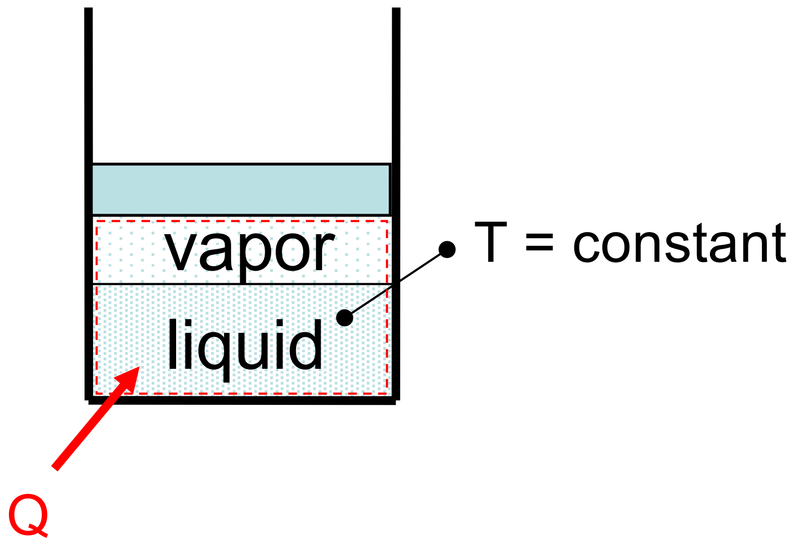
   2. Property Tables or Relations (ideal gas, incompressible)

Entropy only Depends on the States and Not the Process



Questions

Suppose that there is a two-phase mixture undergoing a quasi-static, constant temperature heat addition process with no friction as shown below. Does the entropy increase, decrease, or remain the same?



Can the entropy of a closed system decrease during a process?

Can the entropy of a closed system increase if the process is adiabatic?

Entropy Property Evaluations

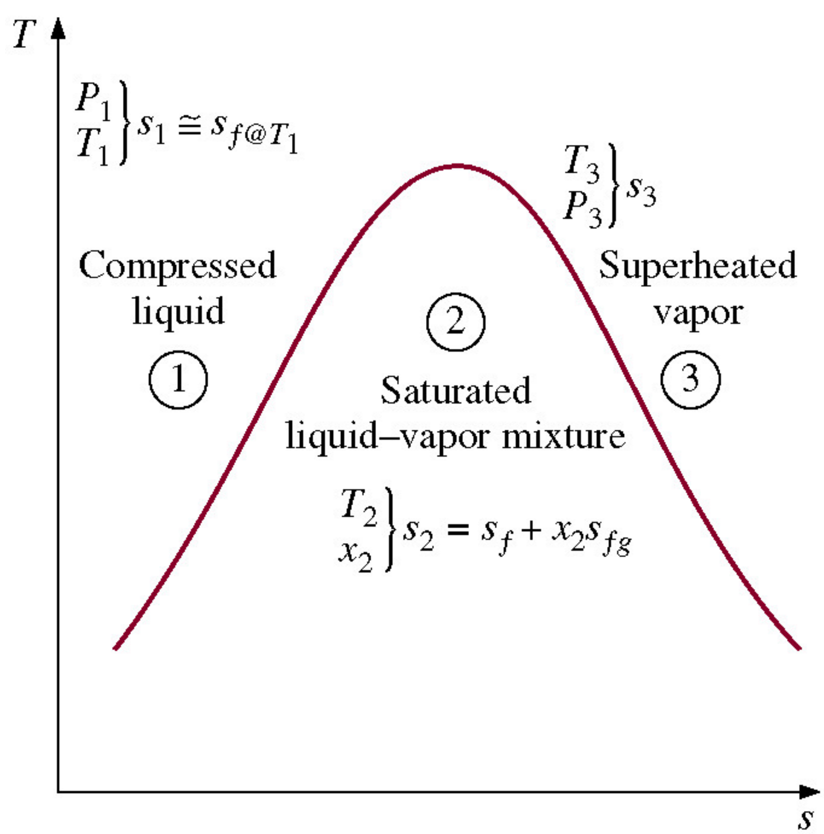
The entropy change of a specified mass during a process can be determined from property evaluations according to

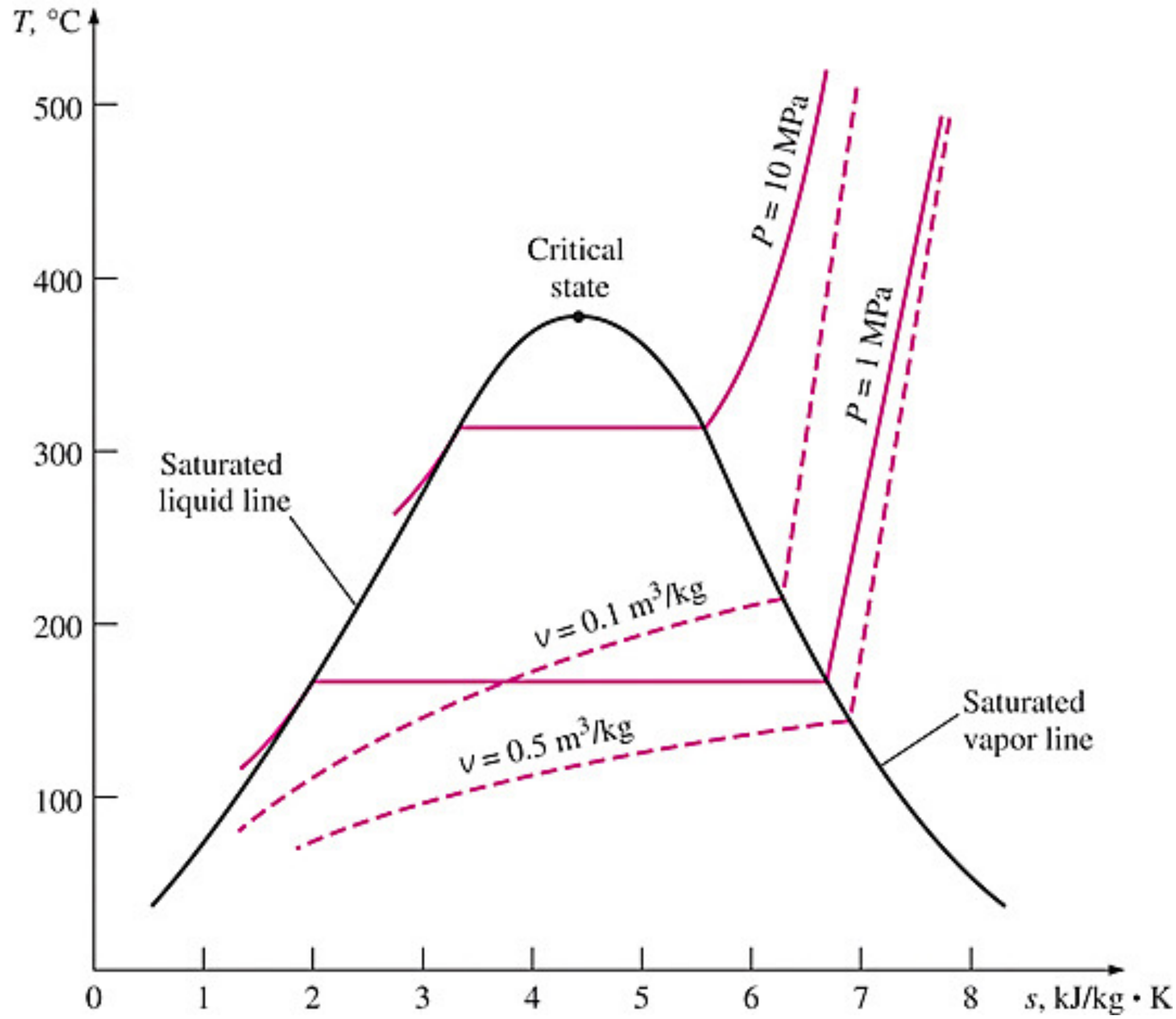
$$\Delta S = S_2 - S_1 = m(s_2 - s_1) = m\Delta s$$

where s is the specific entropy evaluated using property data or property relations

Entropy Evaluations for Pure Substances

Entropy values are given in property tables for two-phase mixtures, superheated vapors, & compressed liquids. T-s diagrams help to depict conditions.





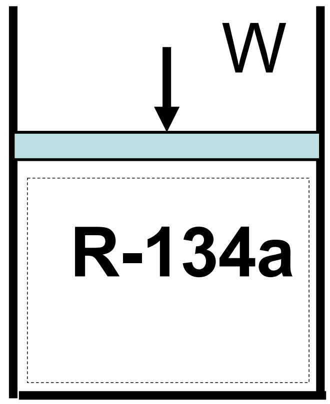
Rules for Evaluating Conditions

Given	CL	2-phase	SHV
T,s	$s<s_{f,T}$	$s_{f,T}<s<s_{g,T}$	$s>s_{g,T}$
P,s	$s<s_{f,P}$	$s_{f,P}<s<s_{g,P}$	$s>s_{g,P}$

Example

Given: R-134a is compressed in a piston-cylinder
Initially: T_1 = -4°C, saturated vapor
Finally: P_2 = 5 bar

System:



Find:

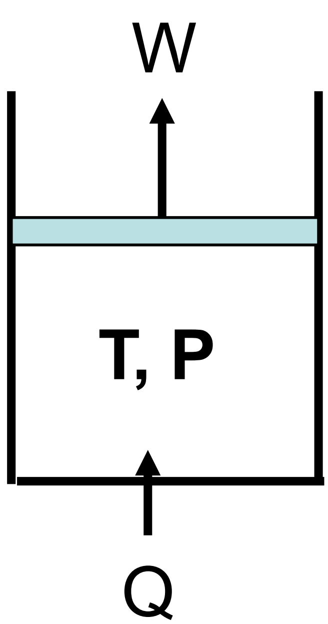
a) change in entropy for the R134a if T_2 = 40°C,
b) the final temperature if there is no change in entropy
c) show the processes on a T-s diagram for both cases

Lectures 25 and 26
More Entropy Evaluations

- T-ds Equations
- Evaluating Entropy Changes for Liquids, Solids, and Ideal Gases

T-ds Relations

- a way to obtain specific entropy from other properties
- consider an internally reversible process for a closed system



Notes on Entropy Evaluations

1) from definition:

$$ds = \left(\frac{\delta q}{T}\right)_{\text{int}.rev.}$$

$$ds = \frac{du}{T} + \frac{Pdv}{T} = \frac{dh}{T} - \frac{v\,dP}{T}$$

2) Δs depends only on beginning and end states and not on the process path, i.e., we do not need a reversible process to evaluate Δs.

3) Δs evaluation: same types of approaches as for other properties

- "real" substances: tables
- "ideal" gases: integration of T-ds equations with ideal gas assumption
- liquids or solids: integration of T-ds equations with incompressible assumption

Entropy Evaluations for Liquids and Solids

For any process between states 1 and 2

$$s_2 - s_1 = \int_1^2 \frac{du}{T} + \int_1^2 \frac{Pdv}{T}$$

For an incompressible substance

$$du = cdT, \quad v = \text{constant}$$

Then

$$s_2 - s_1 = \int_{T_1}^{T_2} C \frac{dT}{T}$$

For a “small” temperature change, the specific heat is relatively constant. For a constant or averaged specific heat

$$s_2 - s_1 = C_{avg} \ln \frac{T_2}{T_1}$$

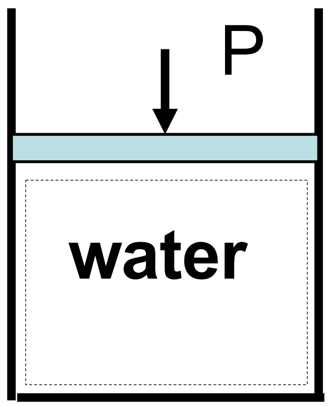
Example

Given: Liquid water is compressed

Initially: $T_1 = 40^oC$, saturated liquid

Finally: $T_2 = 80^oC$, $P_2 = 50$ bar

System:



Find:

a) change in entropy based on compressed liquid table

b) change in entropy based on saturated table

c) change in entropy using constant specific heat

Entropy Evaluations for Ideal Gases

For any process between states 1 and 2

$$s_2 - s_1 = \int_1^2 \frac{du}{T} + \int_1^2 \frac{Pdv}{T}$$

and

$$s_2 - s_1 = \int_1^2 \frac{dh}{T} - \int_1^2 \frac{vdP}{T}$$

For an ideal gas

$$du = C_v dT, \quad dh = C_p dT,$$

$$\frac{P}{T} = \frac{R}{v}, \quad \frac{v}{T} = \frac{R}{P}$$

Then

$$s_2 - s_1 = \int_{T_1}^{T_2} C_v \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

and

$$s_2 - s_1 = \int_{T_1}^{T_2} C_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

Entropy Evaluations for Ideal Gases

Two ways to evaluate integrals:

1. constant (i.e. average) values for C_v and C_p

$$\Delta s = C_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$\Delta s = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

2. tabular data

$$\Delta s = s_2^o - s_1^o - R\ln\frac{P_2}{P_1}$$

where
$$s_T^o = \int_{T_{ref}}^{T} C_p \frac{dT}{T}$$

Example

Given: 1 kg of O_2, initially at a pressure and temperature of 2 bars and 300 K, undergoes a process such that the final pressure and temperature are 1.5 bar and 1500 K.

Find: a) change in entropy using tabular values

b) change in entropy using constant specific heat

Review Questions

What assumptions are needed to apply each of the following equations? Possible assumptions include: closed system/open system, internally reversible, externally reversible, adiabatic, negligible kinetic energy effects, negligible potential energy effects, isothermal, ideal gas, incompressible, constant specific heat, etc.

1. $\Delta s = \frac{1}{m}\int_1^2 \frac{\delta Q}{T}$

2. $\Delta s = \int_1^2 c\,\frac{dT}{T}$

3. $\Delta s = \int_1^2 \frac{du}{T} + \int_1^2 \frac{Pdv}{T}$

4. $\Delta s = \int_1^2 \frac{C_p dT}{T} - \mathrm{R}\cdot\ln\frac{\mathrm{P}_2}{\mathrm{P}_1}$

5. $\Delta s = s_2^o - s_1^o - \mathrm{R}\cdot\ln\frac{\mathrm{P}_2}{\mathrm{P}_1}$

Lectures 27, 28, and 29

- Internally reversible processes
- Increase in entropy principle
- Entropy balances

Internally Reversible Processes

Recall that

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

T is absolute temperature at the boundary inside the system where heat transfer occurs

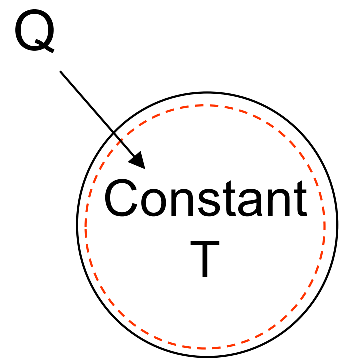
→ For an internally reversible process, entropy can only change if there is heat transfer

Isentropic Processes

- A process where the entropy remains constant
  - Closed System: final entropy = initial entropy
  - SSSF Open System: exit entropy = inlet entropy
- **Adiabatic + Internally Reversible → Isentropic**
- Used as a basis for defining the efficiencies of devices that produce work (e.g., turbines) and require work (e.g., pumps & compressors)

ΔS for a Isothermal Heat Transfer

Examples include a reservoir or constant pressure phase-change process



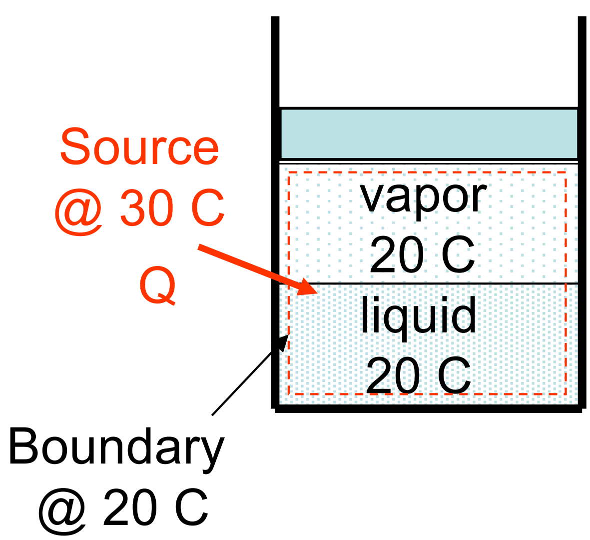
After the heat transfer occurs

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,rev} = \frac{Q_{12}}{T}$$

Example – Constant Pressure Phase Change

Given: 100 kJ of heat is added in a "slow" constant pressure process from a source at 30 C to a two-phase mixture at 20 C

Find: Change in entropy (kJ/K) for the mixture



Assumption: internally reversible process

$$S_2 - S_1 = \frac{Q}{T}$$

$$= \frac{100\,kJ}{(20+273)\,K}$$

$$= 0.341\,kJ/K$$

Trick: choose boundary of system just inside of cylinder so that temp. is nearly uniform (i.e., int. rev. heat addition)

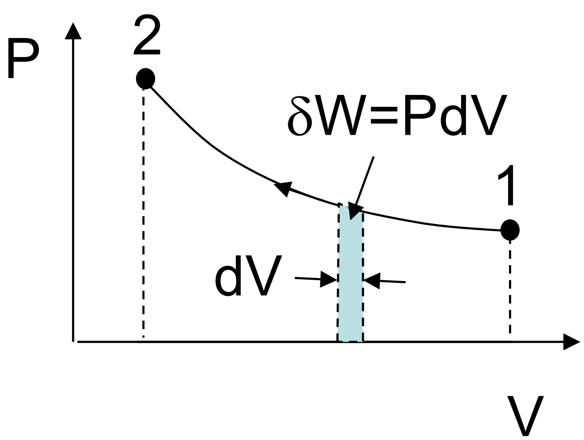
Internally Reversible Heat Transfer

$$\delta Q_{\text{int},rev} = T \cdot dS \Rightarrow Q_{\text{int},rev} = \int_{T_1}^{T_2} T \cdot dS$$

Analogy Between Internally Reversible Boundary Work and Heat Transfer

Reversible Boundary Work

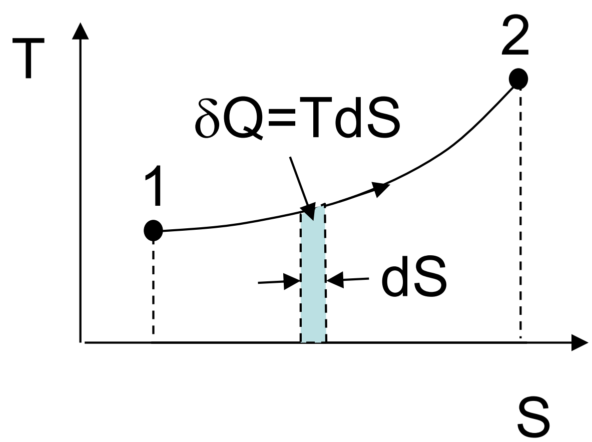
$$W_{\text{int rev}} = \int PdV$$



* depict work transfer on P-V diagrams

Reversible Heat Transfer

$$Q_{\text{int rev}} = \int TdS$$



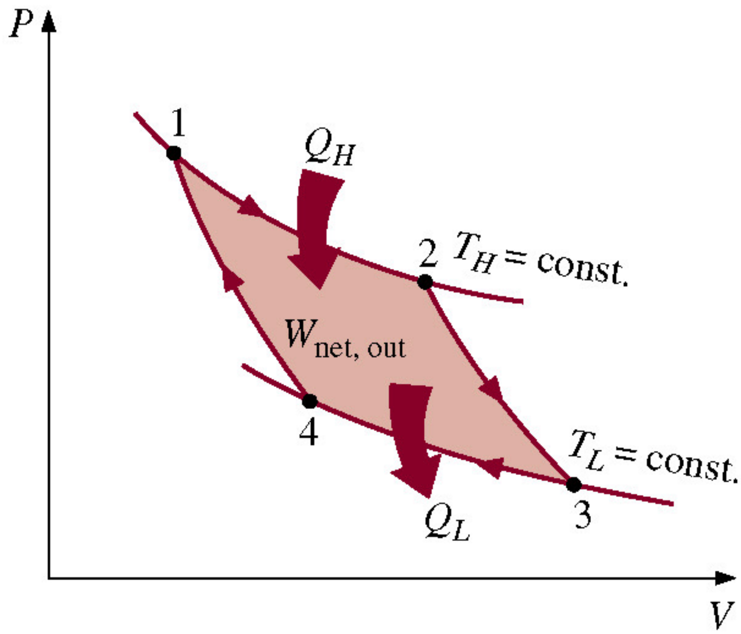
* depict heat transfer on T-S diagrams

* require quasi-equilibrium processes with no friction*

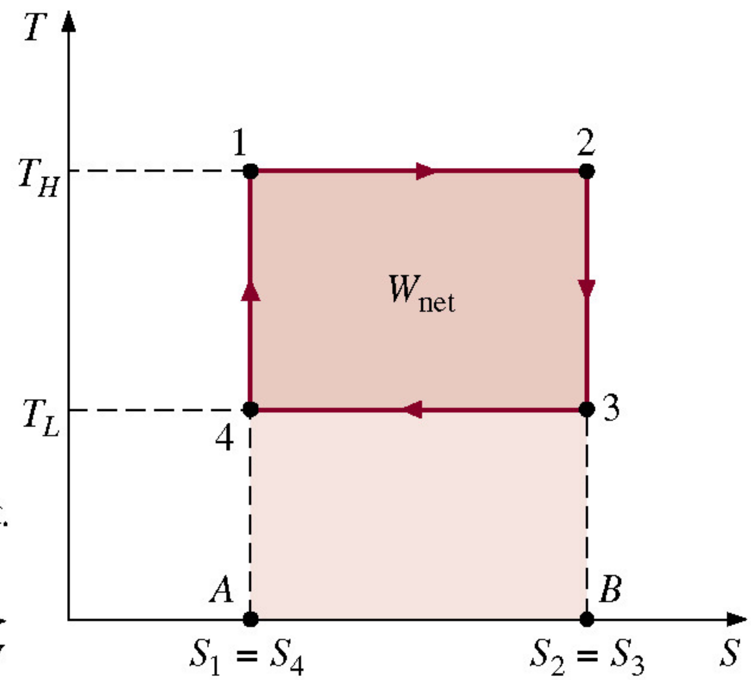
Carnot Heat Engine Cycle

1→2: Reversible, Isothermal Heat Addition
2→3: Reversible, Adiabatic Expansion
3→4: Reversible, Isothermal Heat Rejection
4→1: Reversible, Adiabatic Compression

P-V Diagram



T-S Diagram



Increase in Entropy Principal

It can be shown from the Clausius inequality that the change in entropy of a real process is always greater than the entropy change if the process had been reversible so that

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

the equality holds for int. rev. processes only

or

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + \sigma$$

σ is entropy generation due to irreversibilities

Comments on Entropy Generation

1. $\sigma = 0$ for internally reversible processes
2. $\sigma > 0$ for irreversible processes
3. $\sigma < 0$ is impossible → we can calculate σ to evaluate whether a process or cycle is possible

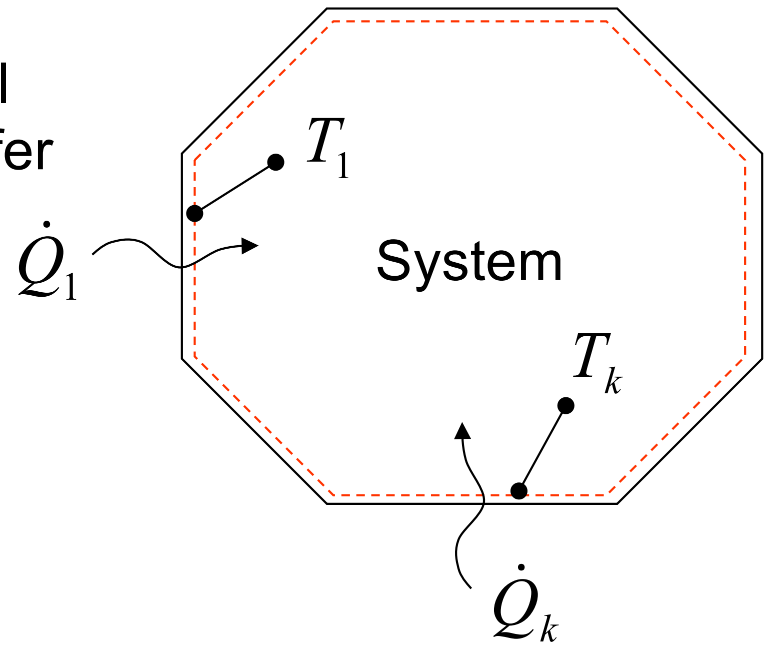
Consider an **adiabatic** system

$$S_2 - S_1 = \sigma$$

The entropy can only increase due to entropy generation

General Closed System Entropy Balances

Consider the general case with heat transfer at N boundaries



At any instant

$$\frac{dS}{dt} = \sum_{k=1}^{N} \frac{\dot{Q}_k}{T_k} + \dot{\sigma}$$

For a finite process

$$\Delta S = \sum_{k=1}^{N} \int_{t_1}^{t_2} \frac{\dot{Q}_k dt}{T_k} + \sigma = \sum_{k=1}^{N} \int \frac{\delta Q_k}{T_k} + \sigma$$

Special Cases

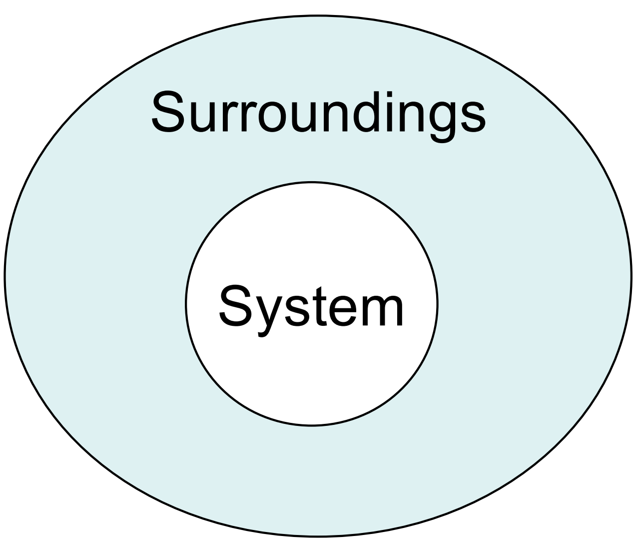
1. Steady State: $$\frac{dS}{dt} = 0 \quad \Rightarrow \quad \dot{\sigma} = -\sum_{k=1}^{N} \frac{\dot{Q}_k}{T_k}$$

2. Constant Boundary Temperatures $$\Delta S = \sum_{k=1}^{N} \frac{Q_k}{T_k} + \sigma$$

3. Internally Reversible, Uniform Temperatures Single Heat Transfer Loc. σ $$\Delta S = \int_{t_1}^{t_2} \frac{\dot{Q}dt}{T} = \int \frac{\delta Q}{T}$$

4. Adiabatic $$\Delta S = \sigma$$

5. System + Surroundings



Combination is adiabatic (w.r.t. their interactions)

$$\Delta S_{sys} + \Delta S_{surr} = \sigma$$

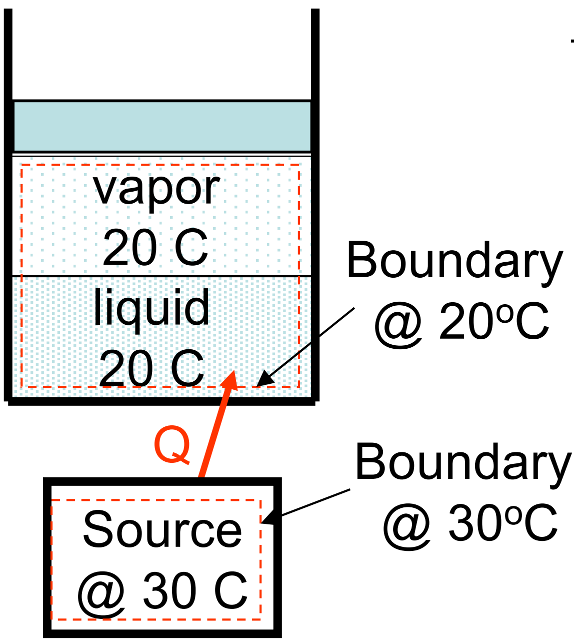
Questions

1. Use an entropy balance to show that the entropy change of a closed system undergoing a process can be negative.

2. Use an entropy balance to show that the entropy generation in a Carnot heat engine must be zero in order to have an efficiency of $1 - T_C/T_H$.

Example – Constant Pressure Phase Change

Given: 100 kJ of heat is added in a "slow" constant pressure process from a source at 30^{o}C to a two-phase mixture at 20^{o}C



Find: Total entropy generation (kJ/K) due to heat transfer

Assumptions: 1) no friction, 2) quasi-static expansion, 3) heat transfer irreversibilities (i.e., temperature gradients) only occur near boundaries between source & cylinder)

Example

Given: Air in a piston-cylinder device with heat input

Initially: $T_1 = 20^oC$, $P_1 = 2$ bar

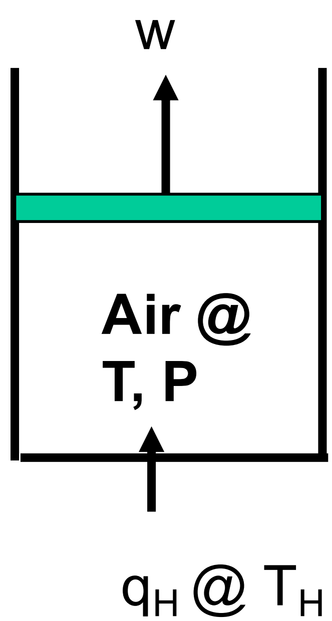
Finally: $T_2 = 40^oC$, $P_2 = 2$ bar $= P_1$

Heat source: $T_H = 300^oC$, $q_H = 25$ kJ/kg

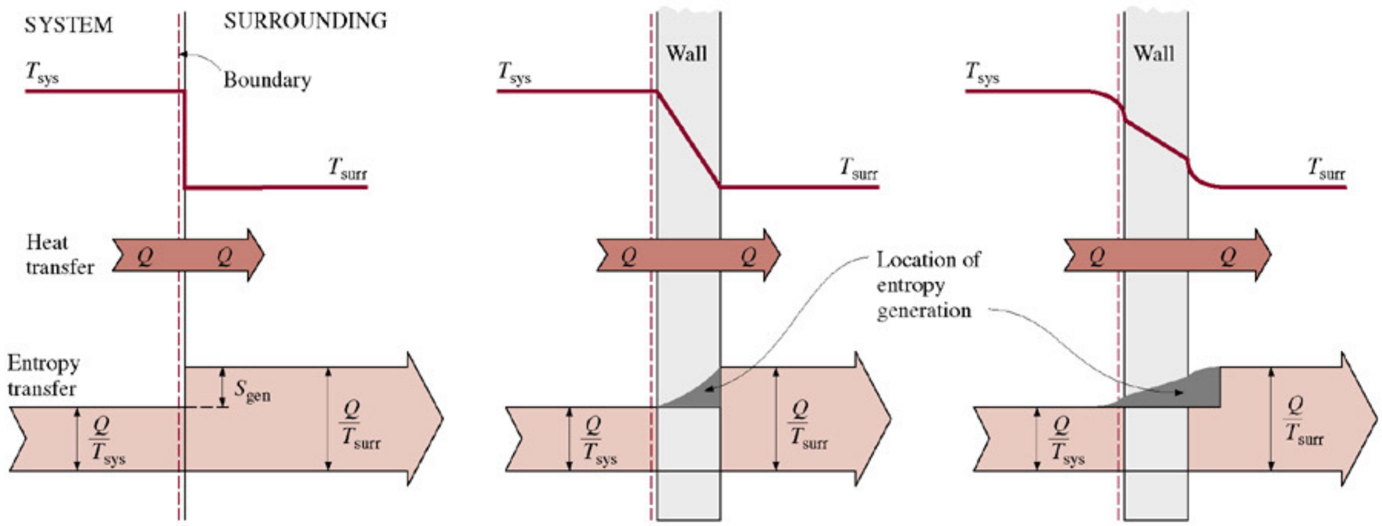
Find: a) specific work output, w=?

b) whether the process is possible?

System:



Entropy Generation in Heat Transfer



(a) The wall is ignored (b) The wall is considered (c) The wall as well as the variations of temperature in the system and the surroundings are considered

Neglecting the change in entropy of the wall (left figure) and assuming uniform and constant T_{sys} and T_{surr}, then for a finite time

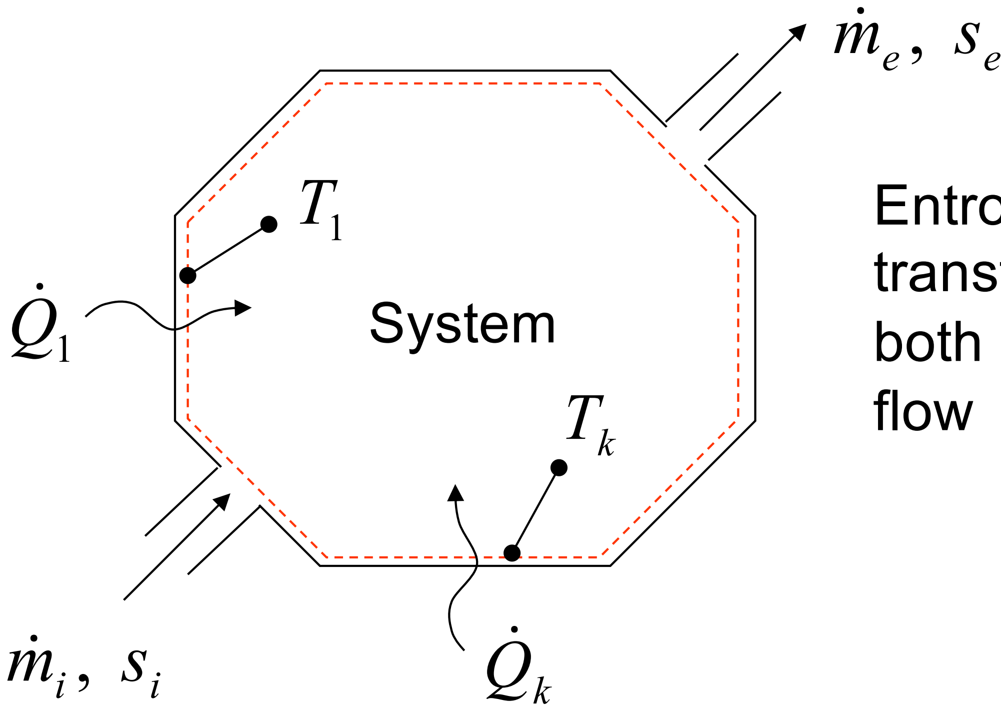
$$\Delta S_{sys} = -\frac{Q}{T_{sys}}, \quad \Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$\Delta S_{sys} + \Delta S_{surr} = \sigma = Q\left(\frac{1}{T_{surr}} - \frac{1}{T_{sys}}\right)$$

On a rate basis,

$$\dot{\sigma} = \dot{Q}\left(\frac{1}{T_{surr}} - \frac{1}{T_{sys}}\right)$$

Open System Entropy Balances



Entropy can be transferred due to both heat and mass flow

$$\frac{dS}{dt} = \sum_{k=1}^{N} \frac{\dot{Q}_k}{T_k} + \sum_{inlets} \dot{m}_i s_i - \sum_{exits} \dot{m}_e s_e + \dot{\sigma}$$

<u>Special Case</u>: Steady State, Steady Flow

$$\dot{\sigma} = \sum_{exits} \dot{m}_e s_e - \left(\sum_{inlets} \dot{m}_i s_i + \sum_{k=1}^{N} \frac{\dot{Q}_k}{T_k} \right)$$

- Entropy changes from inlet to outlet due to heat transfer, mass transfer, or irreversibilities

- For single inlet, single outlet systems:
  Int. Rev. + Adiabatic → Isentropic

Examples

Lecture 30
Isentropic Processes

- Definition
- Incompressible substances
- Ideal gases

Isentropic Processes

- A process where the entropy remains constant
  - Closed System: final entropy = initial entropy
  - SSSF Open System: exit entropy = inlet entropy
- **Adiabatic + Internally Reversible → Isentropic**
- Used as a basis for defining the efficiencies of devices that produce work (e.g., turbines) and require work (e.g., pumps & compressors)

Isentropic Processes for Liquids and Solids

For an incompressible substance undergoing an isentropic process

$$s_2 - s_1 = \int_{T_1}^{T_2} C\frac{dT}{T} = 0 \quad \Longrightarrow \quad T_2 = T_1$$

- No temperature change for reversible & adiabatic processes involving an incompressible liquid, such as the following
  - ideal liquid pump or turbine (i.e., only press. changes)
  - frictionless flow of a liquid through an insulated pipe (recall the hydroelectric dam example)
- No work input for closed piston-cylinder devices that are adiabatic and reversible with incomp. liq.

$$w = -\left(u_2 - u_1\right) = -\int_{T_1}^{T_2} C\,dT = 0$$

- Work for a SSSF pump or turbine that is adiabatic and reversible with incomp. liq. (& Δke = Δpe = 0)

$$w = -\left(h_2 - h_1\right) = -\int_{T_1}^{T_2} C\,dT - v(P_2 - P_1) = -v(P_2 - P_1)$$

Isentropic Processes for Ideal Gases

Case 1: T–v relation for constant specific heats

For an isentropic process, $\Delta s=0$. So with constant C_v

$$\Delta s = C_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} = 0$$

or

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1} \quad \Longrightarrow \quad \frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{-\frac{R}{C_v}}$$

But, R = C_p – C_v, so

$$\left.\frac{T_2}{T_1}\right|_{s_2=s_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

where

$$k = \frac{C_p}{C_v}$$

Isentropic Processes for Ideal Gases

Case 2: T–p relation for constant specific heats

For an isentropic process, $\Delta s=0$, so with constant C_p

$$\Delta s = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 0$$

or

$$\ln\frac{T_2}{T_1} = \frac{R}{C_P}\ln\frac{P_2}{P_1} \quad \Longrightarrow \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_P}}$$

But, $R=C_p - C_v$, so

$$\left.\frac{T_2}{T_1}\right|_{s_2=s_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

where

$$k = \frac{C_p}{C_v}$$

Isentropic Processes for Ideal Gases

Case 3: Variable specific heats given T and P

For an isentropic process, $\Delta s=0$

$$\Delta s = s_2^o - s_1^o - R\ln\frac{P_2}{P_1} = 0$$

or

$$s_2^o - s_1^o = R\ln\frac{P_2}{P_1} \quad \Rightarrow \quad \left(\frac{P_2}{P_1}\right)_{s_2=s_1} = \frac{\exp(s_2^o/R)}{\exp(s_1^o/R)}$$

Define a relative pressure as

$$P_r = \exp(s^o/R)$$

Then

$$\left(\frac{P_2}{P_1}\right)_{s_2=s_1} = \frac{P_{r,2}}{P_{r,1}}$$

Isentropic Processes for Ideal Gases

Case 4: Variable specific heats given T and v

For an isentropic process with an ideal gas

$$\left(\frac{P_2}{P_1}\right)_{s_2=s_1} = \frac{P_{r,2}}{P_{r,1}}$$

But for an ideal gas

$$R = \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \quad \Rightarrow \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}\frac{v_1}{v_2}$$

So

$$\left(\frac{v_2}{v_1}\right)_{s_2=s_1} = \frac{T_2 / P_{r,2}}{T_1 / P_{r,1}}$$

Or

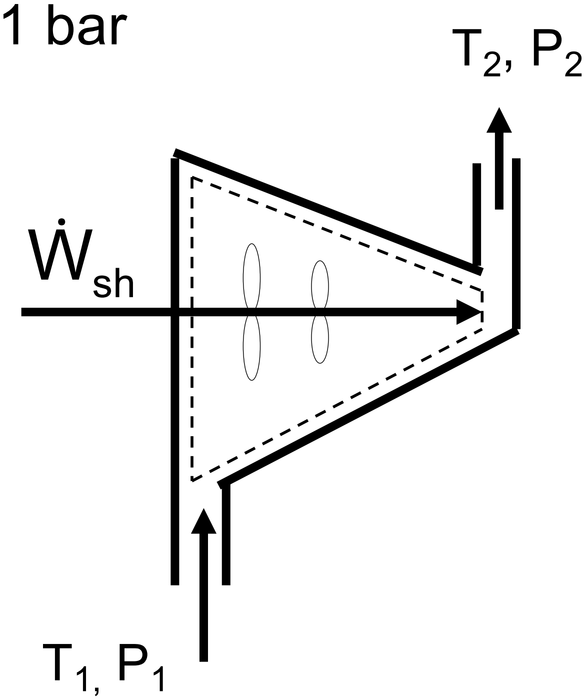
$$\left(\frac{v_2}{v_1}\right)_{s_2=s_1} = \frac{v_{r,2}}{v_{r,1}}$$

Example

Given: Internal reversible & adiabatic air compressor

Inlet: $T_1 = 27^oC$, $P_1 = 1$ bar

Outlet: $P_2 = 5$ bar



System:

Find: Exit temperature T_2

Assumptions:

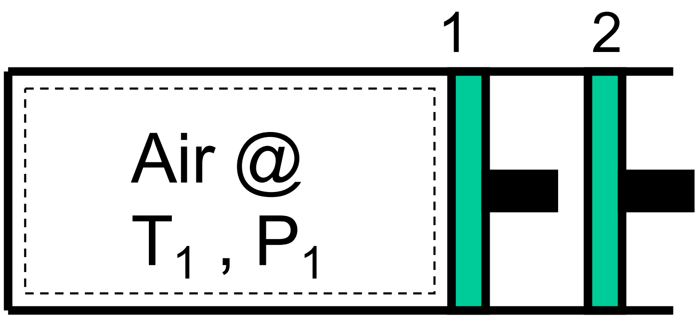
Example

Given: Air expands in an internally reversible and adiabatic piston-cylinder

Initial: T_1 = 157°C, P_1 = 0.307 MPa, V_1 = 760 cm^3

Final: V_2 = 1733 cm^3

System:



Find:

- final temperature, T_2
- mass of air, m_{air}
- final pressure, P_2
- work output, W

Assumptions:

- closed system
- adiabatic
- internally reversible
- ΔKE=0, ΔPE=0
- air is ideal gas

Lectures 31
Isentropic Efficiencies

- Turbines
- Compressors and Pumps
- Nozzles

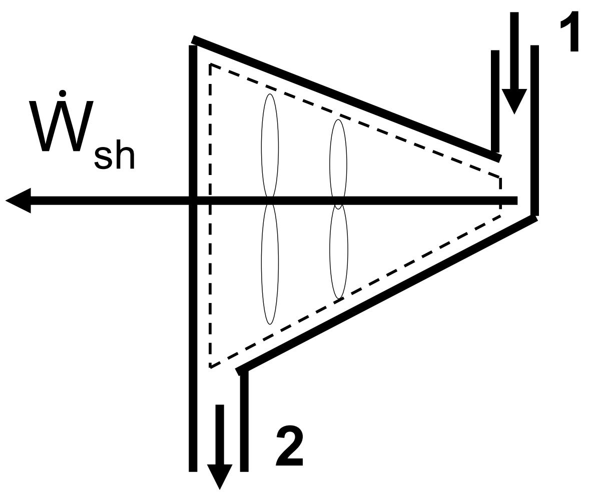
Isentropic Efficiencies

Work Producing Device: actual work output relative to work output for an isentropic device (i.e., actual/maximum work output of an adiabatic device)

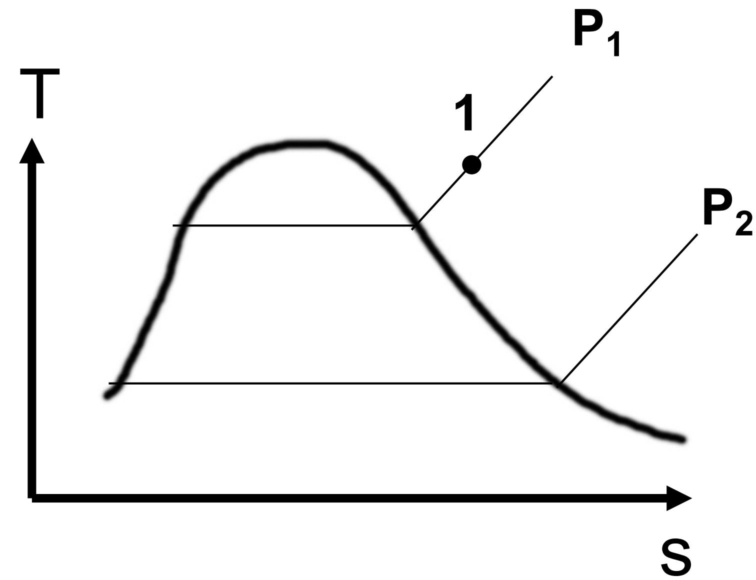
Work Consuming Device: isentropic work input relative to actual work input (i.e., minimum/actual work input for an adiabatic device)

Thrust Producing Device: actual exit kinetic energy relative to isentropic exit kinetic energy (i.e., actual/maximum KE output of an adiabatic device)

Isentropic Turbine Efficiency



Steam Turbine Example



In general, turbine isentropic efficiency is

$$\eta_T = \frac{actual\ work\ output}{isentropic\ work\ output} = \frac{w_a}{w_s}$$

where the isentropic work is evaluated between the actual inlet condition and the outlet pressure.

For an adiabatic turbine with $\Delta ke = \Delta pe = 0$,

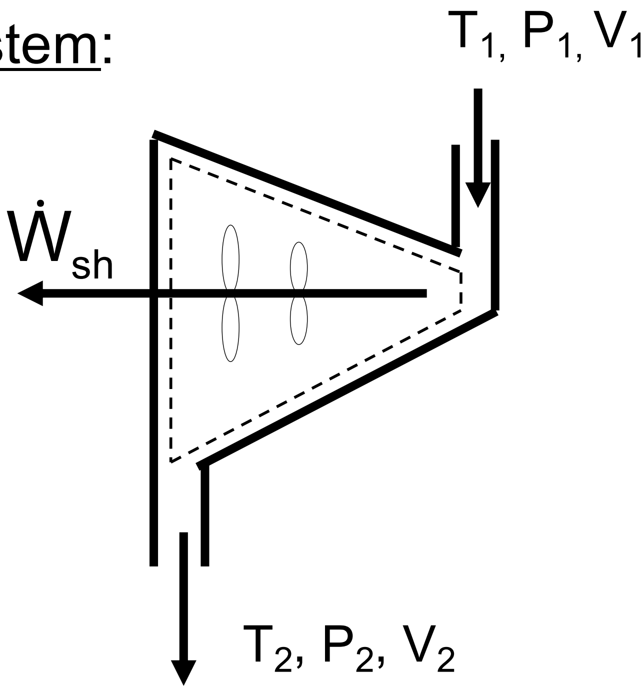
$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}}$$ typically $0.5 < \eta_T < 0.9$

Often given inlet conditions, outlet pressure, and isentropic efficiency → calculate exit state and work output

Example

Given: steam turbine, work output = 8 MW
inlet: P_1=6 MPa, T_1=600°C
outlet: P_2=70 kPa, T_2=100°C

System:

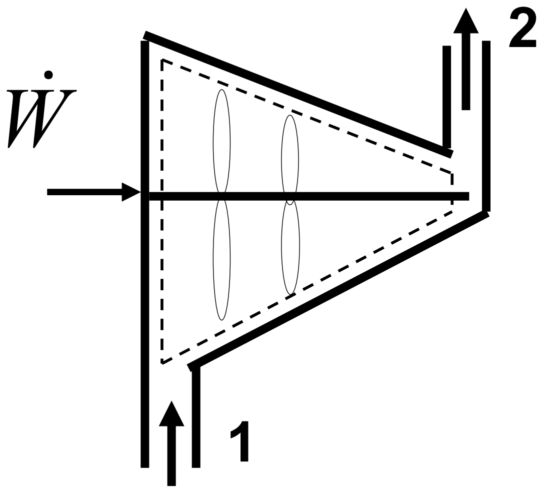


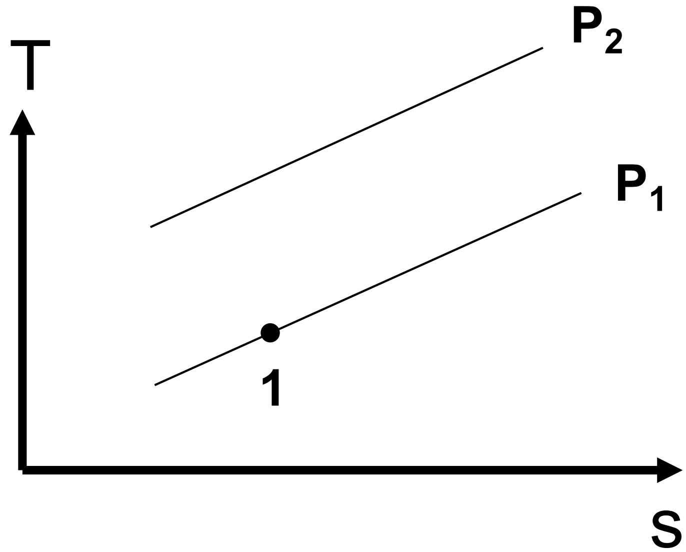
Find:
(a) mass flow rate
(b) isentropic efficiency

Assumptions: 1) SSSF,
2) Δke=Δpe=0, 3) adiabatic

Isentropic Compressor Efficiency

Air Compressor Example





Compressor isentropic efficiency is the inverse of turbine efficiency

$$\eta_C = \frac{isentropic\ work\ input}{actual\ work\ input} = \frac{w_s}{w_a}$$

For an adiabatic compressor with $\Delta ke = \Delta pe = 0$,

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

typically $0.7 < \eta_C < 0.9$

Often given inlet conditions, outlet pressure, and isentropic efficiency → calculate exit state and work input

Example

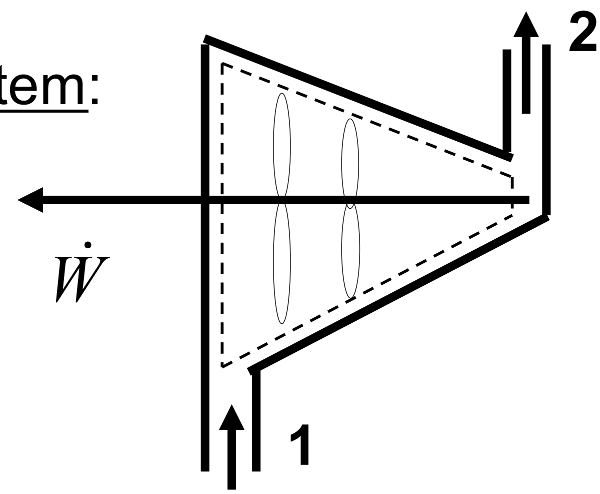
Given: Adiabatic compressor

P_1 = 100 kPa, P_2 = 1 MPa

Saturated water vapor @ P_1

Find: Specific work input and exit temperature for an isentropic efficiency of 0.8

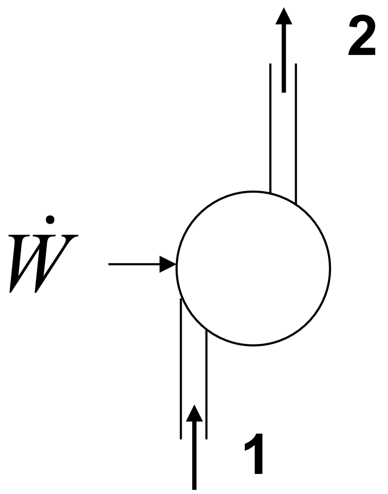
System:

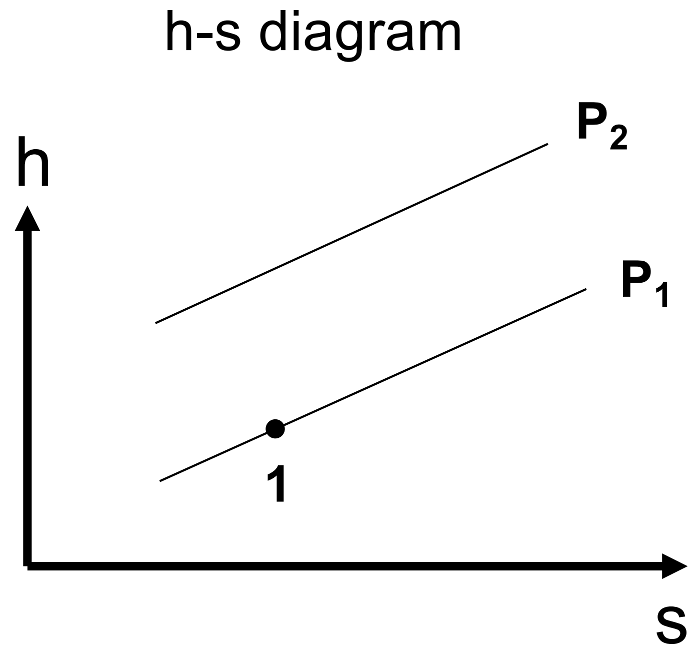


Assumptions:

SSSF, Δke=0, Δpe=0, adiabatic

Isentropic Pump Efficiency





Pump isentropic efficiency has the same definition as compressor efficiency

$$\eta_P = \frac{isentropic\ work\ input}{actual\ work\ input} = \frac{w_s}{w_a}$$

For an adiabatic pump with Δke = Δpe = 0 and an incompressible liquid having a constant specific heat

$$\eta_P = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{v(P_2 - P_1)}{C(T_2 - T_1) + v(P_2 - P_1)}$$

Example

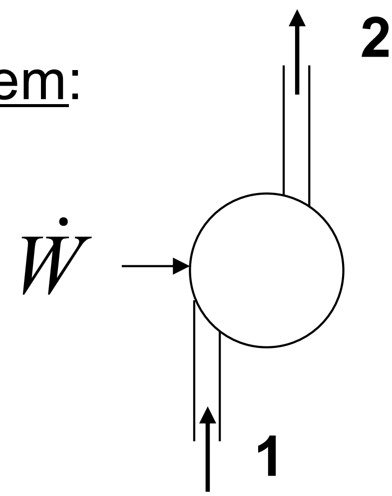
Given: Adiabatic pump

P_1 = 100 kPa, P_2 = 1 MPa

Saturated liquid @ P_1

Find: Specific work input and exit temperature for an isentropic efficiency of 0.8

System:

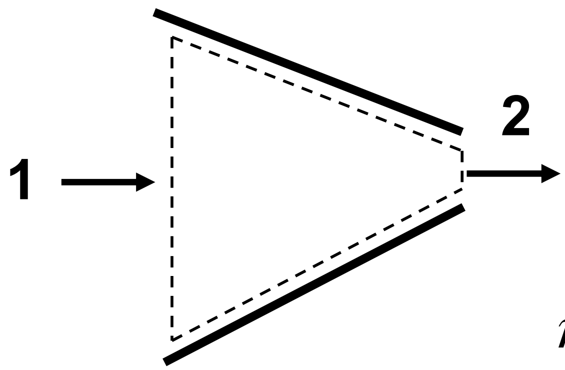


Assumptions:

SSSF, Δke=0, Δpe=0,

adiabatic

Isentropic Nozzle Efficiency



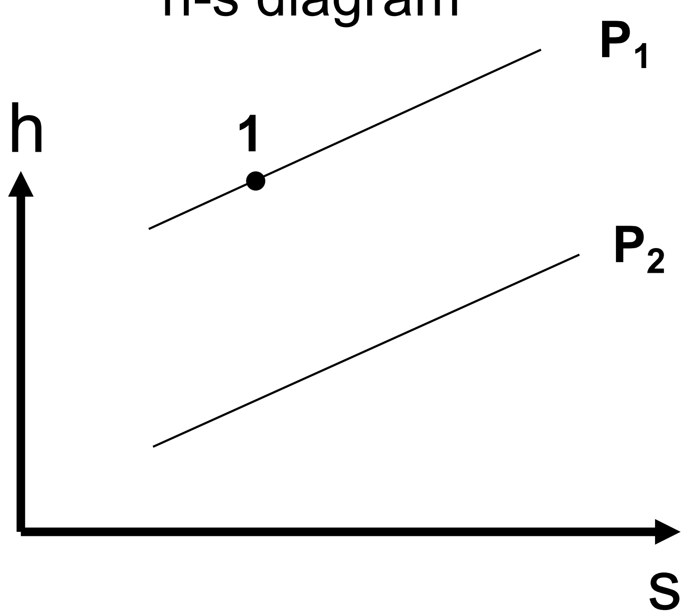
The purpose of a nozzle is to produce kinetic energy

$$\eta_N = \frac{actual\ exit\ \ ke}{isentropic\ exit\ \ ke} = \frac{V_2^2}{V_{2s}^2}$$

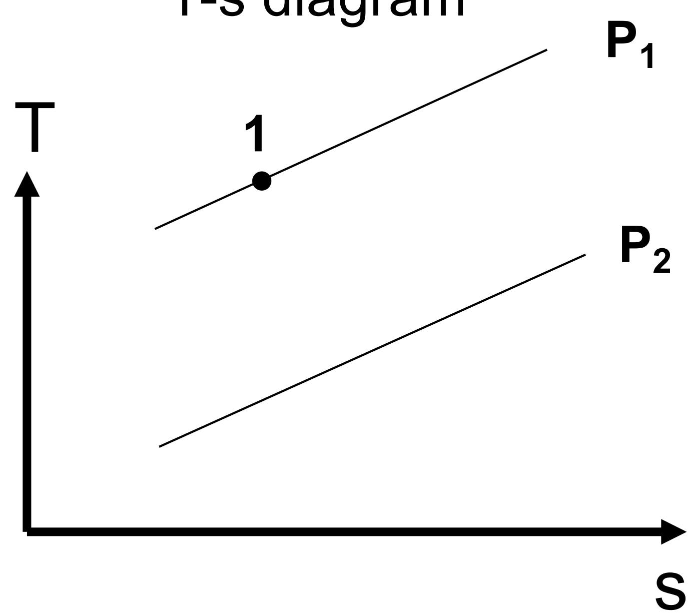
For an adiabatic nozzle with Δpe = 0 and $ke_1 << ke_2$

$$\frac{V_2^2}{2} \sim h_1 - h_2 \quad \Rightarrow \quad \eta_N = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

h-s diagram



T-s diagram



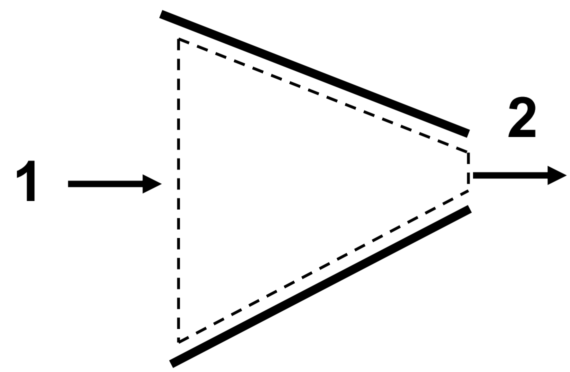
Example

Given: air nozzle, η_N=0.92

inlet: P_1 = 200 kPa, T_1=950 K

outlet: P_2=80 kPa

System:



Find:

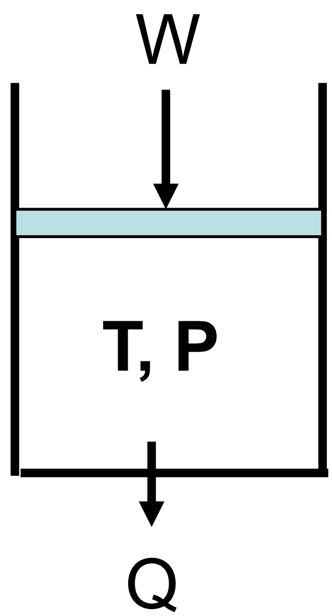
1. exit velocity, V_2
2. exit temperature, T_2

Assumptions: SSSF, adiabatic, Δpe=0, ke_1<<ke_2, constant specific heat, ideal gas

Lectures 32
Reversible Work

- Closed Systems
- Open Systems, SSSF

Closed Systems



Recall that for a quasistatic process with no friction

$$W = m\int_1^2 P dv$$

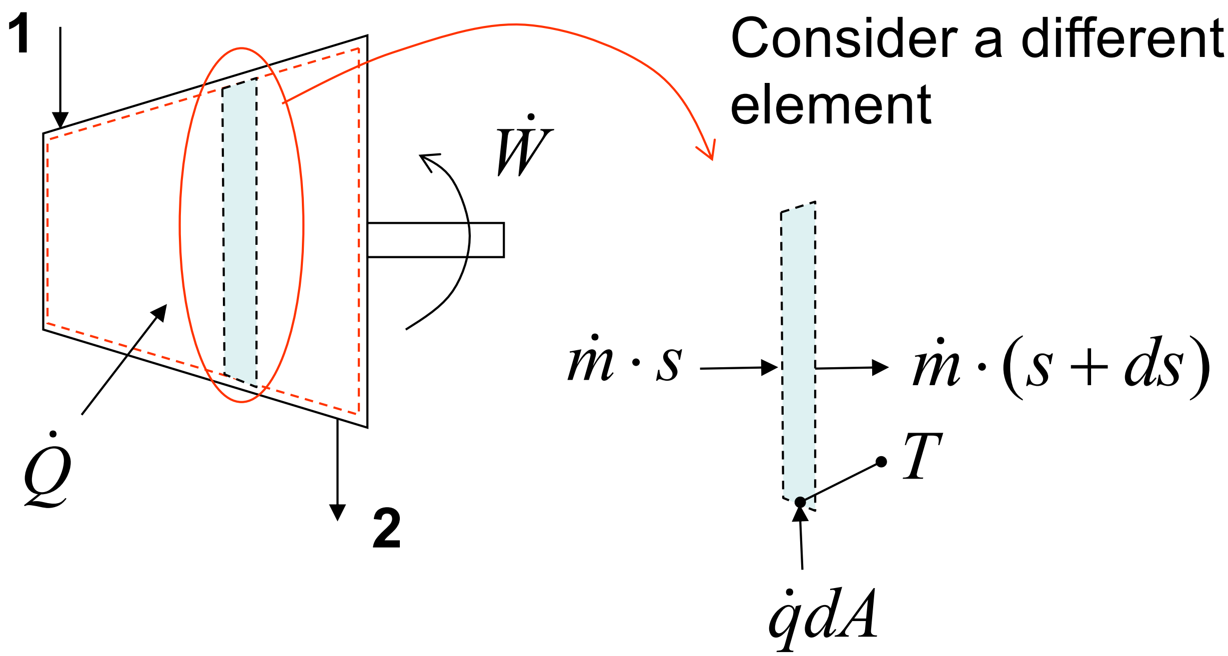
Note that (quasi-static + no friction) = internally reversible

For a given pressure change, which one requires more work: a gas or a liquid?

Open Systems, SSSF

A similar equation for internally reversible work can be developed for a SSSF open system using the 1st and 2nd Laws of Thermodynamics

Consider any work producing or consuming device (e.g., turbine or compressor) with heat transfer & changes in KE, PE



For an internally reversible process, the 2nd Law yields

$$\dot{m} \cdot ds = \frac{\dot{q}_{\text{int},rev} dA}{T} \quad \Rightarrow \quad \dot{q}_{\text{int},rev} dA = \dot{m} \cdot T ds$$

or $$\dot{Q}_{\text{int},rev} = \int^{A} \dot{q}_{\text{int},rev} dA = \int_{1}^{2} \dot{m} \cdot T ds$$

$$\dot{Q}_{\text{int},rev} = \dot{m}\int_1^2 T ds$$

and recall that

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

$$\dot{Q}_{\text{int},rev} = \dot{m}\left[\left(h_2 - h_1\right) - \int_1^2 v dP\right]$$

Now look at an overall energy balance for this SSSF internally reversible device

$$\dot{Q}_{\text{int},rev} - \dot{W}_{\text{int},rev} = \dot{m}\left[\left(h_2 - h_1\right) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

Combining the 1st and 2nd Law relations gives

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\int_1^2 v dP + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

Important Special Case

Internally reversible SSSF process with negligible changes in potential and kinetic energy (e.g., turbines, compressors, pumps, fans)

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\int_1^2 v\,dP$$

How do you depict work on a P-v diagram for this case?

For a given pressure change, which produces more work in a turbine: a liquid or a gas?

For a given pressure change, which requires more work : a pump (liquid) or a compressor (gas)?

Polytropic Processes

Internally reversible SSSF process with negligible changes in potential and kinetic energy (e.g., turbines, compressors, pumps, fans) and a process that obeys

$$Pv^n = \text{constant} = P_1 v_1^n = P_2 v_2^n$$

Therefore

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\int_1^2 v\,dP = -P_1^{1/n} v_1 \int_1^2 \frac{dP}{P^{1/n}}$$

or

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\frac{n}{n-1}\left(P_2 v_2 - P_1 v_1\right) \quad \text{n} \neq 1$$

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -P_1 v_1 \ln\left(\frac{P_2}{P_1}\right) \quad \text{n} = 1$$

Ideal Gas Special Cases

1. Ideal gas with constant specific heats operating in an adiabatic, internally reversible SSSF process with negligible changes in potential and kinetic energy (e.g., turbines, compressors, pumps, fans)

Reversible + Adiabatic = Isentropic

For an isentropic process with an ideal gas having constant specific heats,

$$\left.\frac{T_2}{T_1}\right|_{s_2=s_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1} \Rightarrow P_2 v_2^k = P_1 v_1^k$$

or more generally $Pv^k = \text{constant}$

Special case of a polytropic process with n = k

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\frac{k}{k-1}\left(P_2 v_2 - P_1 v_1\right)$$

$$= \frac{kR}{k-1}\left(T_2 - T_1\right)$$

Ideal Gas Special Cases

2. Ideal gas with constant specific heats operating in an isothermal, internally reversible SSSF process with negligible changes in potential and kinetic energy (e.g., turbines, compressors, pumps, fans)

$$Pv = RT = \text{constant}$$

Special case of a polytropic process with n = 1

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -P_1 v_1 \ln\left(\frac{P_2}{P_1}\right) = -RT \ln\left(\frac{P_2}{P_1}\right)$$

Incompressible Liquid

Incompressible liquid operating in an internally reversible SSSF process with negligible changes in potential and kinetic energy (e.g., pumps)

$$\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int},rev} = -\int_1^2 v\,dP = v(P_1 - P_2)$$

Note: Doesn't need to be adiabatic and isothermal

Example

Given: Isentropic pumping or compression

P_1 = 100 kPa, P_2 = 1 MPa

(a) Saturated water liquid @ P_1

(b) Saturated water vapor @ P_1

Find: Specific work needed

System sketch:

Assumptions: SSSF, Δke=0, Δpe=0, reversible and adiabatic process