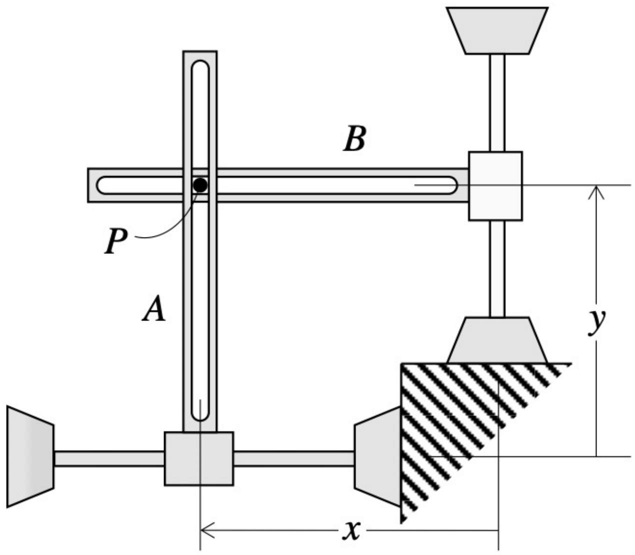
Chapter 1

Particle Kinematics Homework

Homework H1.A

Given: Pin P moves within slots cut into links A and B. The horizontal position of link A is given by $x = 10 + t^2$, whereas the vertical position of link B is given by $y = 15 - t^3$, where t is given in seconds and x and y are given in mm.

Find: Determine the velocity and acceleration of P at $t = 3$ s.



Problem H1.B

Given: A particle P travels on a path described by the Cartesian coordinates of $y = cx(b - x)$, where x and y have the units of meters.The x-component of velocity, $\dot{x}$, for P is constant.

Find: For this problem:

(a) Make a sketch of the path of P over the range of $0 < x < b$.

(b) Determine the Cartesian components of the velocity and acceleration of P at $x = 0$. Add a sketch of the velocity and acceleration vectors for P to your path drawn above.

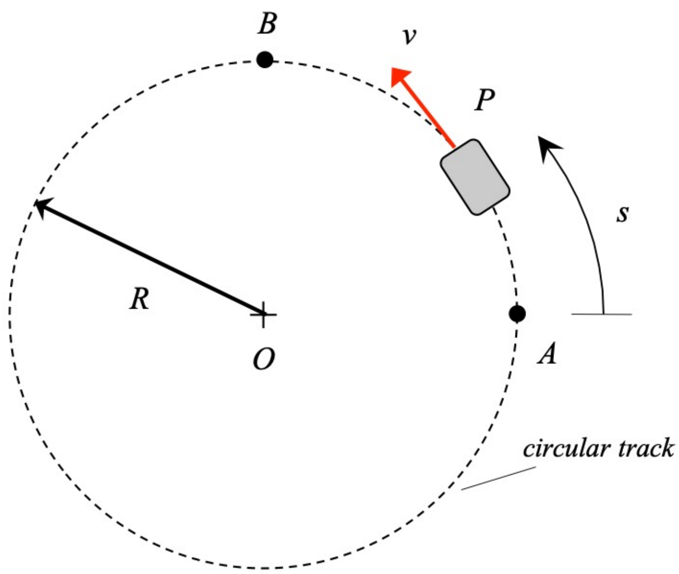
(c) Determine the Cartesian components of the velocity and acceleration of P at $x =$ b/2. Add a sketch of the velocity and acceleration vectors for P to your path drawn above.

Use the following parameters in your analysis: $b = 2$ m, $c = 5$ and $\dot{x} = 4$ m/s.

Problem H1.C

Given: An automobile P is traveling along on a circular track of radius R. A position A on the track, the automobile has a speed of v_A. At this position, the driver of automobile applies the brakes with the speed of the automobile changing with distance s traveled along the track according to the following equation: $v(s) = v_A \cos(bs)$, where s is given in meters.

Find: Determine the magnitude of the acceleration for the driver when the automobile reaches position B on the track where B is a quarter of the distance around the track from position A.



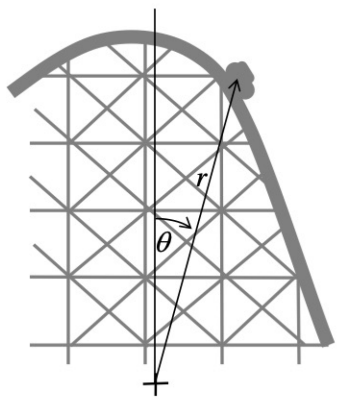
Use the following parameters in your analysis: $b = 0.001/\text{m}$, $R = 500$ m and $v_A = 80$ m/s.

Homework H1.D

PART I

Given: A roller coaster car travels along a short part of the track given by the equation $r = 30\cos(2\theta)$, where r is in meters and θ is in radians.

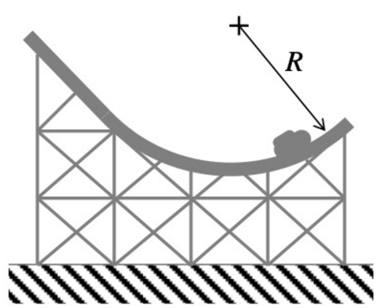
Find: What is the velocity and acceleration of the car when $\theta = \pi/10$, $\dot{\theta} = 1.25$ rad/s, and $\ddot{\theta} = 0.25$ rad/s^2.



PART II

Given: Along a circular loop at the bottom of the track, the cars will move at a speed of v_0 that is changing at a rate of $\dot{v}$. To ensure the safety of the passengers, the magnitude of the acceleration should not exceed 5g.

Find: What is the minimum allowable radius R of this part of the track?



Use the following parameters in your analysis: $v_0 = 30$ m/s and $\dot{v} = -5$ m/s^2.

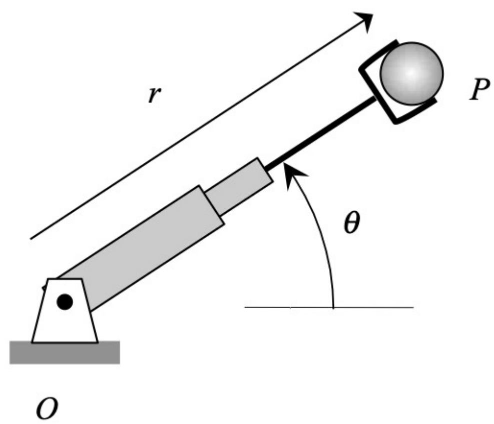
Problem H1.E

Given: A rotating and telescoping robotic arm is gripping a small sphere P in its end effector. The arm is rotating counterclockwise with a constant angular speed of $\dot{\theta}$. The arm is extending such that the radial distance from O to P is related to the rotation angle θ by the following equation:

$$r(\theta) = R_0 + R_1 \cos 2\theta$$

where r and θ are given in terms of meters and radians, respectively.

Find: Determine the velocity and acceleration of the sphere P. Write your answers as vectors in terms of the polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$.



Use the following parameters in your analysis: $R_0 = 2$ m, $R_1 = 0.5$ m, $\theta = \pi$ rad and $\dot{\theta} = 2$ rad/s.

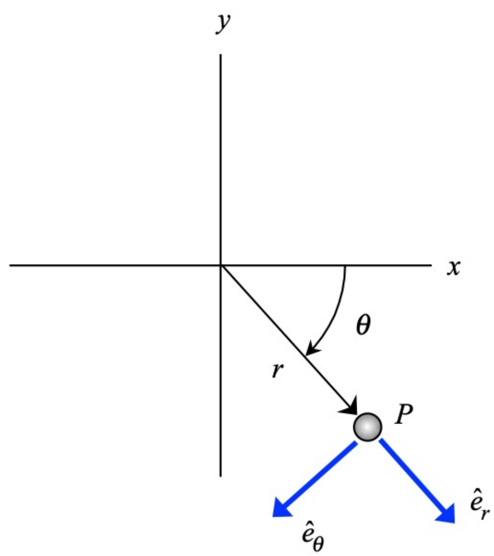
Homework H1.F

Given: Particle P moves with its position in the xy-plane given by $r(t) = r_0 + bt^2$ and $\theta(t) = ct$, where r is in terms of m and θ is in radians.

Find: For the position of P when $t = 2$ s:

(a) determine the velocity and acceleration vectors of P in terms of their $\hat{e}_r$ and $\hat{e}_\theta$ components.

(b) determine the velocity and acceleration vectors of P in terms of their $\hat{i}$ and $\hat{j}$ components.



Use the following parameters in your work: $r_0 = 2$ m, $b = 0.5/\mathrm{s}^2$ and $c = (\pi/2)/\mathrm{s}$.

Homework H1.G

Given: Particle P travels within the x-y plane along a path given by $y(x) = x^2/2 - 10x$, where x and y are given in feet. The x-component of the position for P is changing at a constant rate of $\dot{x}$.

Find: For this problem:

(a) Make a sketch of the path of particle P.

(b) Determine the velocity and acceleration of P.

(c) Show the velocity and acceleration vectors of P in your sketch of P's path.

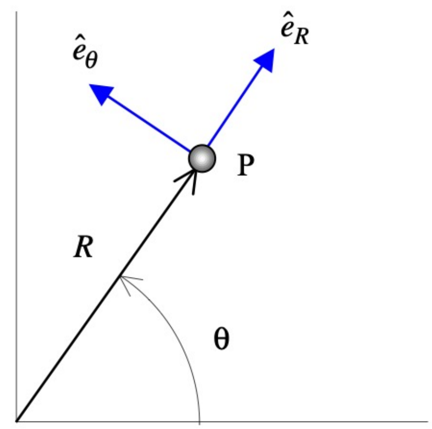
(d) Determine the rate of change of speed of P.

Use the following parameters in your analysis: $\dot{x} = 5$ ft/s and $x = 10$ ft.

HomeworkH1.H

Given: At the instant shown, the polar components for the velocity and acceleration of particle P are known as $\vec{v}_P$ and $\vec{a}_P$, respectively.

Find: For this instant, determine the speed, rate of change of speed and radius of curvature for the path of P.

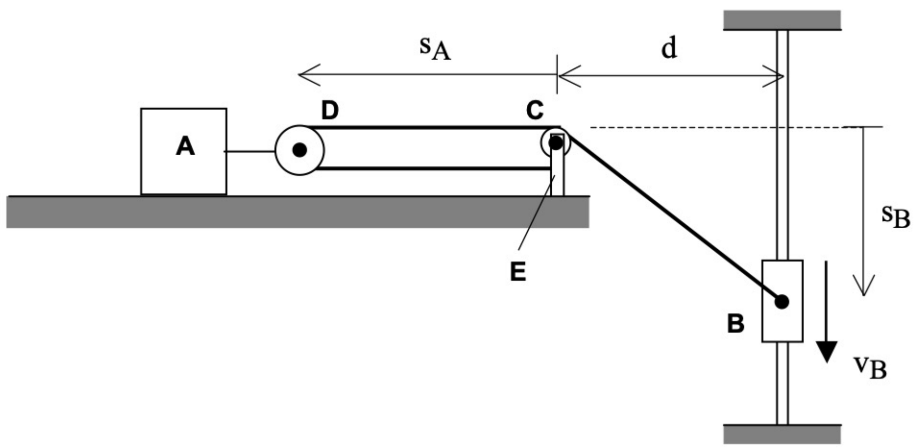


Use the following parameters in your analysis: $\vec{v}_P = (30\hat{e}_R - 40\hat{e}_\theta)$ ft/s and $\vec{a}_P = (-4\hat{e}_\theta)$ ft/s^2.

Homework H1.I

Given: An inextensible cable is attached to block B, is wrapped around pulleys C and D and is attached to a fixed support at E. Block B is constrained to move on a vertical guide, and block A is constrained to slide along a horizontal surface. Block B is moving downward with a constant speed of v_B.

Find: Determine the speed of block A.



Use the following parameters in your analysis: $d = 3$ ft, $s_B = 4$ ft and $v_B = 15$ ft/s.

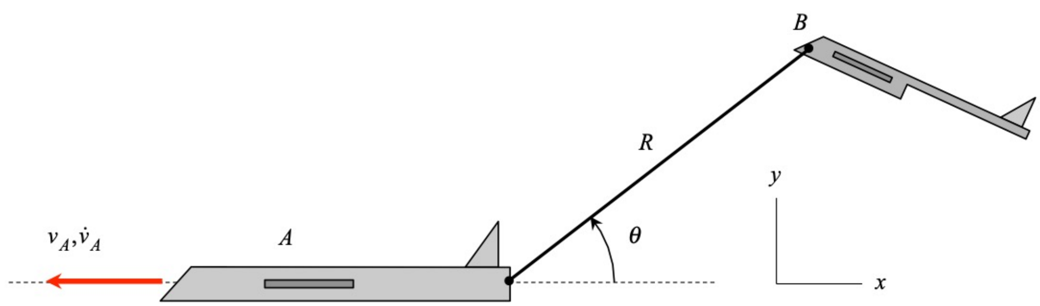
Problem H1.J

Given: Aircraft A is traveling along a straight-line path with a speed of v_A that is increasing by an amount of $\dot{v}_A$. The aircraft is towing a glider B with a cable that has a length of R. The angle θ of the towline is increasing by a constant amount of $\dot{\theta}$.

Find: For this problem:

(a) Determine the velocity vector of the point on glider B to which the cable is attached.

(b) Determine the acceleration vector of the point on glider B to which the cable is attached.



Use the following parameters in your analysis: $R = 80$ m, $v_A = 600$ m/s, $\dot{v}_A = 5$ m/s^2 and $\theta = 30°$ and $\dot{\theta} = 0.1$ rad/s.

Chapter 2

Planar Rigid Body Kinematics Homework

Homework H.2.A

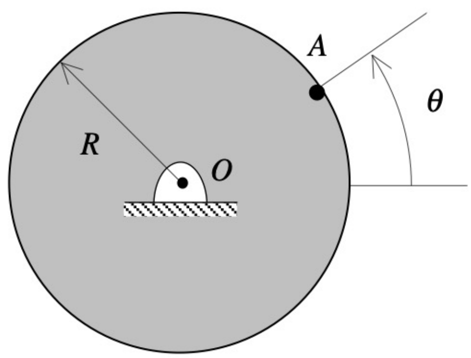
Given: The orientation angle θ for a circular disk of radius R is given by: $\theta(t) = \frac{\pi}{3}\sin\pi t$ (with θ in radians and t in seconds).

Find: Determine the velocity and acceleration vectors for point A on the outer edge of the disk for:

(a) $t = 0$ s

(b) $t = 1/3$ s

Make sketches of these vectors for the two instants in time noted above.

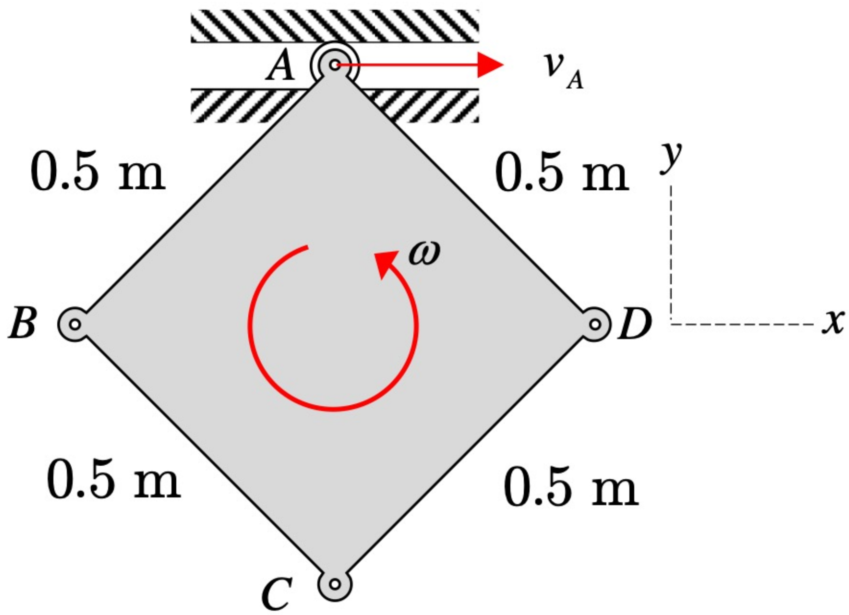


Use the following parameter in your analysis: $R = 0.75$ m.

Homework H.2.B

Given: Point A of plate ABCD moves to the right with a speed of v_A as the plate rotates with an angular speed of ω.

Find: For the instant shown, determine the velocity of point C on the plate. Write your answer as a vector.

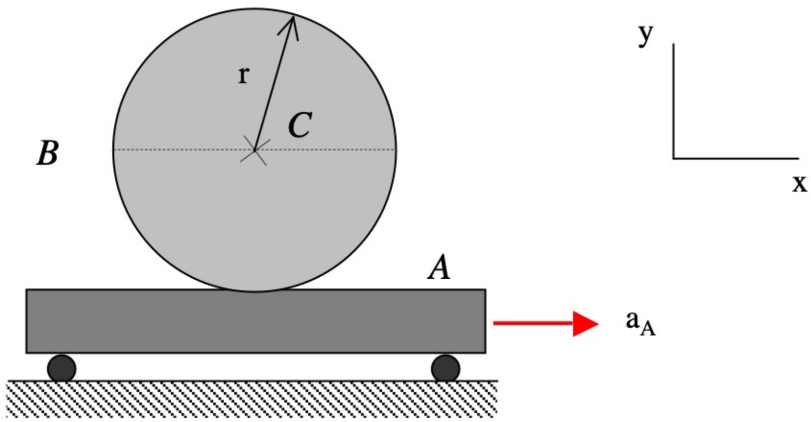


Use the following parameters in your analysis: $v_A = 6$ m/s and $\omega = 5$ rad/s (CCW).

Homework H.2.C

Given: The wheel shown below rolls without slip on cart A. The cart has velocity $\vec{v}_A = v_A\hat{i}$ and acceleration $\vec{a}_A = a_A\hat{i}$. The center of the wheel has velocity $\vec{v}_C = v_C\hat{i}$ and acceleration $\vec{a}_C = a_C\hat{i}$.

Find: Determine the velocity and acceleration of point B.

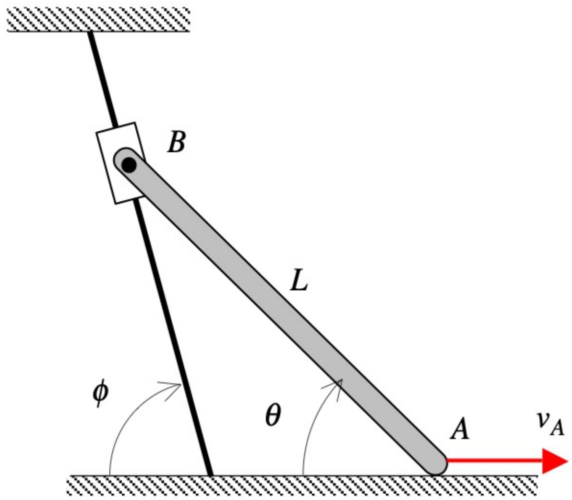


Use the following parameters in your analysis: r = 0.6 m, v_A = 8 m/s, v_C = 5 m/s, a_A = -4 m/s^2 and a_C = 8 m/s^2.

Homework H.2.D

Given: Thin bar AB (having a length of L) moves in a way that ends A and B slide along straight, fixed guides as shown in the figure. End A has a constant speed of v_A to the right as the bar moves.

Find: Determine the velocity and acceleration of end B.



Use the following parameters in your analysis: $L = 2$ ft, $v_A = 4$ ft/s, $\theta = 30°$ and $\phi = 53.13°$.

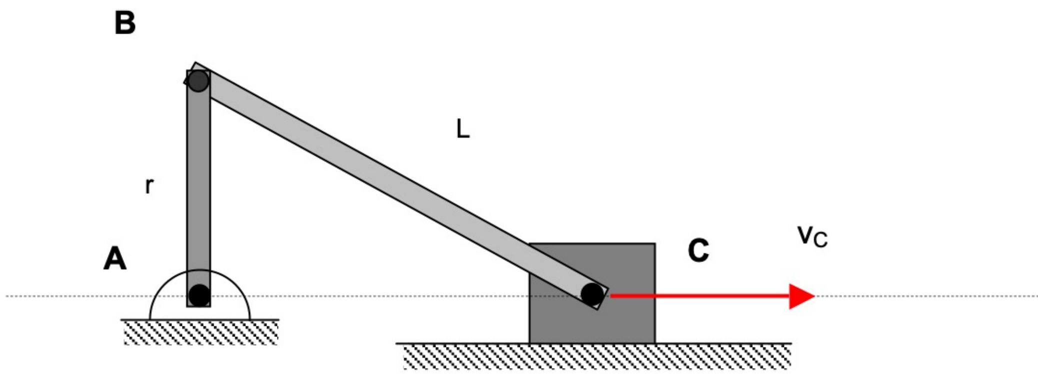
Homework H.2.E

Given: A mechanism is made up of two links AB and BC pinned together at point B, and with link AB pinned to ground at point A. Link BC is pinned to a block that is constrained to move on a horizontal plane. At the instant shown, AB is vertical, and point C is moving to the right with a speed of v_C with this speed changing at a rate of $\dot{v}_C$.

Find: For this position:

(a) Determine the angular velocity of links AB and BC. Write your answers as vectors.

(b) Determine the angular acceleration of links AB and BC. Write your answers as vectors.



Use the following parameters in your analysis: r = 1 ft, L = 1.5 ft, v_C = 10 ft/s and $\dot{v}_C$ = -20 ft/s^2.

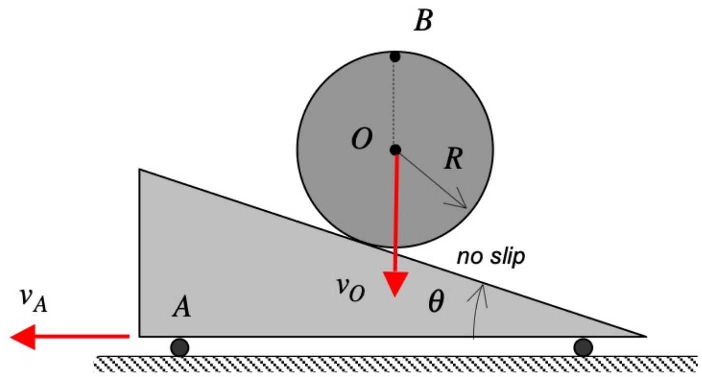
Homework H.2.F

Given: Wedge A moves to left with a speed of v_A. A circular disk is able to roll without slipping on the inclined surface of the wedge. At the instant shown, the velocity of the center of the disk O is moving straight down with an unknown speed of v_O.

Find: For this problem:

(a) Determine the angular velocity of the disk and the velocity of O. Write your answers as vectors.

(b) Determine the velocity of point B on the disk that is directly above O on the outer surface of the disk. Write your answer as a vector.



Use the following parameters in your analysis: $R = 1.5$ ft, $v_A = 30$ ft/s and $\theta = 20°$.

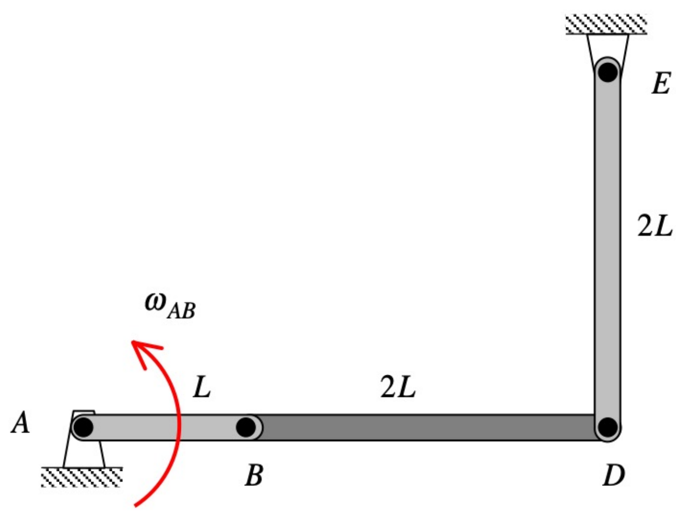
Homework H.2.G

Given: A mechanism is made up of links AB, BD and DE. At the instant shown, links AB and BD are aligned horizontally, and link DE is vertical.

Find: For this position:

(a) Locate the instant center for link BD.

(b) Determine the angular velocities of links BD and DE. Write your answers as vectors.



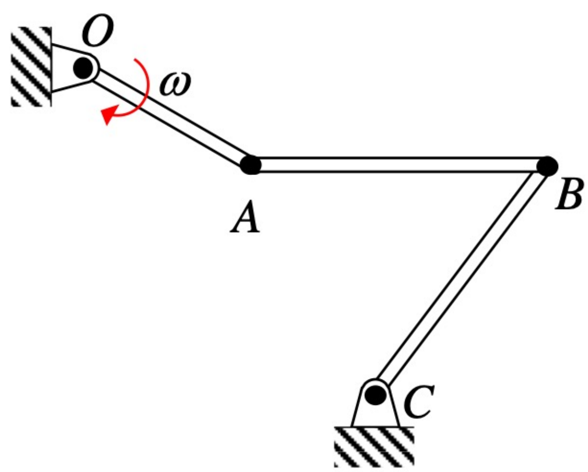
Use the following parameters in your analysis: L = 2 ft and ω_{AB} = -4 rad/s.

Homework H.2.H

Consider the four mechanisms shown below. Respond to the questions posed. Feel free to draw directly on these sheets and submit for your homework solution. Use a straight edge when making your drawings.

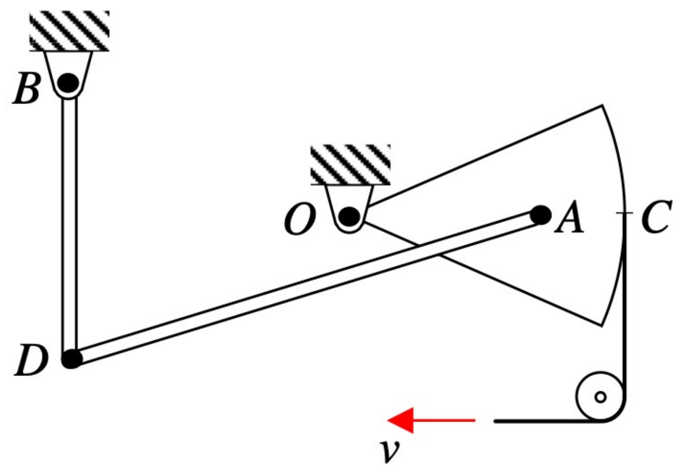
MECHANISM NO. 1

Link OA is rotating in the clockwise sense. Determine the location of the instant center (IC) of link AB. From the location of this IC, determine the sense of rotation for links AB and BC. Justify your answers in words.



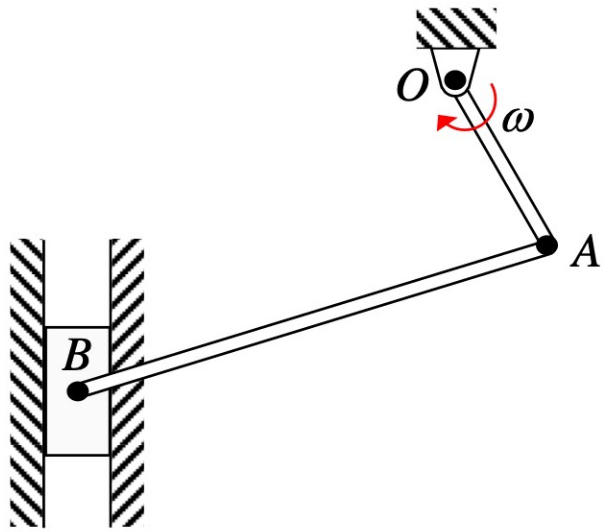
MECHANISM NO. 2

Determine the location of the instant center (IC) of link AD. From the location of this IC, determine the sense of rotation for links AD and DB. Justify your answers in words.



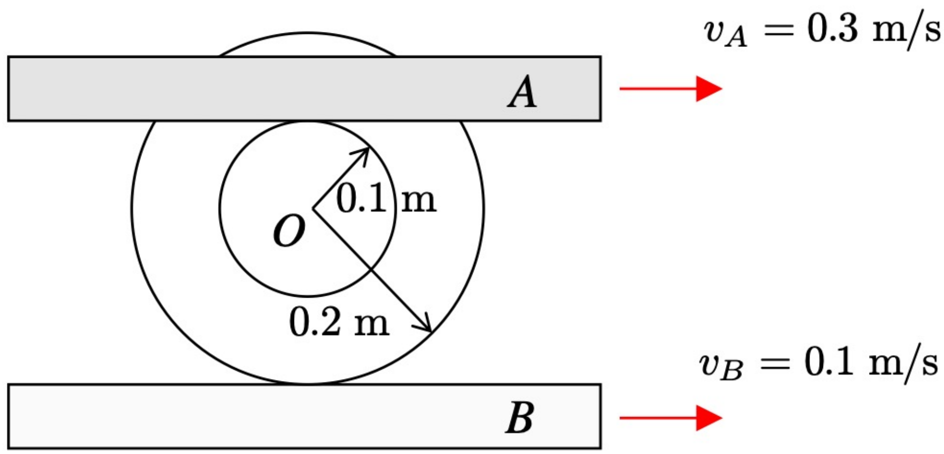
MECHANISM NO. 3

Link OA is rotating in the clockwise sense. Determine the location of the instant center (IC) of link AB. From the location of this IC, determine the sense of rotation for link AB and the sense of translation for the piston. Justify your answers in words.



MECHANISM NO. 4

Racks A and B are moving in the directions shown with the speeds provided. Assume that the gear does not slip on the racks. Determine the location of the instant center (IC) of the gear. From the location of this IC, determine the sense of rotation for the gear. Justify your answers with numbers and words.



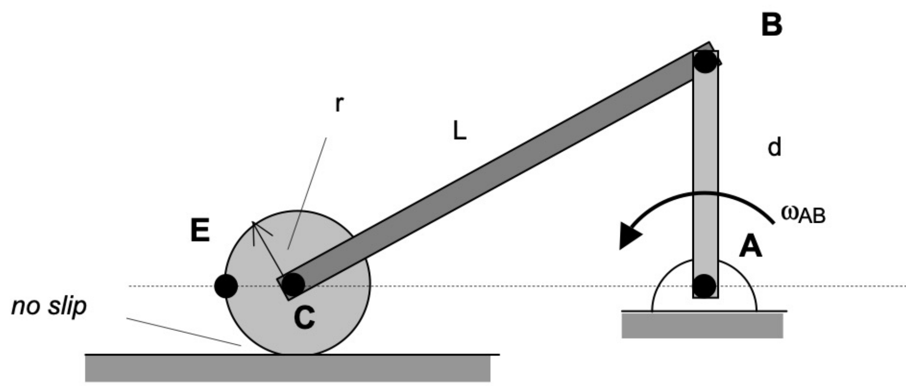
Homework H.2.I

Given: A mechanism is made up of links AB and BC and a wheel pinned to BC at the wheel's center C. The wheel rolls without slipping on a horizontal surface. Link AB rotates counterclockwise with a constant rate of ω_{AB}. At the instant shown, link AB is vertical.

Find: For this position:

(a) Determine the angular velocity of link BC and the wheel. Write your answers as vectors.

(b) Determine the angular acceleration of link BC and the wheel. Write your answers as vectors.



Use the following parameters in your analysis: $r = 0.5$ m, $L = 2$ m, $d = 1$ m and $\omega_{AB} = 7$ rad/s. You may solve the problem using the method of instant centers and/or by adopting a vector approach.

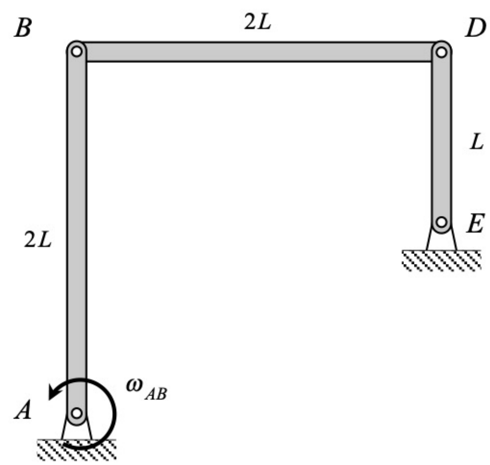
Homework H.2.J

Given: Rigid links AB, BD and DE are joined together to form the mechanism shown. Link AB is known to be rotating at a constant rate of ω_{AB}. At the instant shown, links AB and DE are vertical, and link BD is horizontal.

Find: For the position shown:

(a) Where is the instant center for link BD? Based on this result, what are the angular velocities of links BD and DE? Write your answers as vectors.

(b) Determine the angular acceleration of links BD and DE. Write your answers as vectors.



Chapter 3

Moving Reference Frame Kinematics Homework

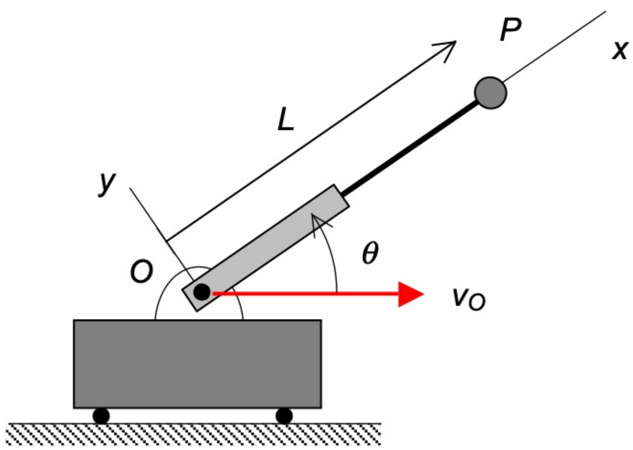
newpage **Homework H.3.A**

Given: A telescoping arm OP is pinned to a cart at end O. The cart moves along a horizontal surface with a constant speed of v_O. The angle of the arm, θ, is increasing at a constant rate of $\dot{\theta}$ and is extending at a rate of $\dot{L}$. The xyz coordinate system is attached to the telescoping arm with its origin at end O of the arm.

Find: For this problem:

(a) Determine the velocity and acceleration of particle P. Express your answers as vectors in terms of their x-y components.

(b) Make a sketch of the velocity and acceleration vectors found above.



Use the following parameters in your analysis: $\theta = 90°$, $v_O = 6$ m/s, $\dot{\theta} = 5$ rad/s, $L = 2$ m, $\dot{L} = 0$ m/s, and $\ddot{L} = 3$ m/s^2.

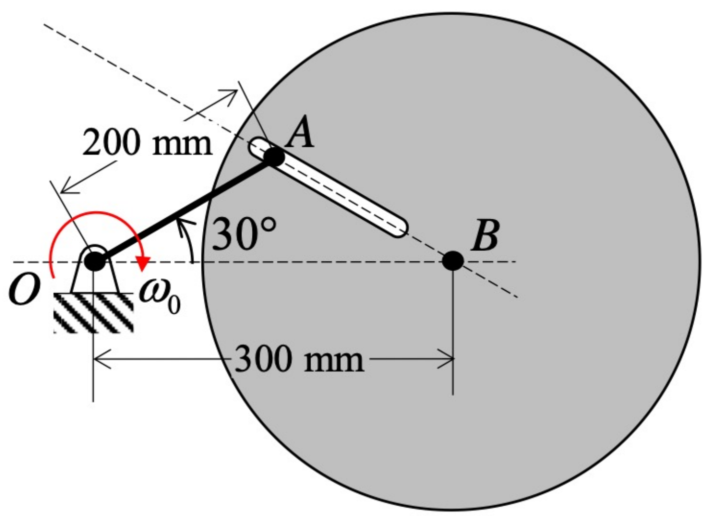
Homework H.3.B

Given: Link OA rotates with a constant angular speed of ω_0. A pin at point A moves within a smooth slot that has been cut in the disk, which is pinned to ground at point B.

Find: Determine:

(a) The angular velocity of the disk.

(b) The acceleration of pin A as seen by an observer on the disk.



Use the following parameters in your analysis: $\omega_0 = 10$ rad/s (CW).

Homework H.3.C

Given: The mechanism shown below is driven by a motor attached at B. End A of link AB is free to slide within a straight slot within link OC. The angular velocity of ω_{AB} is in the direction shown and constant.

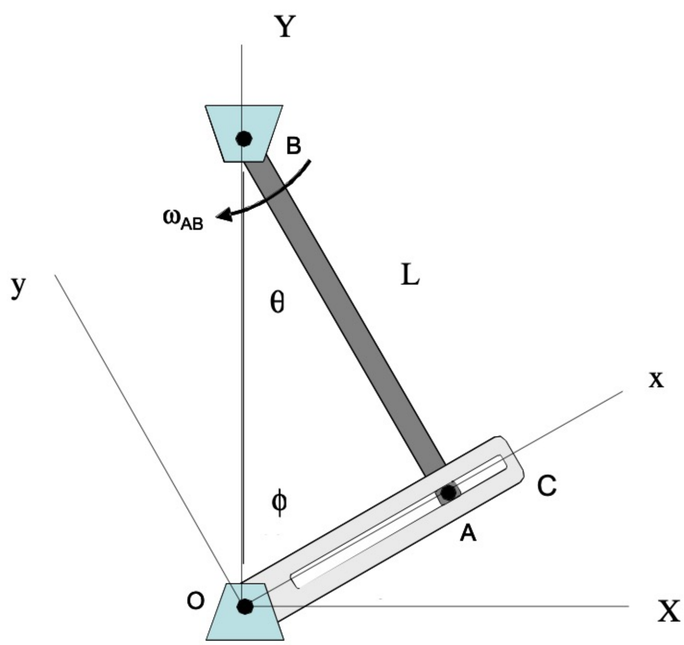
Find: For the position shown, determine:

(a) The angular velocity of OC.

(b) The velocity of A relative to an observer positioned on OC and rotating with OC.

(c) The angular acceleration of OC.

(d) The acceleration of A relative to an observer positioned on OC and rotating with OC.



Use the following parameters in your analysis: $\omega_{AB} = 2$ rad/s, $L = 0.17$ m, $\theta = 30°$ and $\phi = 60°$.

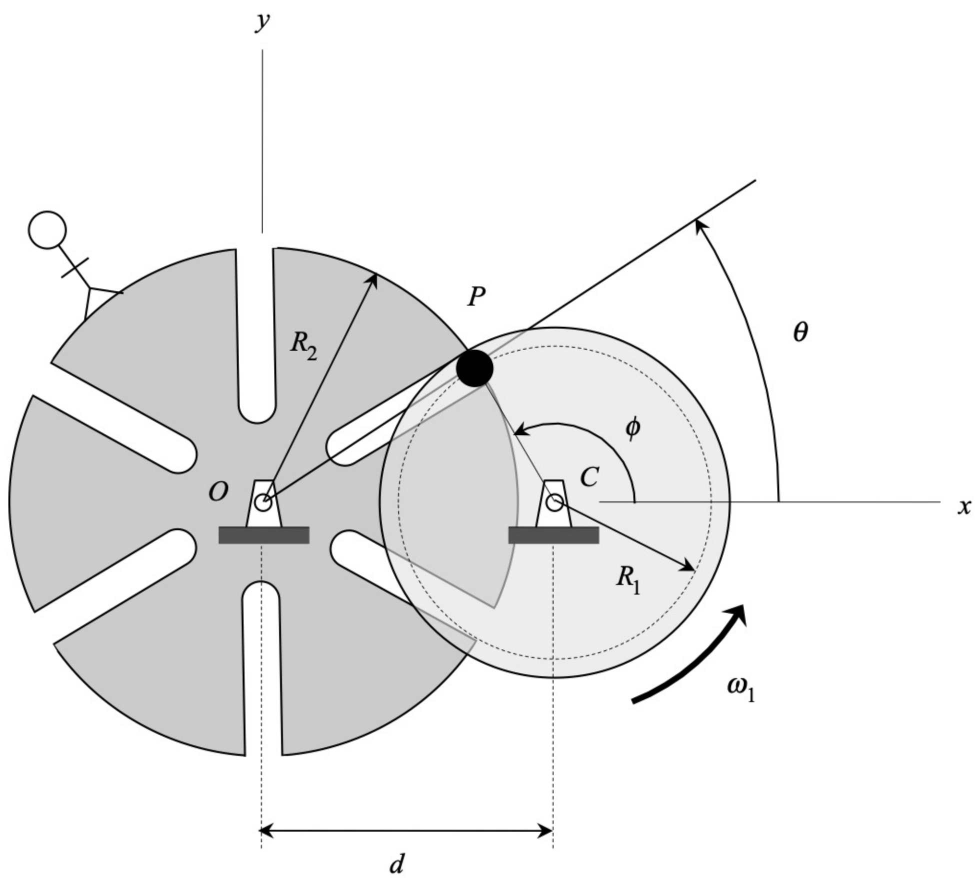
Homework H.3.D

Given: A "Geneva mechanism" is made up of a slotted wheel (on the left) and a disk (on the right), with the wheel and disk having parallel axes. Pin P on the disk is able to slide in a slot in the slotted wheel as the disk turns. The disk is given a constant counter-clockwise rotation rate of ω_1. For the position shown, it is known that $\theta = 30°$ and $\phi = 120°$. An observer also attached to the slotted wheel.

Find: For this problem:

(a) Determine the angular velocity and angular acceleration of the slotted disk for the position shown. Write your answers as vectors.

(b) Determine the angular velocity and angular acceleration of the slotted disk for the position corresponding to $\phi = 150°$. Write your answers as vectors.

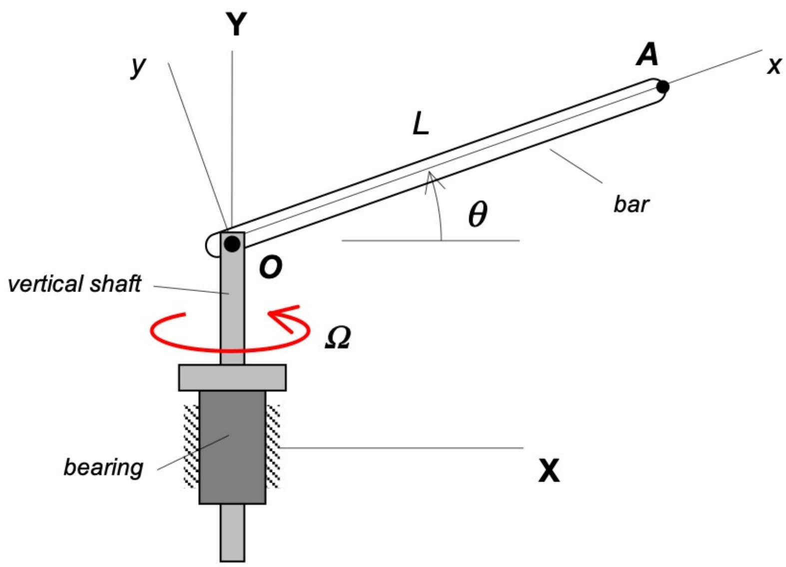


Use the following parameters in your analysis: $R_1 = 2$ in, $d = 4$ in and $\omega_1 = 6$ rad/s.

Homework H.3.E

Given: A shaft rotates about a fixed vertical axis at a constant rate of Ω, as shown below. A straight bar OA, having a length of L, is pinned to point O on the shaft, with O being on the rotation axis of the shaft. At the instant when $\theta = 0°$, bar OA is being raised at a rate of $\dot{\theta}$ from the horizontal plane, with this rate changing at a rate of $\ddot{\theta}$. A set of xyz coordinate axes is attached to bar OA with its origin at O. A second set of coordinate axes, XYZ, are fixed to ground. At the instant when $\theta = 0°$, the xyz and XYZ axes are aligned with each other.

Find: For the instant when $\theta = 0°$, determine the angular velocity and angular acceleration of bar OA.



Use the following parameters in your analysis: Ω = 5 rad/s, $\dot{\theta}$ = 4 rad/s, $\ddot{\theta}$ = -3 rad/s^2 and L = 2 m.

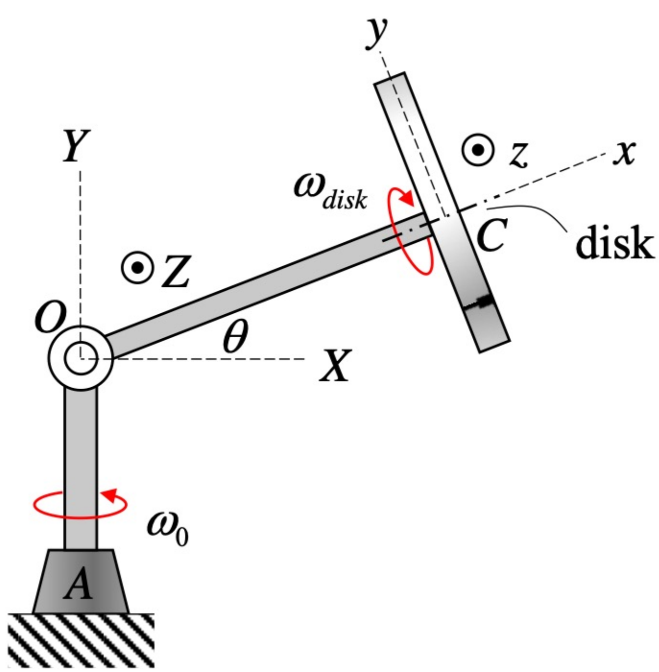
Homework H.3.F

Given: A disk and shaft OC are mounted in a clevis that rotates about a fixed vertical axis at a rate of ω_0. The shaft and disk rotate with respect to the clevis with a rate of ω_{disk} in the direction shown below, with the angle θ held constant. The XYZ coordinate system is fixed with the Y-axis aligned with the fixed rotation axis of the clevis. The xyz coordinate system is attached to the disk with the x-axis aligned with OC for all time. For the position shown below, the z- and Z-axes are aligned.

Find: For the position shown:

(a) Determine the angular velocity of the disk. Write your answer as a vector in terms of its xyz components.

(b) Determine the angular acceleration of the disk. Write your answer as a vector in terms of its xyz components.



Use the following parameters in your analysis: $\theta = 36.87°$, $\omega_0 = 4$ rad/s = constant and $\omega_{disk} = 6$ rad/s = constant.

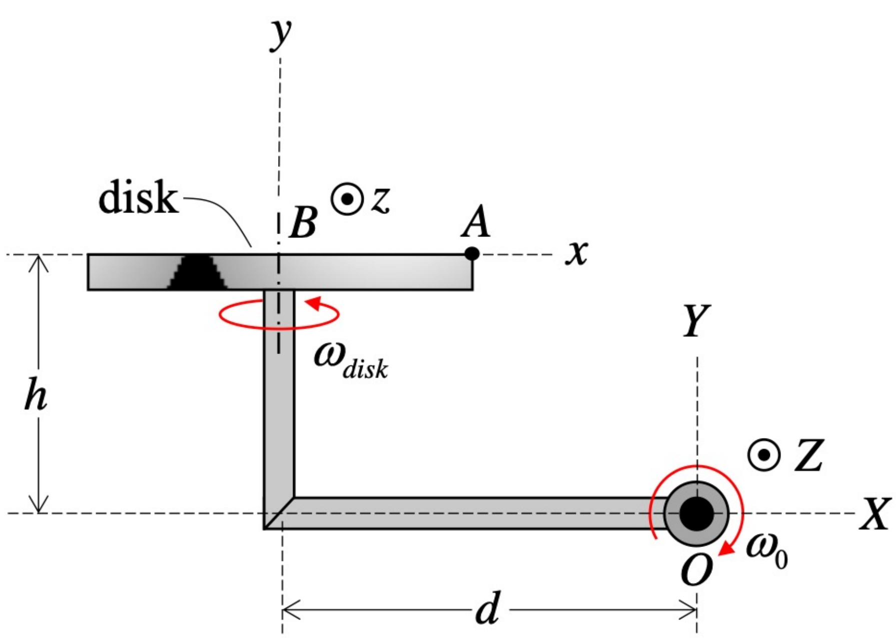
Homework H.3.G

Given: Arm OB rotates about a fixed axis with a constant rate of ω_0. A disk of radius R rotates about its central axis with a constant rate of ω_{disk} relative to the arm OB. The XYZ coordinate system is fixed with the Z-axis aligned with the fixed rotation axis of OB. The xyz coordinate system is attached to the disk with the y-axis aligned with the upper part of the arm for all time. For the position shown below, the xyz axes are aligned with the XYZ axes.

Find: For the position shown:

(a) Determine the angular acceleration of the disk. Write your answer as a vector in terms of its xyz components.

(b) Determine the acceleration of point A on the disk. Write your answer as a vector in terms of its xyz components.



Use the following parameters in your analysis: $\omega_0 = 4$ rad/s, $\omega_{disk} = 3$ rad/s, $d = 18$ cm, $h = 10$ cm and $R = 6$ cm.

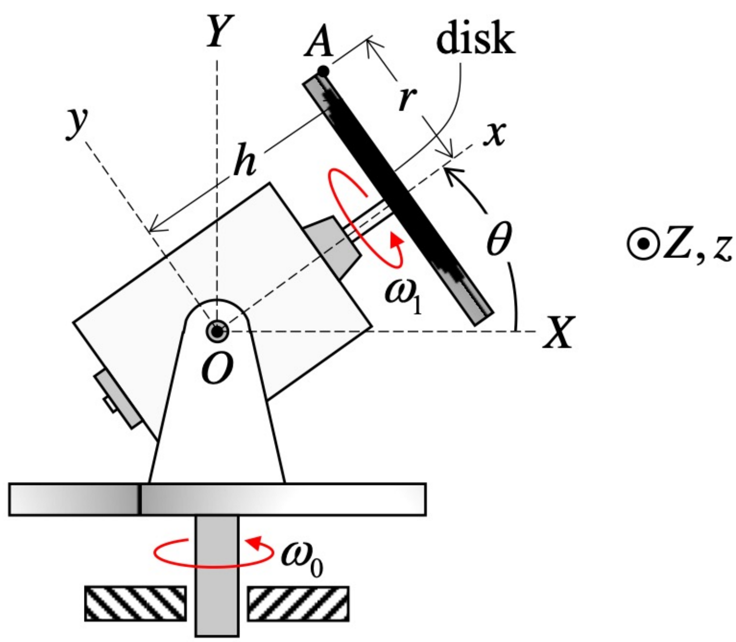
Homework H.3.H

Given: A motor is attached to a platform that is rotating with a constant rate of ω_0 about a fixed vertical axis. The body of the motor pivots about a moving horizontal axis at a constant rate of $\dot{\theta}$ with the shaft of the motor rotating at a constant rate of ω_1.

Find: Determine:

(a) The angular acceleration of the disk attached to the shaft of the motor.

(b) The velocity of point A on the disk when A is at the top of the disk.

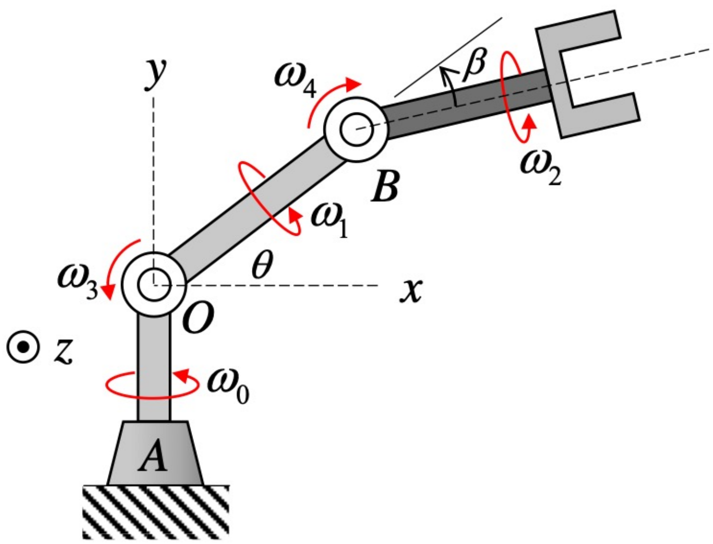


Use the following parameters in your analysis: $\omega_0 = 1$ rad/s, $\theta = 30°$, $\dot{\theta} = 0.3$ rad/s, $\omega_1 = 60$ rad/s, $h = 0.15$ m, and $r = 0.1$ m.

Homework H.3.I

Given: Consider the system shown below.

Find: Determine the angular velocity and angular acceleration of the jaws. Assume that the rotation rates are constant.

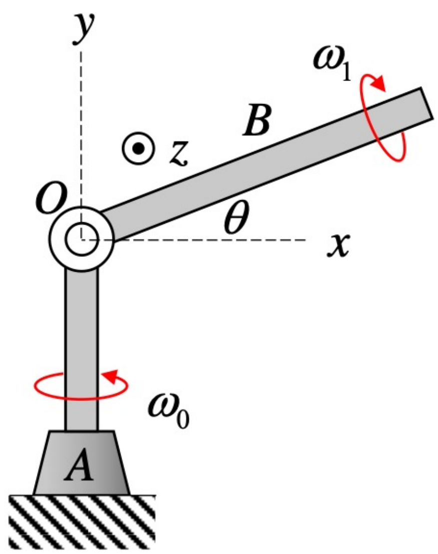


Use the following parameters in your analysis: $\omega_0 = \omega_2 = \omega_4 = 0$ rad/s, $\omega_1 = 2$ rad/s, $\theta = 60°$ and $\dot{\theta} = \omega_3 = 1.5$ rad/s.

Homework H.3.J

Given: Consider the system shown below.

Find: Determine the angular velocity and angular acceleration of shaft B. Assume all rotation rates are constant.



Use the following parameters in your analysis: $\omega_0 = 4$ rad/s, $\omega_1 = 3$ rad/s, $\theta = 30°$ and $\dot{\theta} = 2$ rad/s.

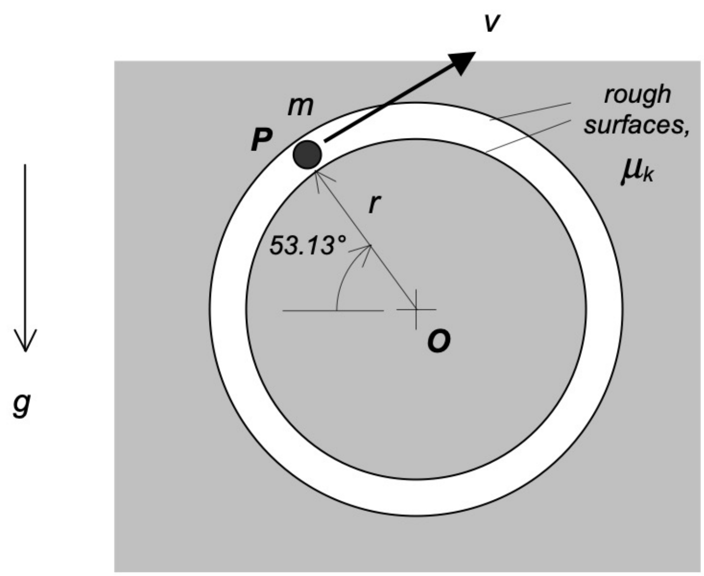
Chapter 4

Particle Kinetics Homework

Homework H.4.A

Given: Particle P (of mass m) moves within a vertical plane inside a rough, circular slot. The coefficient of kinetic friction between particle P and the slot is μ_k, and the radius of the slot is r. At the position shown below, the speed of P is known to be v.

Find: For this position:
(a) Determine the numerical value of the normal contact force of the slot on P;
(b) Determine the rate of change of speed of P.

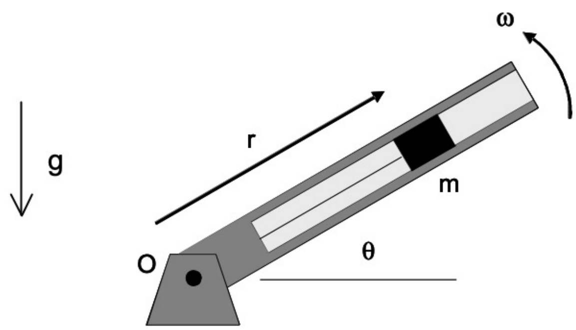


Use the following parameters in your analysis: $m = 8$ kg, $\mu_k = 0.2$, $r = 2$ m and $v = 3$ m/s.

Homework H.4.B

Given: The mechanism, shown below, consisting of a rotating arm, frictionless slot, and block of mass m, is driven at a constant speed ω by a motor attached at point O.

Find: Determine:
(a) The tension in the cable at the instant shown provided $\omega = 10$ rad/s;
(b) The normal force exerted on the block provided $\omega = 10$ rad/s;
(c) The minimum angular velocity ω necessary to keep the cable taut at the instant shown.

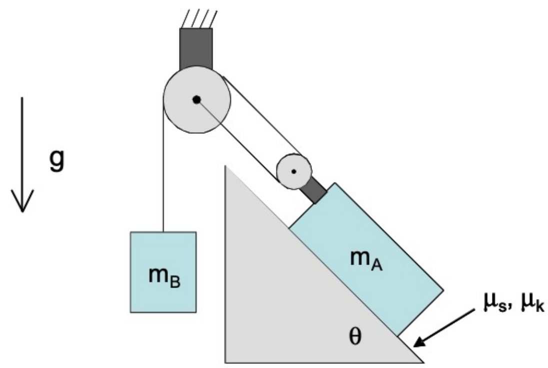


Use the following parameters in your analysis: $r = 0.3$ m, $m = 12$ kg and $\theta = 60°$.

Homework H.4.C

Given: The system shown below is released from rest in the configuration shown.

Find: Determine:
(a) The acceleration of block A;
(b) The acceleration of block B;
(c) The tension in the cable.

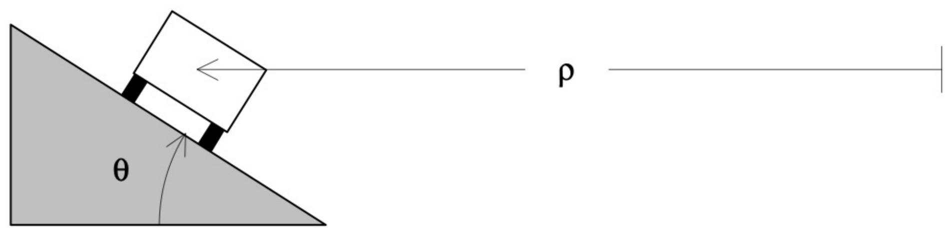


Use the following parameters in your analysis: $m_A = 40$ kg, $m_B = 80$ kg, $\mu_s = 0.3$, $\mu_k = 0.15$ and $\theta = 60°$.

Homework H.4.D

Given: The race car, shown below, travels around a banked curve at Daytona International Speedway.

Find: Determine:
(a) The speed at which the race car can circumvent the curve without the assistance of a lateral friction force;
(b) The maximum speed the race car can circumvent the curve with the assistance of a lateral friction force.

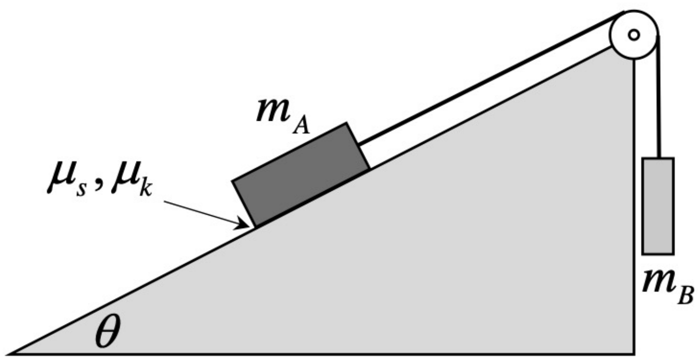


Use the following parameters in your analysis: $\theta = 25°$, $\mu_s = 0.85$ and $\rho = 450$ m.

Homework H.4.E

Given: The system shown is released from rest.

Find: On release, determine:
(a) The acceleration of each block;
(b) The tension in the cable.



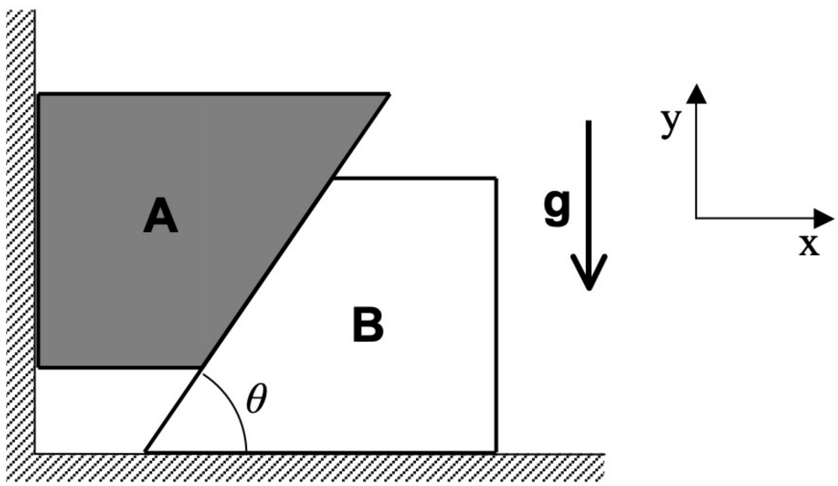
Use the following parameters in your analysis: $\mu_s = 0.30$, $\mu_k = 0.2$, $m_A = 3$ kg, $m_B = 4$ kg, and $\theta = 36.87°$.

Homework H.4.F

Given: Blocks A and B (having masses of m_A and m_B, respectively) are released from rest. Assume that friction is negligible.

Find: Determine:
(a) The accelerations of A and B;
(b) The normal forces acting on A and B.

Hint: The component of acceleration normal to the contact surface is equal for A and B.

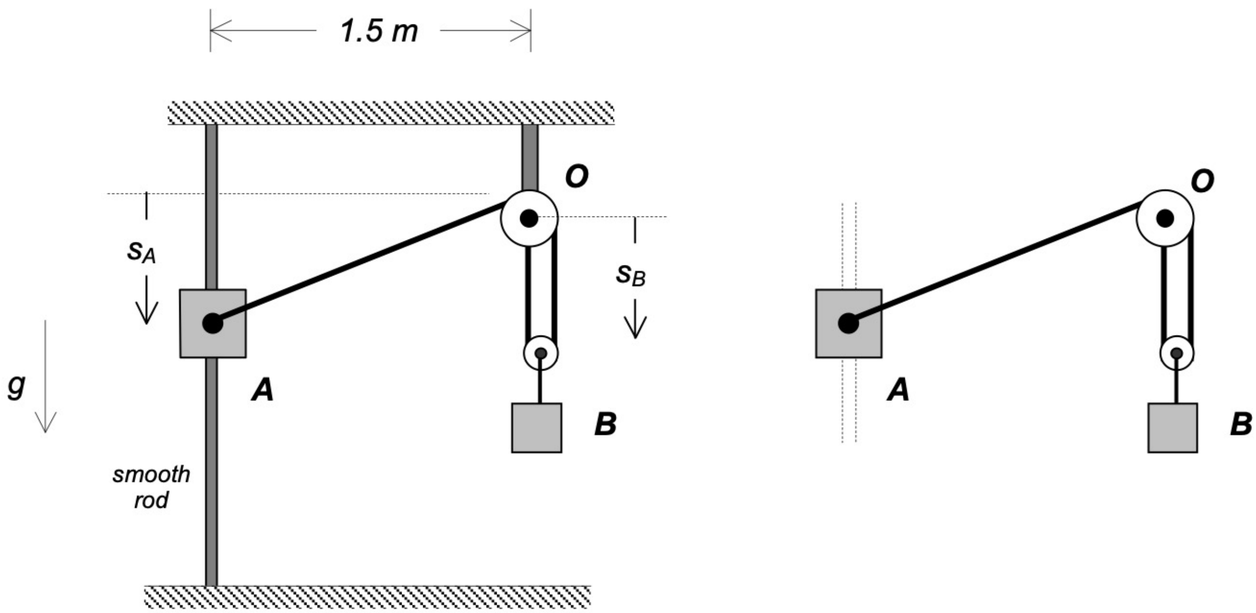


Use the following parameters in your analysis: $m_A = 160$ kg, $m_B = 80$ kg and $\theta = 30°$.

Homework H.4.G

Given: Particles A and B (having masses of m_A and m_B) are interconnected by the cable-pulley system shown in the figure. Both particles are constrained to vertical motion with particle A able to slide on a smooth vertical rod. The system is released at $s_A = 0$ m with A traveling downward with a speed of v_{A1}. Assume the pulleys to be small, massless and frictionless.

Find: Find the speed of particle A when A has reached the position of s_A.

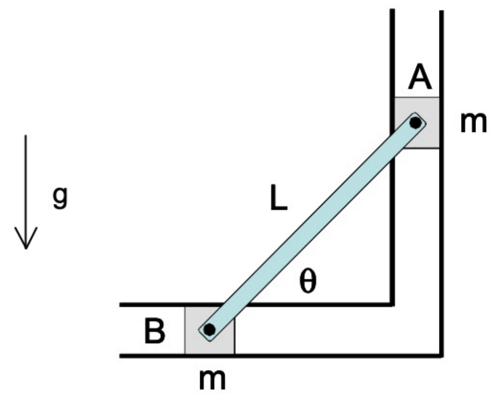


Use the following parameters in your analysis: $m_A = 10$ kg, $m_B = 10$ kg, $v_{A1} = 5$ m/s and $s_A = 2$ m.

Homework H.4.H

Given: The mechanism shown below, consisting of two blocks A and B and a massless bar, is released from rest in the configuration shown.

Find: Determine the speed of block A when it reaches the bottom of the slot.

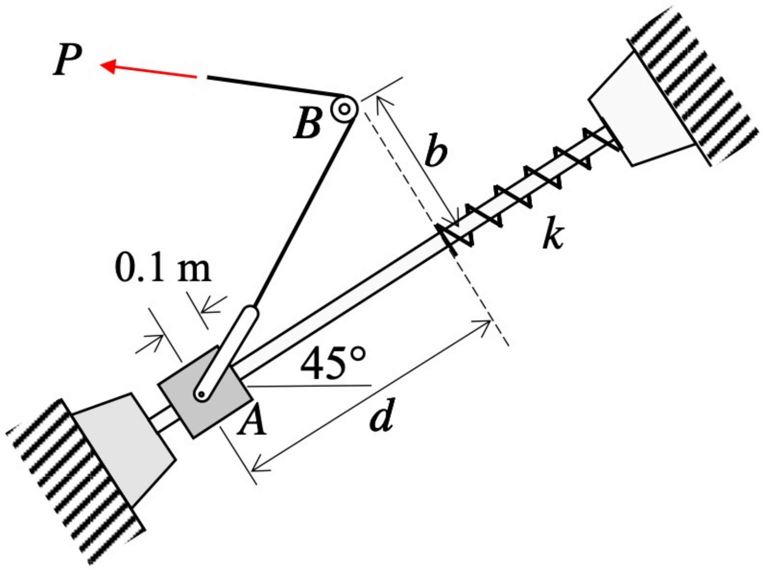


Use the following parameters in your analysis: $m = 6$ kg, $\theta = 53.13°$ and $L = 2$ m.

Homework H.4.I

Given: A constant force P acts at the free end of the cable as block A (having mass m) is pulled up the smooth rod. The system starts out from rest.

Find: Determine the stiffness k of the spring corresponding to a maximum spring compression of Δ_{max}.

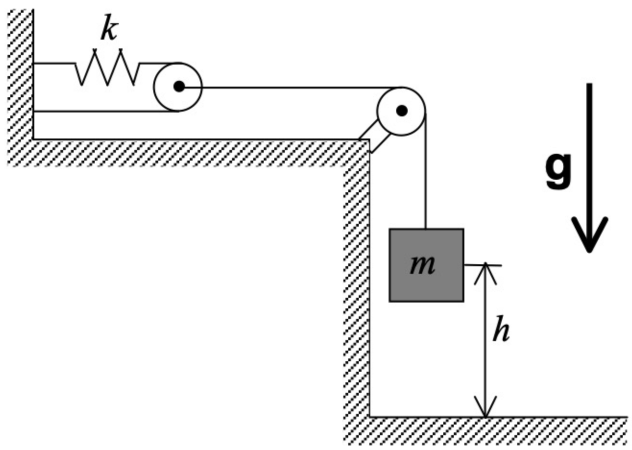


Use the following parameters in your analysis: $m = 28$ kg, $P = 50$ N, $b = 1$ m, $d = 1.2$ m and $\Delta_{max} = 0.02$ m.

Homework H.4.J

Given: A block with mass m is attached to a pulley system. The other end of the cable is attached to a spring with stiffness k, as shown in the diagram. The spring is initially unstretched when the block is at a distance h from the ground. The system is released from rest. Assume all pulleys are massless.

Find: Determine the velocity of the block when it hits the ground.

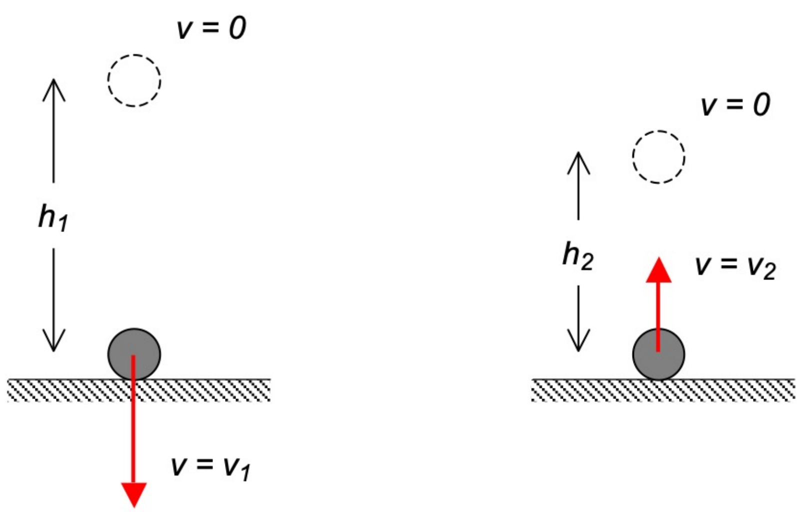


Use the following parameters in your analysis: $m = 200$ kg, $k = 3$ N/m and $h = 0.75$ m.

Homework H.4.K

Given: A particle of mass m is dropped from rest when at a height h_1 above a rigid floor. The particle impacts the floor with a speed of v_1. This impact of the particle with the floor lasts for a short duration of time Δt, and after the impact is complete, the particle rebounds upward with a speed of v_2. The particle continues upward reaching a maximum height of h_2.

Find: For this problem:
(a) Determine the average force acting on the particle by the floor during impact in the presence of gravity;
(b) Determine the average force acting on the particle by the floor during impact in the absence of gravity;
(c) Compare your answers from (a) and (b);
(d) Determine the value of h_2/h_1.

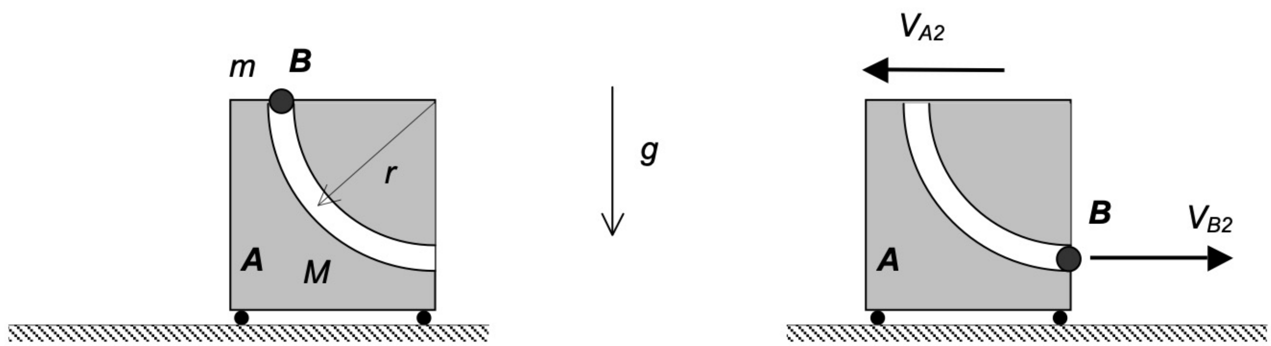


Use the following parameters in your analysis: $\Delta t = 0.002$ s, $m = 15$ kg, $v_1 = 80$ m/s and $v_2 = 50$ m/s.

Homework H.4.L

Given: A smooth, circular slot is cut into block A with block A being constrained to move along a smooth, horizontal surface. The slot is vertical at the top surface of the block with the slot being horizontal at the right edge of the block, as shown in the figures below. In Position 1, block A is stationary, and a particle B is released from rest into the upper opening of the slot. At Position 2 shown below, particle B is exiting the slot at the right edge of the block. The masses of A and B are M and m, respectively. The radius of the circular slot is r.

Find: Determine the velocity of block A and the velocity of particle B at position 2.



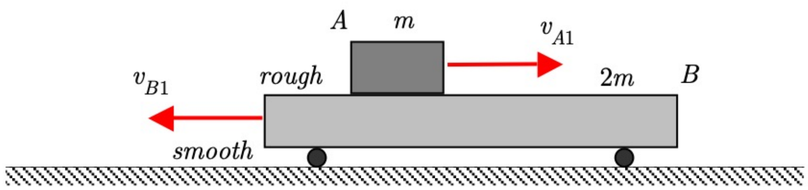
POSITION 1 **POSITION 2**

Use the following parameters in your analysis: $m = 20$ kg, $M = 40$ kg and $r = 2$ m.

Homework H.4.M

Given: Block B (of mass $2m$) is able to slide along a smooth horizontal surface. Block A (of mass m) is able to slide along the rough top surface of block B, as shown in the figure. Initially, A is traveling to the right with a speed of v_{A1}, and block B is traveling to the left with a speed of v_{B1}.

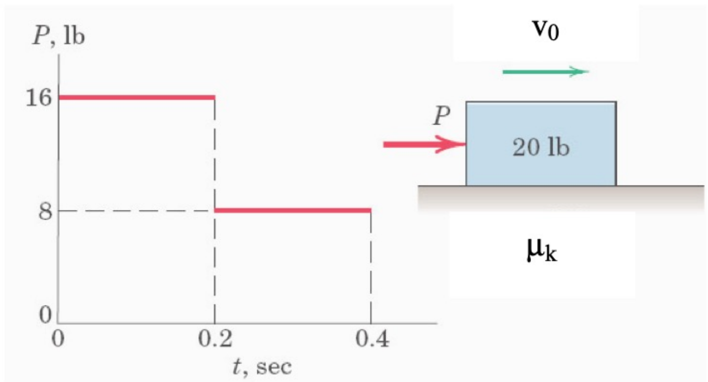
Find: Determine the velocity of block B when block A has to come rest relative to block B.



Homework H.4.N

Given: A block is sliding to the right on a rough, horizontal surface with a speed of v_0 when a force P is applied.

Find: Determine the speed of the block at time t.

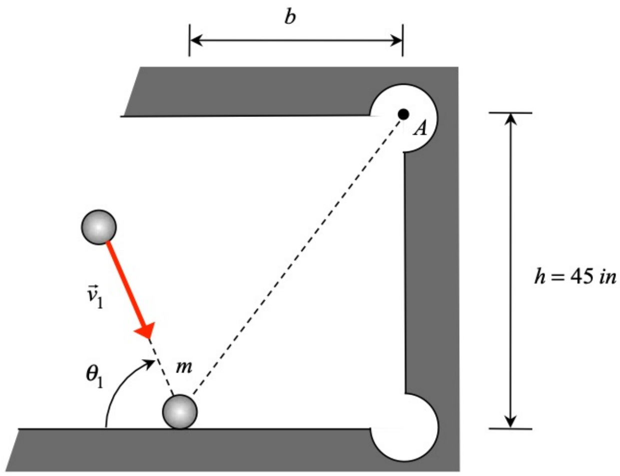


Use the following parameters in your analysis: $v_0 = 2$ ft/s, $\mu_k = 0.3$ and $t = 0.4$ s.

Homework H.4.O

Given: A pool ball is given a velocity of $\vec{v}_1$ at an angle of θ_1 measured from the bumper cushion, as shown in the figure. The coefficient of restitution for the ball impacting the bumper cushion is known to be e. It is desired to drop the ball into the corner pocket at A.

Find: Determine the angle θ_1.

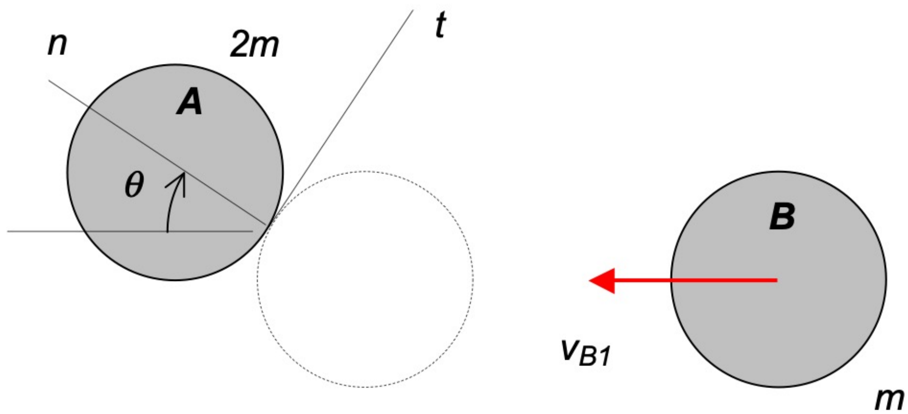


Use the following parameters in your analysis: $e = 0.6$ and $b = 40$ in.

Homework H.4.P

Given: Disks A and B have masses of $2m$ and m, respectively. Disk B is traveling in the direction shown with a speed of v_{B1} when it strikes the stationary disk A ($v_{A1} = 0$ ft/s). Let e represent the coefficient of restitution of impact between A and B.

Find: Determine the velocity of disk A after impact. Write your answer as a vector in terms of its n and t components.

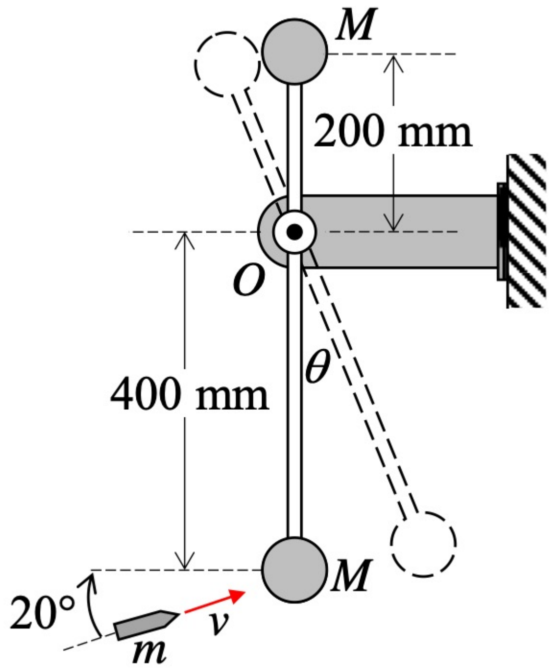


Use the following parameters in your analysis: $e = 0.6$, $\theta = 60°$ and $v_{B1} = 80$ ft/s.

Homework H.4.Q

Given: A bullet strikes the lower particle of a stationary pendulum with a speed of v. After impact, the bullet sticks to the particle.

Find: Determine:
(a) The angular speed of the pendulum immediately after impact;
(b) The maximum rotation angle through which the pendulum swings after impact.

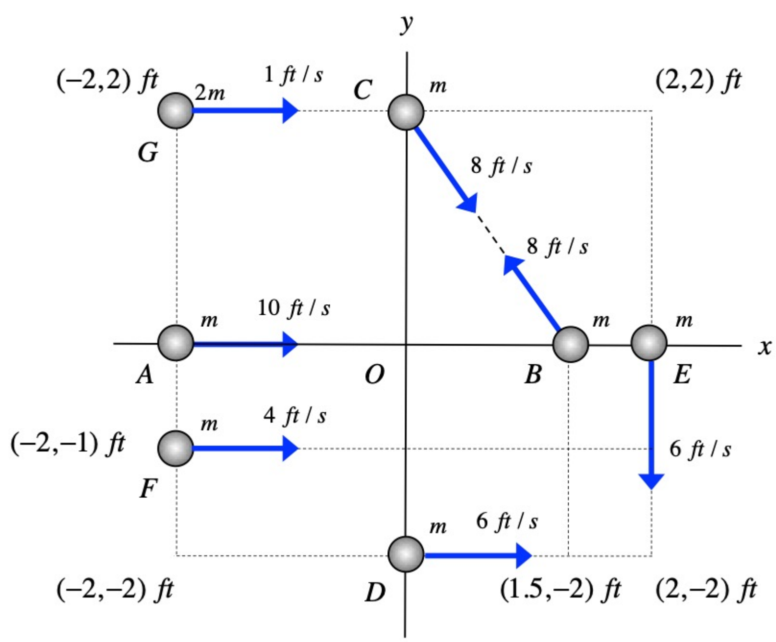


Use the following parameters in your analysis: $v = 100$ m/s, $m = 50$ g and $M = 4$ kg.

Homework H.4.R

Given: Seven particles, A through G, move within a single plane. The mass of each particle is shown in the figure below, along with the velocity and position of each particle.

Find: Determine the total angular momentum about the fixed point O for the entire system of seven particles.



Use the following parameters in your analysis: $m = 10$ slugs.

Homework H.4.S

Given: Particle P (of mass m) travels in the x-y plane with a velocity of $\vec{v}_P$.

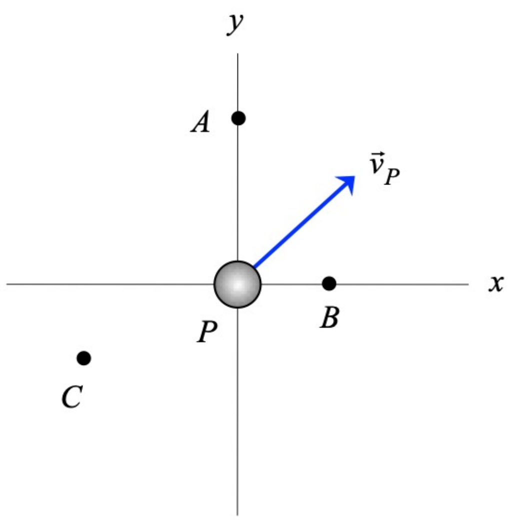
Find: For this problem:

(a) The angular momentum of P about point A, $\vec{H}_A$.

(b) The angular momentum of P about point B, $\vec{H}_B$.

(c) The angular momentum of P about point C, $\vec{H}_C$.

Write your answers as vectors.

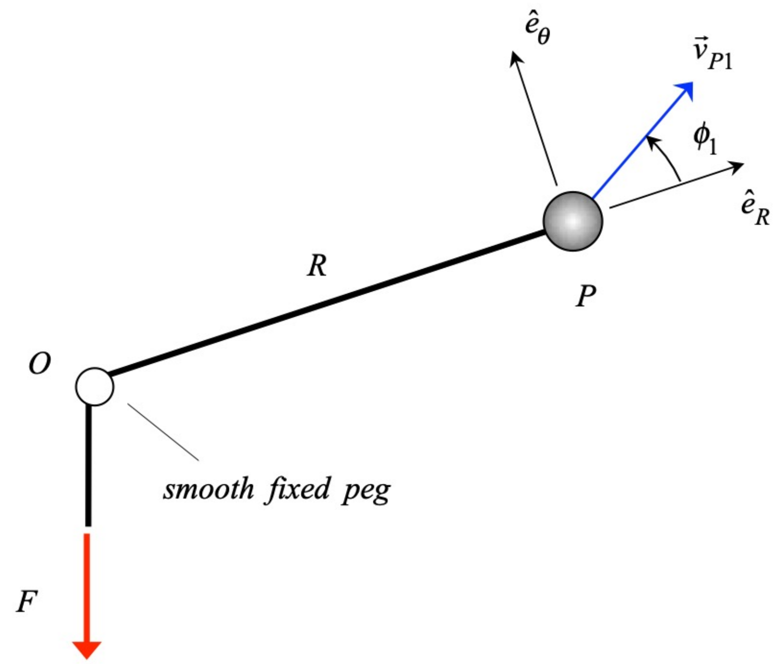


Use the following parameters in your analysis: $m = 5$ kg, $\vec{v}_P = \left(20\hat{\imath} + 15\hat{\jmath}\right)$ m/s and the following (x, y) coordinates for points A, B, C and P: $A : (0, 1.2)$ m, $B : (0.9, 0)$ m, $C : (-0.4, -0.3)$ m, and $P : (0, 0)$ m.

Homework H.4.T

Given: A rope is attached to particle P (having a mass of m) with the rope being pulled through a hole in a fixed, smooth peg by a constant force F applied at the other end of the rope. At the initial state, P has a speed of v_{P1} and is at a distance $R = R_1$ from the peg. The particle moves on a smooth horizontal plane.

Find: Determine the value of $\dot{R}_2$ when $R = R_2$.



HORIZONTAL PLANE

Use the following parameters in your analysis: $m = 6$ kg, $\phi_1 = 30°$, $R_1 = 3$ m, $R_2 = 4$ m, $F = 60$ N and $v_{P1} = 20$ m/s.

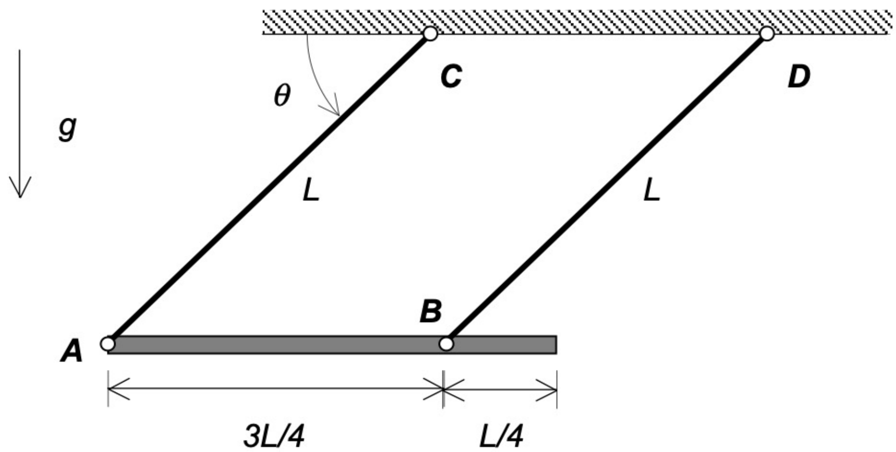
Chapter 5

Rigid Body Kinetics Homework

Homework H5.A

Given: A thin, homogeneous bar of mass M is supported by parallel cables AC and BD, as shown below in the figure. The bar is released from rest with the cables at an angle of θ measured from the horizontal.

Find: Determine the tension in cables AC and BD on release.

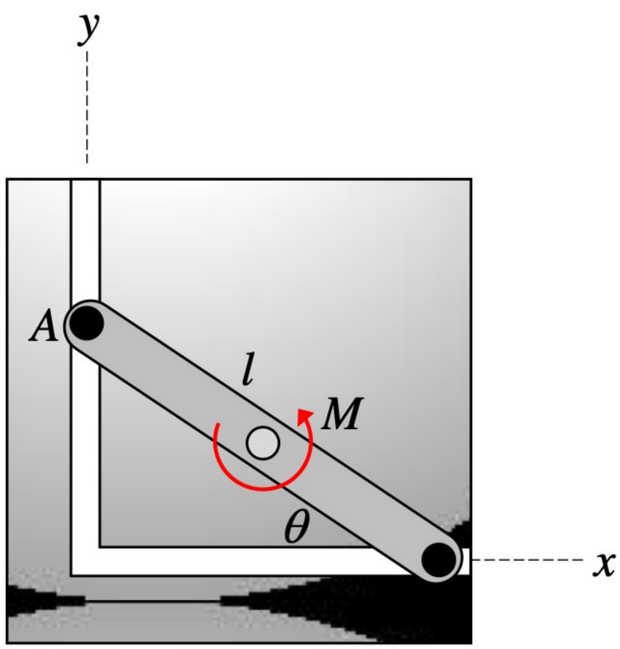


Use the following parameters in your analysis: $M = 300$ kg, $L = 2.5$ m and $\theta = 30°$.

Homework H5.B

Given: An applied moment M acts on a slender, uniform bar of mass m. Assume all surfaces to be smooth.

Find: Determine the reaction forces acting on the bar at A and B on release.

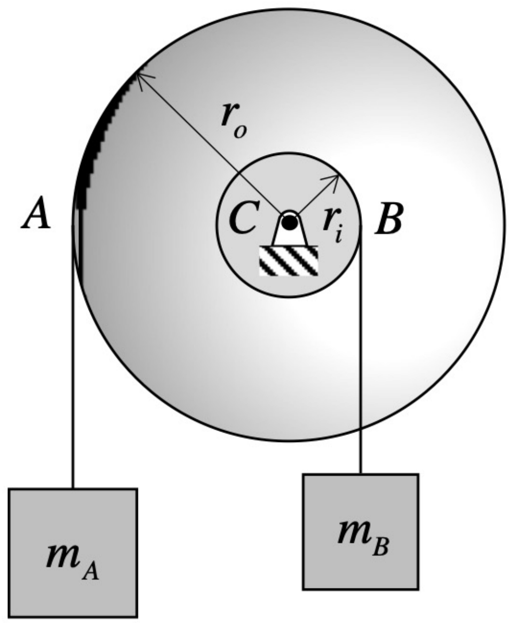


Use the following parameters in your analysis: $\theta = 45°$.

Homework H5.C

Given: A pulley of mass M is connected to two blocks as shown. Assume that the axle at C is frictionless, and that the pulley can be crudely modeled as a homogeneous disk.

Find: Determine the angular acceleration of the pulley and the acceleration of each block.

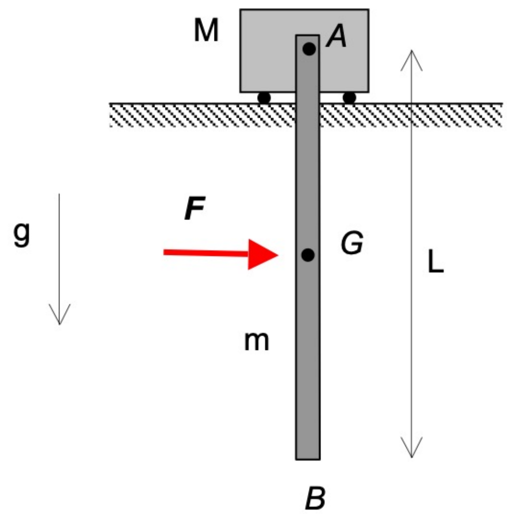


Use the following parameters in your analysis: $M = 10$ kg, $m_A = 12$ kg, $m_B = 9$ kg, $r_i = 0.06$ m and $r_o = 0.2$ m.

Homework H5.D

Given: A homogeneous, thin bar AB (of length L and mass m) is pinned to a block at A. The block has a mass of M and is constrained to move along a smooth horizontal surface. A horizontal force F is applied to the bar's center of mass G at an instant when the bar is vertical and at rest.

Find: Determine the angular acceleration of the bar for the instant when the force F is applied to the bar.

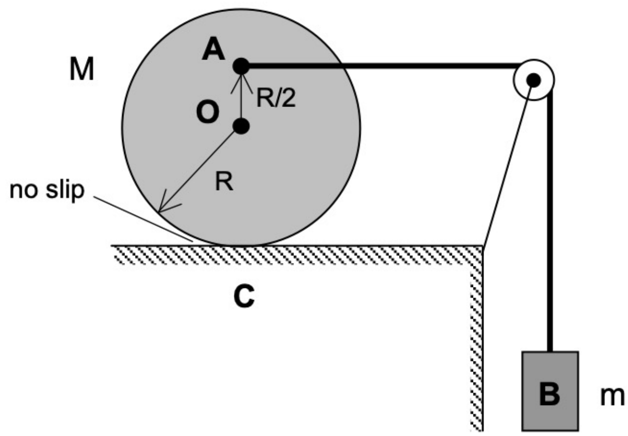


Use the following parameters in your analysis: $L = 3$ m, $m = 50$ kg, $M = 100$ kg and $F = 200$ N.

Homework H5.E

Given: An inextensible cable connects block B with point A on a rigid, homogeneous disk. At the instant shown, point A is directly above the disk's center O. Block B has a mass of m, and the disk has a mass of M. The disk is able to roll without slipping on a rough horizontal surface with point C being the contact point. The system is released from rest.

Find: Determine the angular acceleration of the disk immediately after release.

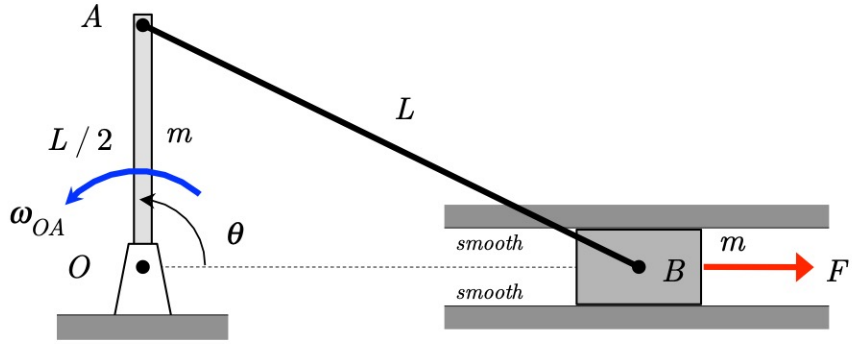


Use the following parameters in your analysis: $M = 20$ kg, $m = 40$ kg, and $R = 0.1$ m.

Homework H5.F

Given: The mechanism shown below is made up of: a thin homogeneous link OA (of mass m and length $L/2$), link AB (having negligible mass and of length L) and slider B (of mass m). Link OA is pinned to ground at end O. Slider B is constrained to move along a straight guide. Link AB connects link OA to B, as shown. A force F acts to the right on slider B. At the instant shown, $\theta = 90°$ and link OA is rotating in the counterclockwise sense with an angular speed of ω_{OA}. Note that the system moves in a horizontal plane.

Find: Determine the acceleration of slider B.



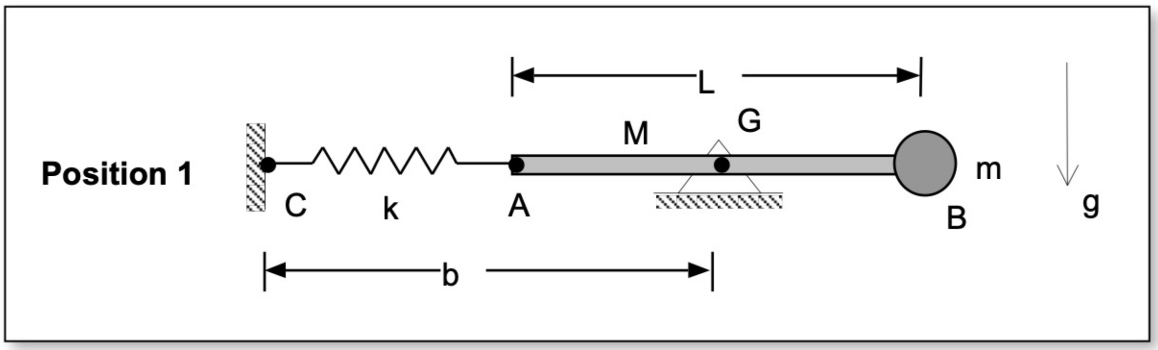
HORIZONTAL PLANE

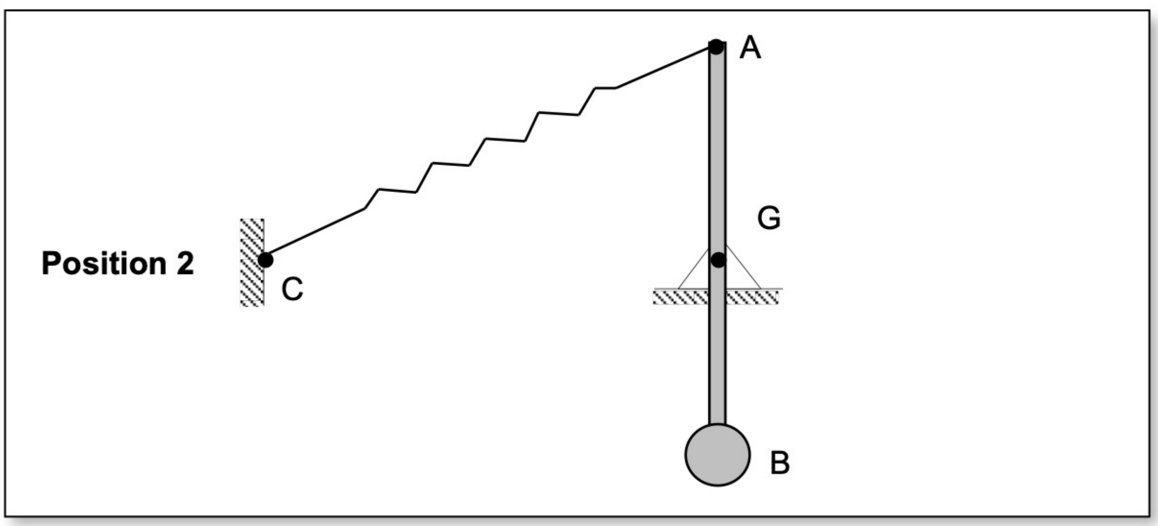
Use the following parameters in your analysis: $m = 5$ kg, $L = 0.5$ m, $F = 75$ N and $\omega_{OA} = 2$ rad/s.

Homework H5.G

Given: A thin, homogeneous bar having a mass of M and length L is pinned to ground at its mass center G. Particle B, having a mass of m, is rigidly attached to the right end of the bar. A spring, having a stiffness of k, is attached between end A of the bar and pin C on a wall. The pin G is a distance of b from the wall. When the bar is horizontal (Position 1 shown below), the spring is unstretched.

Find: If the bar is released from rest in Position 1 above, find the angular velocity of the bar in Position 2 when the bar is in a vertical position.



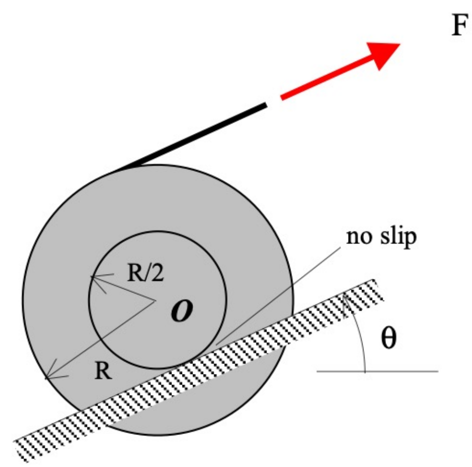


Use the following parameters in your analysis: M = 15 kg, m = 25 kg, k = 100 N/m, L = 3 m and b = 2.5 m.

Homework H5.H

Given: The compound wheel shown below rolls without slipping up the incline on its hubs and is pulled by a constant force F applied to a cord wrapped around its outer rim. The wheel starts from rest, has a mass of m, and has a radius of gyration about its center of mass O of k_O. Assume that the cable does not slip on the wheel.

Find: Determine the angular velocity of the wheel after its center O has moved a distance of d up the incline.

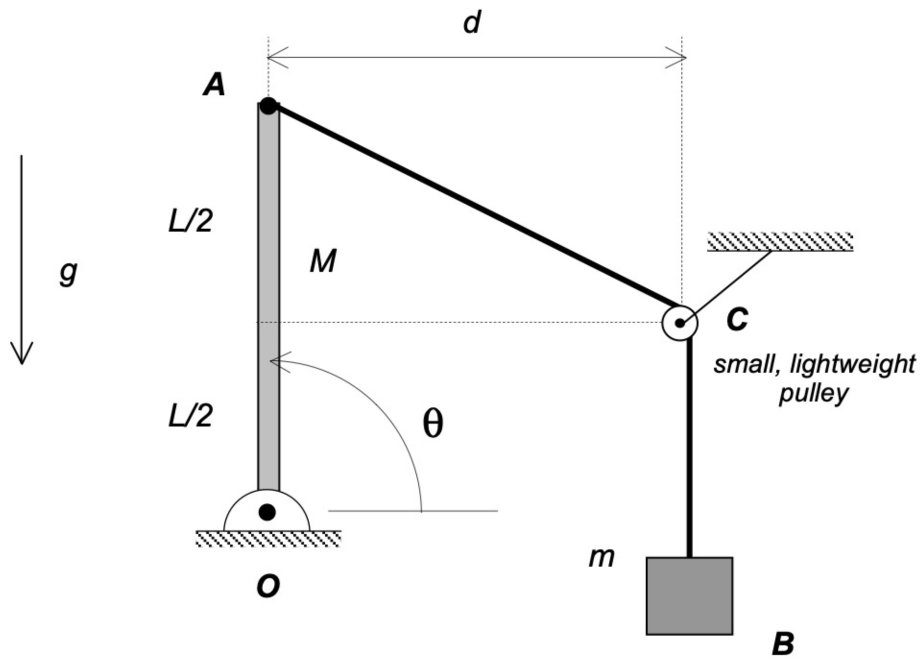


Use the following parameters in your analysis: $m = 40$ kg, $R = 0.2$ m, $d = 2$ m, $F = 100$ N, $k_O = 0.15$ m, and $\theta = 30°$.

Homework H5.I

Given: A thin, homogeneous bar OA (having a length of L and a mass of M), is pinned to ground at O. An inextensible cable is attached to end A of the bar. This cable is also attached to block B, with B having a mass of m. The cable is hung over a small pulley C where the pulley has negligible mass. The system is released from rest when $\theta = 90°$. Assume that the cable remains taut for all time.

Find: Determine the velocity of block B when bar OA has reached the position of $\theta = 0°$.

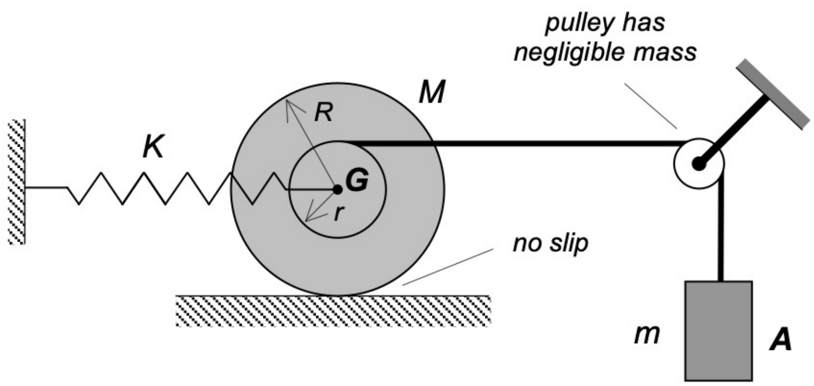


Use the following parameters in your analysis: $m = 20$ kg, $M = 100$ kg, $L = 3$ m and $d = 4$ m.

Homework H5.J

Given: A drum has a mass of M, outer radius of R, inner radius of r, a centroidal radius of gyration of k_G and a centroid at the geometric center G. A cable is wrapped around the inner radius of the drum and is connected to particle A having a mass of m. The cable is also wrapped over an ideal pulley. A spring of stiffness K is attached between G of the drum and ground. The system is released from rest with the spring unstretched.

Find: Determine the speed of A after it has dropped through a distance of d.

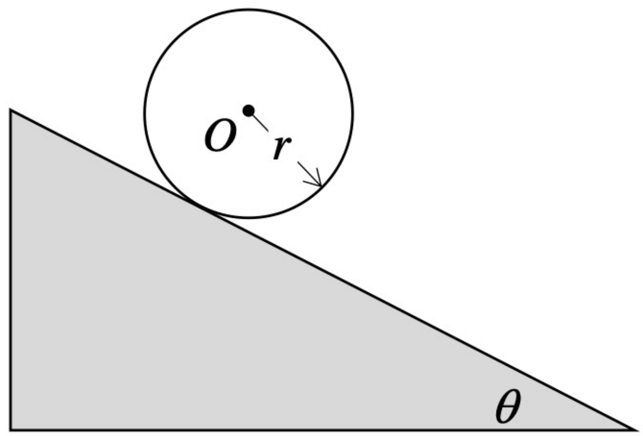


Use the following parameters in your analysis: M = 30 kg, m = 10 kg, R = 0.5 m, r = 0.25 m, k_G = 0.3 m, K = 10 N/m and d = 2 m.

Homework H5.K

Given: The center of the homogeneous disk of mass m shown below has an initial velocity of v_0 up the slope.

Find: Find the velocity of the disk after an elapsed time of Δt, assuming that the disk rolls without slip.

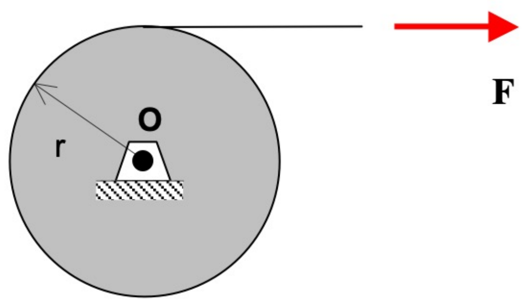


Use the following parameters in your analysis: $m = 10$ kg, $r = 0.5$ m, $\theta = 30°$, $v_0 = 30$ m/s and $\Delta t = 12$ s.

Homework H5.L

Given: A uniform disk (of mass m and radius r) is pinned at its center of mass O. Force F is applied to a rope wrapped around the disk. The disk is released from rest. Assume that the rope does not slip on the disk.

Find: At time t, determine:
(a) the angular velocity of the disk; and
(b) the reaction force components on the disk at pin O.

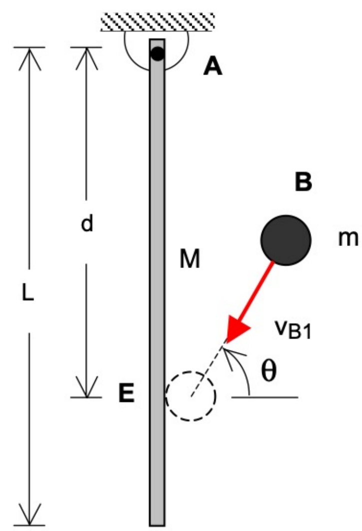


Use the following parameters in your analysis: $m = 15$ kg, $r = 0.4$ m, $F = 200$ N and $t = 2$ s.

Homework H5.M

Given: Particle B (having a mass of m) initially travels on a smooth HORIZONTAL surface with a speed of v_{B1}. This particle strikes point E on a stationary thin homogeneous bar (having a mass of M), with end A of the bar being pinned to ground. The coefficient of restitution between the particle and the bar at point E is known to be e. Furthermore, the contact surface between B and point E is smooth.

Find: Determine the angular velocity of the bar immediately after being impacted by the particle.

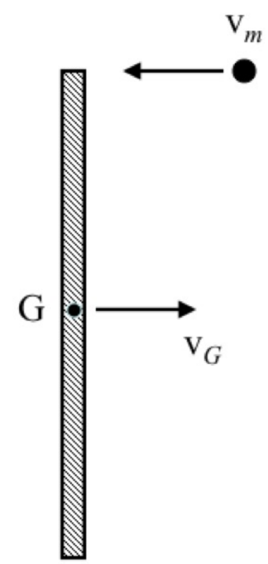


Use the following parameters in your analysis: $L = 3$ m, $d = 2$ m, $m = 10$ kg, $M = 15$ kg, $e = 0.6$, $\theta = 60°$ and $v_{B1} = 20$ m/s.

Homework H5.N

Given: A homogeneous, slender bar of mass M and length L is sliding across a frictionless horizontal surface with a speed v_G when it is suddenly struck at its end by a particle of mass m moving with velocity v_m in the direction shown. Assume that the particle sticks to the bar upon impact.

Find: Determine the velocity of the bar's center of mass G and the angular velocity of the bar post-impact.

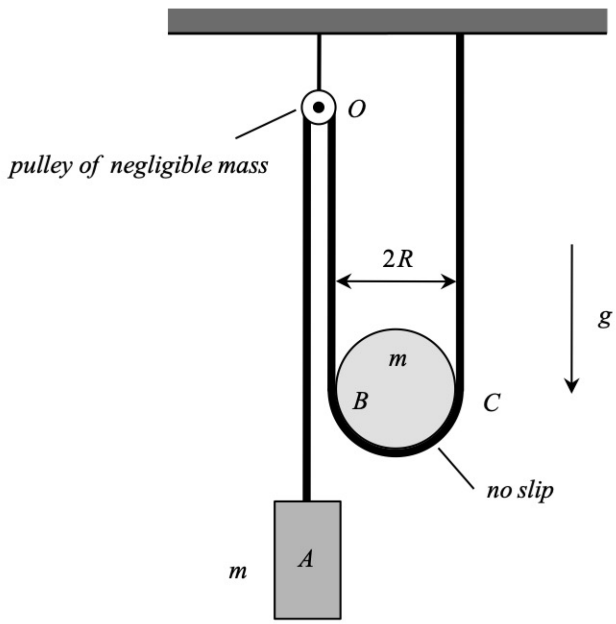


Use the following parameters in your analysis: $M = 4$ kg, $L = 0.75$ m, $v_G = 5$ m/s, $m = 0.4$ kg and $v_m = 25$ m/s.

Homework H5.O

Given: A homogeneous disk having a mass m and outer radius R is supported by the cable-pulley system shown, where block A (with a mass of m) is attached to the free end of the cable. The disk does not slip on the cable as the system moves.

Find: Determine the acceleration of block A on release.

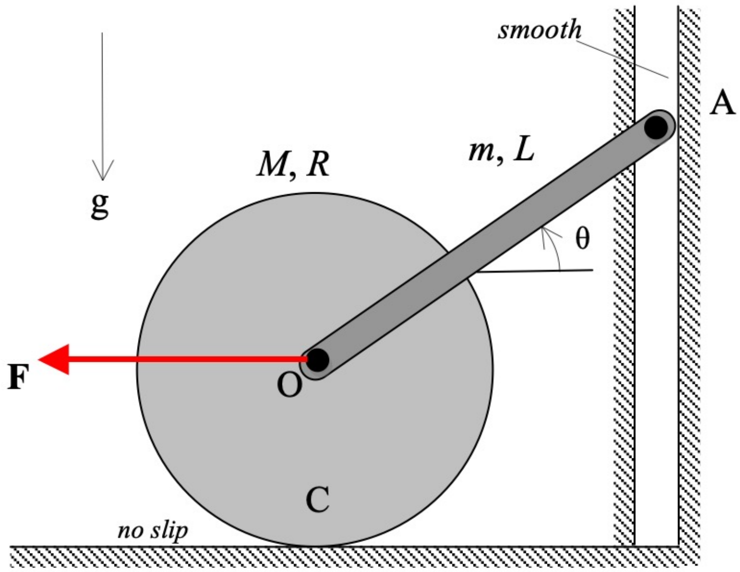


Use the following parameters in your analysis: $m = 20$ kg and $R = 0.2$ m.

Homework H5.P

Given: A homogeneous disk (mass M and radius R) is attached to a homogeneous, thin rod OA (mass m) at its center O. A constant force F is applied at O and point A is confined so that it moves along a smooth, vertical slot. The disk rolls without slipping. If the system is released from rest with $\theta = \theta_1$.

Find: Determine the velocity of point A when $\theta = \theta_2$.



Use the following parameters in your analysis: $M = 12$ kg, $m = 6$ kg, $R = 1$ m, $L = 2.5$ m, $F = 80$ N, $\theta_1 = 53.13°$ and $\theta_2 = 0°$.

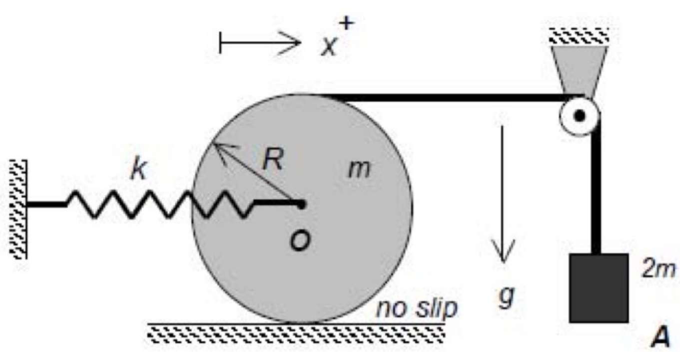
Chapter 6

Vibrations Homework

Homework 6.A

Given: A homogeneous drum (of mass m and outer radius R) rolls without slip on a rough horizontal surface. A spring of stiffness k is attached between the center O of the drum and ground such that the spring remains horizontal at all times. Block A (of mass $2m$) is connected to an inextensible cable that is wrapped around the outer radius of the drum, as shown in the figure. Let x represent the motion of O (measured positively to the right), and let the spring be unstretched when $x = 0$. Assume that the cable does not go slack at any time.

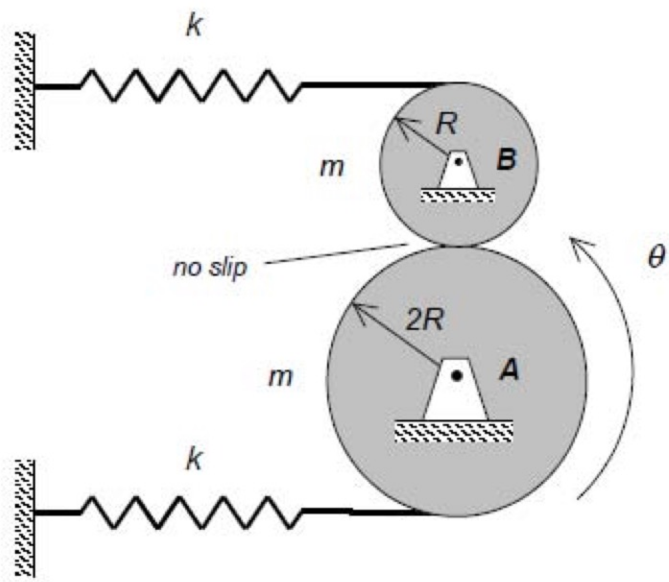
Find: For this problem, derive the single differential equation of motion (EOM) for the system in terms of the coordinate x, its time derivatives, and, at most, the following parameters: g, m, R, and k.



Homework 6.B

Given: Gears A and B (having mass m, and outer radii of $2R$ and R, respectively) mesh without slipping. Two identical springs, each of stiffness k, are wrapped around the outer radii of the gears, as shown in the figure below. Let θ represent the rotation of gear A, and let both springs be unstretched when $\theta = 0$. Treat the gears as homogeneous disks.

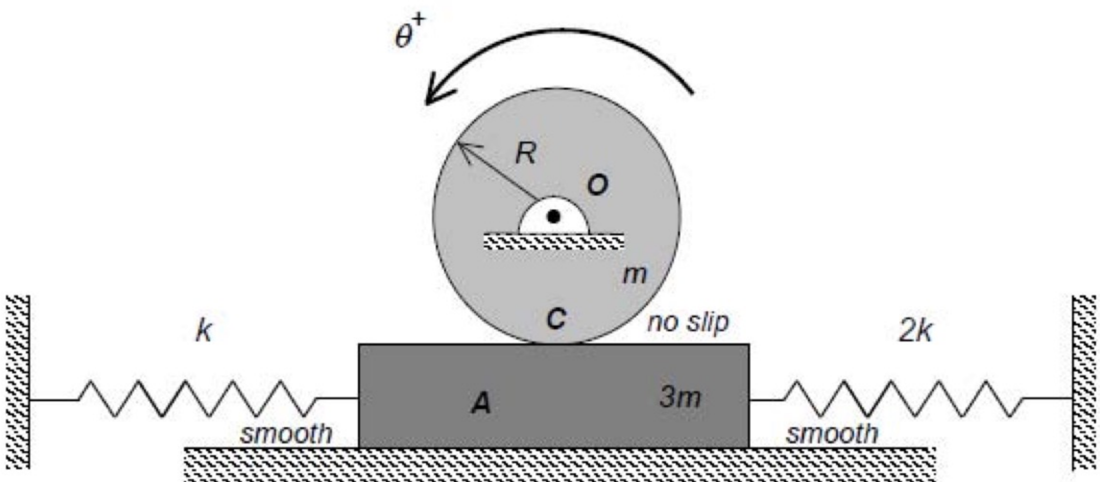
Find: Derive a single differential equation of motion for the system in terms of the coordinate θ.



Homework 6.C

Given: A homogeneous drum having a mass of m and outer radius R is pinned to ground at its center O. This drum is in geared contact with block A. Block A, having a mass of $3m$, is able to slide along a smooth horizontal surface and in such a way that the block does not slip in its contact with drum. Two springs, having stiffnesses of k and $2k$, are attached between block A and ground, as shown in the figure below. Let θ represent the rotation of the drum with θ being measured positive counterclockwise. When $\theta = 0$ rad the springs are unstretched.

Find: For this problem:
(a) Draw individual free body diagrams of the drum and block;
(b) Derive the single differential equation of motion (EOM) for the system in terms of the coordinate θ, its time derivatives, and, at most, the following parameters: m, R, and k;
(c) Based on the EOM derived above, determine the natural frequency of the system. Express the answer in both rad/s and Hz; and
(d) Assuming the system is released when the springs are unstretched with $\dot{\theta}(0) = \omega_0$ (CCW), determine the response of the system $\theta(t)$, for $t > 0$.

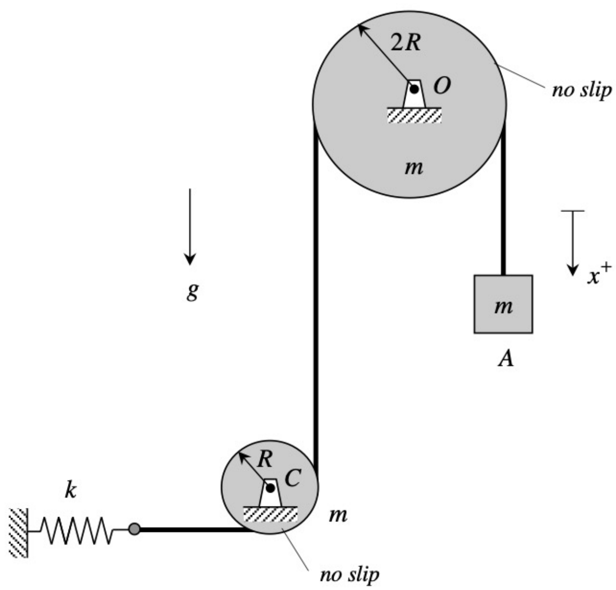


Use the following parameters in your analysis: $m = 0.75$ kg, $k = 5500$ N/m, $R = 0.25$ m, and $\omega_0 = 0.5$ rad/s.

Homework 6.D

Given: Two homogeneous disks, each having a mass of m and with outer radii of $2R$ and R, are supported by smooth shafts at their centers O and C, respectively. An inextensible cable connects block A (mass m) to a spring of stiffness k, with the spring being attached to ground at its left end. The cable is pulled over the disks, and the cable does not slip on the disks as the system moves. Let x represent the motion of block A, where $x = 0$ when the spring is unstretched.

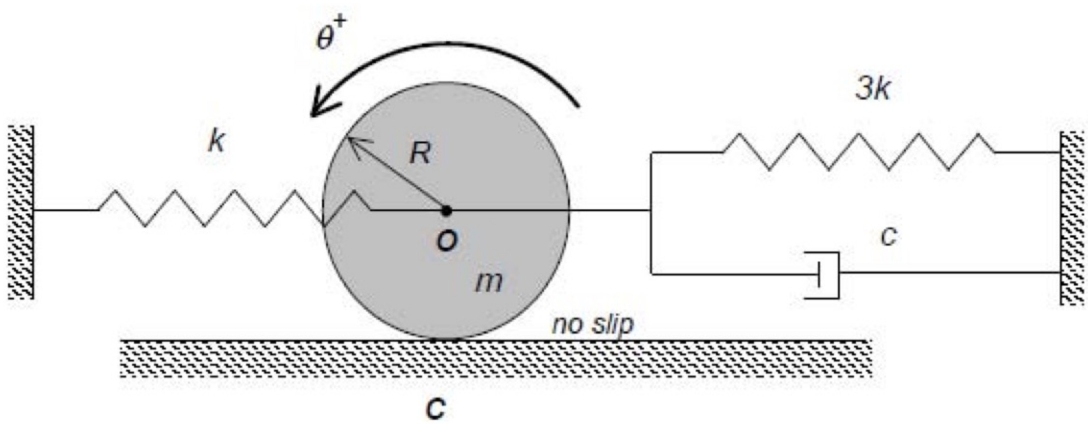
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate x;
(b) Write down the dynamical EOM of the system in terms of the coordinate $z = x - x_{st}$, where x_{st} is the x-position of A when the system is in static equilibrium; and,
(c) Determine the undamped natural frequency ω_n, for the system.



Homework 6.E

Given: A homogeneous disk having a mass of m and outer radius of R rolls without slipping on a rough, horizontal surface. A spring of stiffness k is connected between the center O of the disk and ground on the left side of the disk. A second spring of stiffness $3k$ and a dashpot with damping constant c is connected between O and ground on the right side of the disk, as shown in the figure below. Let θ represent the rotation of the disk measured positive counterclockwise, and let $\theta = 0°$ describe when the springs are unstretched.

Find: For this problem:
(a) Draw a free body diagram of the disk;
(b) Derive the single differential equation of motion for the system in terms of the coordinate θ, its time derivatives, and, at most, the following parameters: m, R, c, and k;
(c) Determine the value of the damping constant c required for the system to be underdamped with a damping ratio of $\zeta = 0.5$; and
(d) Suppose that the mass of the drum is doubled (to a mass of $2m$). Determine the new value of the damping constant c required for the system to be underdamped with a damping ratio of $\zeta = 0.5$.

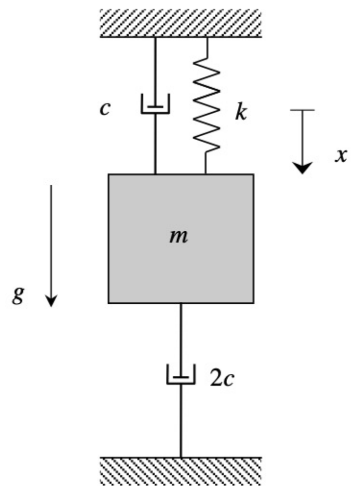


Use the following parameters in your analysis: m = 24 kg, k = 900 N/m, and R = 0.5 m.

Homework 6.F

Given: A block of mass m is attached to a grounded spring (of stiffness k) and two dashpots (having damping coefficients c and $2c$), as shown in the figure. Let x represent the motion of the block, with $x = 0$ when the spring is unstretched.

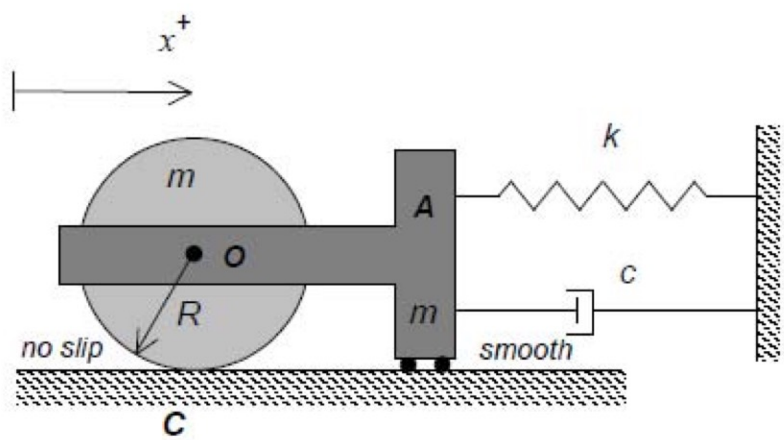
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate x;
(b) Determine the static equilibrium position of the block, x_{st};
(c) Rewrite the EOM of the system in terms of the variable $z = x - x_{st}$, where z represents the position of the block relative to its static equilibrium position; and,
(d) Determine undamped natural frequency ω_n, the damping ratio ζ and the damped natural frequency ω_d for the system in terms of, at most, the parameters of the problem: m, c and k.



Homework 6.G

Given: A homogeneous wheel of mass m and outer radius R rolls without slipping on a horizontal surface. Block A (also having a mass of m) is pinned to the center O of the wheel and is able to slide without friction on the same horizontal surface. A spring (of stiffness k) and a dashpot (of damping constant c) are connected between block A and ground. Let x represent the motion of block A measured positively to the right. When $x = 0$ m, the spring is unstretched.

Find: For this problem:
(a) Draw individual free body diagrams for block A and the wheel;
(b) Derive the single differential equation of motion for the system in terms of the coordinate x, its time derivatives, and the following parameters: m, R, c, and k;
(c) Determine numerical values for: the undamped natural frequency ω_n, the damping ratio ζ, and the damped natural frequency ω_d; and
(d) Determine the response of the system $x(t)$ for $t > 0$, assuming the system is released when the springs are unstretched with $\dot{x}(0) = v_0$.

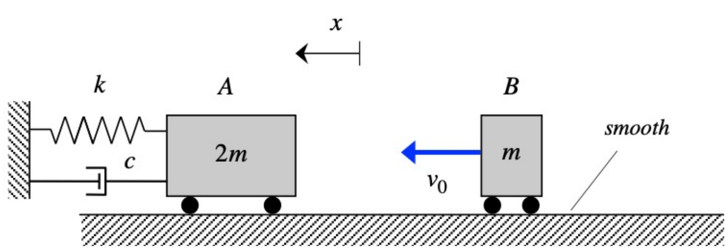


Use the following parameters in your analysis: $m = 4$ kg, $k = 2250$ N/m, $R = 0.1$ m, $c = 60$ kg/s, and $v_0 = 8$ m/s.

Homework 6.H

Given: Particle A (having a mass of $2m$) is attached to a grounded spring of stiffness k and a dashpot with a damping coefficient c. Let x represent the motion of A, with $x = 0$ when the spring is unstretched. At an instant when A is at rest and with the spring being unstretched/uncompressed (that is, when $x = 0$), particle B (of mass m), traveling with a speed of v_0, strikes A. On the impact with A, block B immediately sticks to block A.

Find: For this problem:
(a) Determine the speed of A immediately after B sticks to it.
(b) Derive the dynamical equation of motion (EOM) of A+B in terms of the coordinate x describing the motion after the two blocks stick together;
(c) Determine the undamped natural frequency ω_n, the damping ratio ζ and the damped natural frequency ω_d for the system;
(d) Determine the response $x(t)$ of the system after A and B stick together.

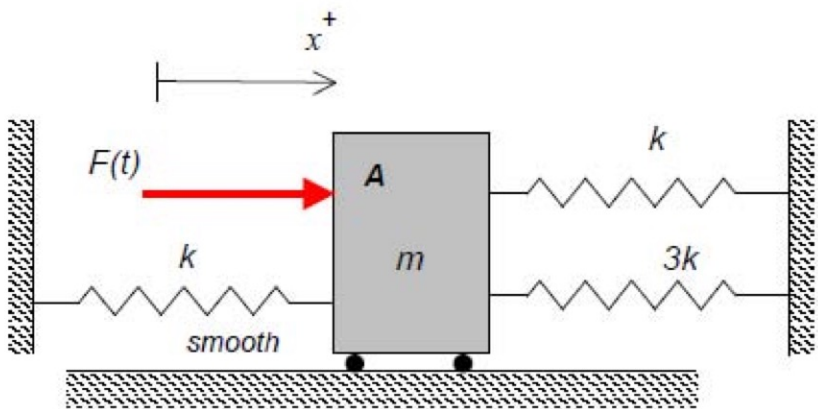


Use the following parameters in your analysis: $m = 10$ kg, $k = 3000$ N/m and $c = 360$ kg/s.

Homework 6.I

Given: Block A, having a mass of m, is able to slide along a smooth horizontal surface. Three springs are connected between block A and ground, as shown in the figure below. A force $F(t) = F_0 \sin \omega t$ acts horizontally on block A. Let x represent the motion of block A measured positively to the right, and let $x = 0$ m designate the state at which the springs are unstretched.

Find: For this problem:
(a) Draw a free body diagram of block A;
(b) Derive the single differential equation of motion for the system in terms of the coordinate x; and
(c) Derive the particular solution $x_p(t)$ for the equation of motion derived above.

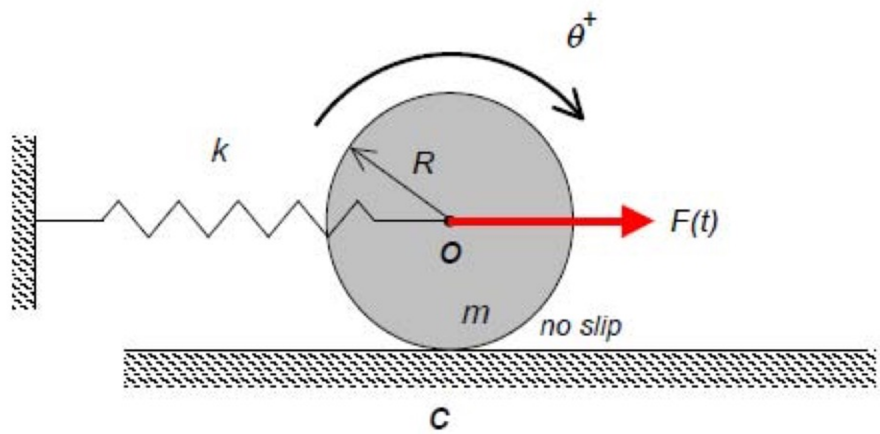


Use the following parameters in your analysis: $m = 10$ kg, $k = 3200$ N/m, $F_0 = 150$ N, and $\omega = 15$ rad/s.

Homework 6.J

Given: A homogeneous disk, having a mass of m and outer radius of R, rolls without slipping on a rough horizontal surface. A spring of stiffness k is connected between the center O of the disk and ground on the left side of the disk. A force $F(t) = F_0 \sin \omega t$ acts horizontally at point O on the disk. Let θ represent the rotation of the disk measured positive clockwise, and let the spring be unstretched when $\theta = 0$ rad.

Find: For this problem:
(a) Draw a free body diagram of the disk;
(b) Derive the single differential equation of motion for the system in terms of the coordinate θ; and
(c) Derive the particular solution $\theta_p(t)$ for the equation of motion obtained above.



Use the following parameters in your analysis: $m = 2$ kg, $k = 4800$ N/m, $R = 0.5$ m, $F_0 = 50$ N, and $\omega = 60$ rad/s.

Homework 6.K

Given: Ground motion during an earthquake event is often represented by a prescribed motion x_B(t) for the ground. Structures attached to the ground experience an excitation due to this ground motion. Consider here a structure of mass m and support stiffness k attached to ground during an earthquake event. Let x represent the motion of this structure shown in the figure.

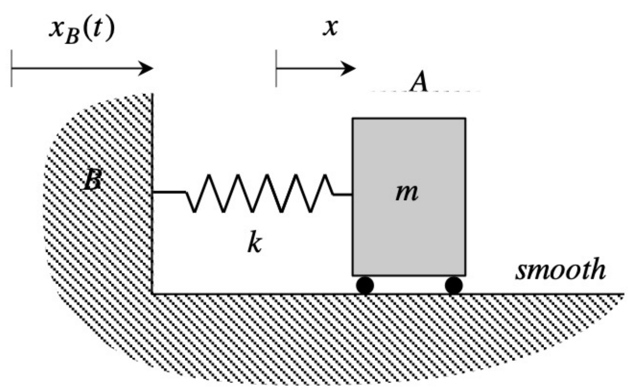
Find: For this problem:

(a) Derive the differential equation of motion (EOM) for the structure in terms of the coordinate x;

(b) Recognizing that the strain in the structure support is an important measure of the response and that strain is related to the relative motion between A and B, write the EOM in (a) above in terms of the relative coordinate z(t) = x(t) - x_B(t). Compare this EOM with that found in (a);

(c) Consider a ground motion of x_B(t) = b sinΩt. Determine the particular solution for the EOMs in (a) and (b); and,

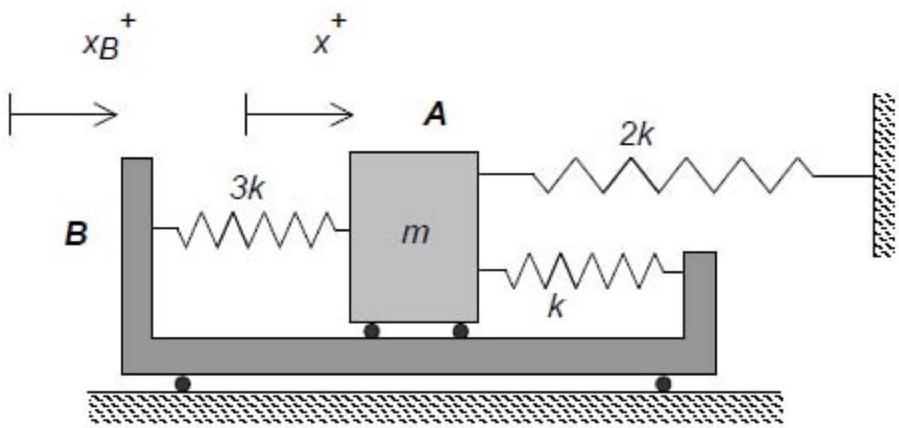
(d) Make sketches of the amplitude of response for these two solutions. Compare the results. Are they consistent?



Homework 6.L

Given: Block A (having a mass of m) is attached to cart B with two springs of stiffnesses $3k$ and k, as shown below. A third spring of stiffness $2k$ is attached between A and ground. Cart B is given a prescribed displacement of $x_B(t) = b\cos\omega t$. The absolute motion of block A is described by the coordinate x. All springs are unstretched when $x = x_B = 0$ m. Consider all of the surfaces to be smooth.

Find: For this problem:
(a) Derive the differential equation of motion for block A in terms of the coordinate x;
(b) Determine the numerical value for the natural frequency of this system; and
(c) Derive the particular solution of the system $x_p(t)$.

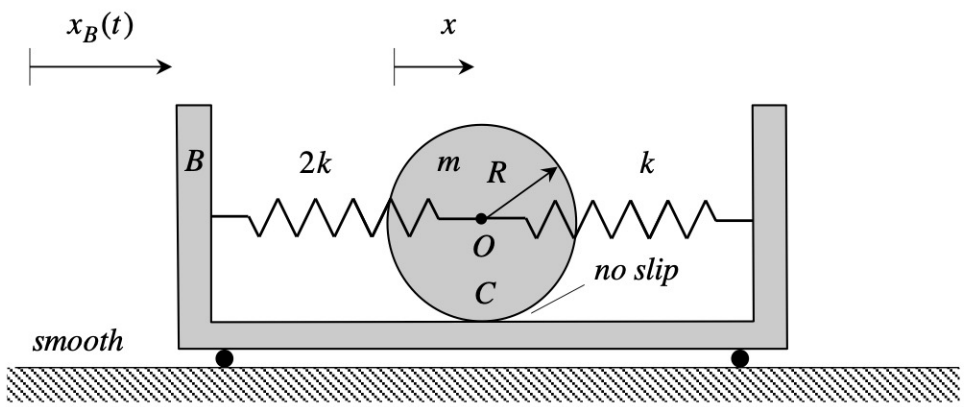


Use the following parameters in your analysis: $m = 12$ kg, $k = 800$ N/m, $b = 0.1$ m, and $\omega = 25$ rad/s.

Homework 6.M

Given: A homogeneous disk of mass m and with an outer radius of R rolls without slipping on a rough horizontal surface on cart B. The disk is connected at its center O with two springs (of stiffnesses $2k$ and k) to B, as shown in the figure. The base B is given a prescribed motion of x_B(t) = b $sin\Omega t$. Let x measure the position of O from its position when the springs are unstretched.

Find: For this problem:
(a) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x;
(b) Determine the natural frequency ω_n of the system;
(c) Determine the amplitude of the motion described by particular solution of the EOM; and,
(d) For the motion found in (c) above, does O move in phase or out of phase with B?

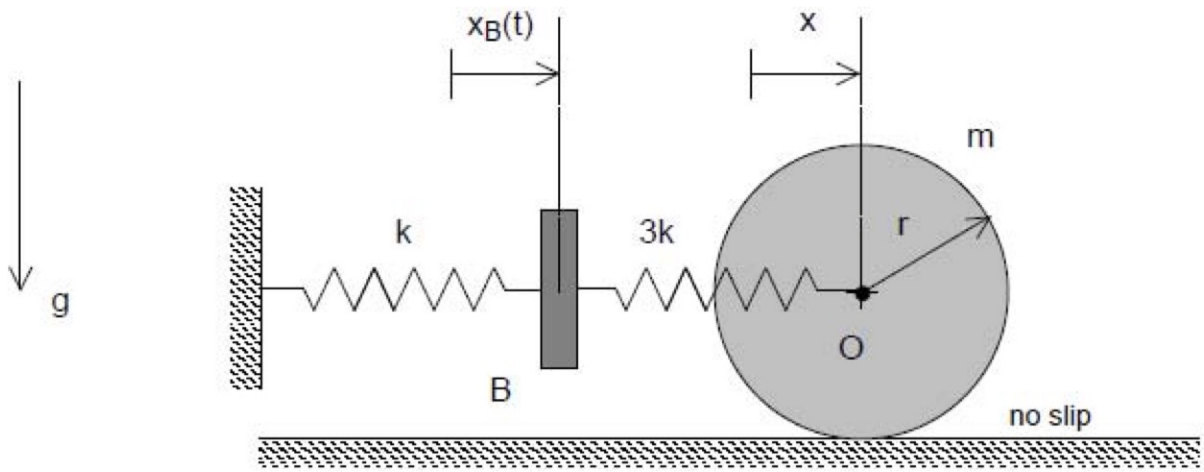


Use the following parameters in your analysis: m = 200 kg, k = 10,000 N/m, b = 0.1 m, and Ω = 15 rad/s.

Homework 6.N

Given: A homogeneous disk (of mass m and outer radius R) rolls without slipping on a rough, horizontal surface. A spring (of stiffness $3k$) is attached between the center O of the disk and a moveable base B. A second spring (of stiffness k) is attached between point B and ground. Base B is given a prescribed motion of $x_B(t) = b \sin \Omega t$. The coordinates x and x_B are both zero when the springs are unstretched.

Find: For this problem:
(a) Derive the differential equation of motion for the disk in terms of the coordinate x;
(b) Determine the numerical value for the natural frequency of this system;
(c) Determine the numerical value of X, if the particular solution of the system is written as $x_p(t) = X \sin \Omega t$; and
(d) Determine if the disk is moving in-phase or out-of-phase with the base B.



Use the following parameters in your analysis: $m = 80$ kg, $k = 640$ N/m, $r = 0.25$ m, $b = 0.16$ m, and $\Omega = 10$ rad/s.

