

## Chapter 1

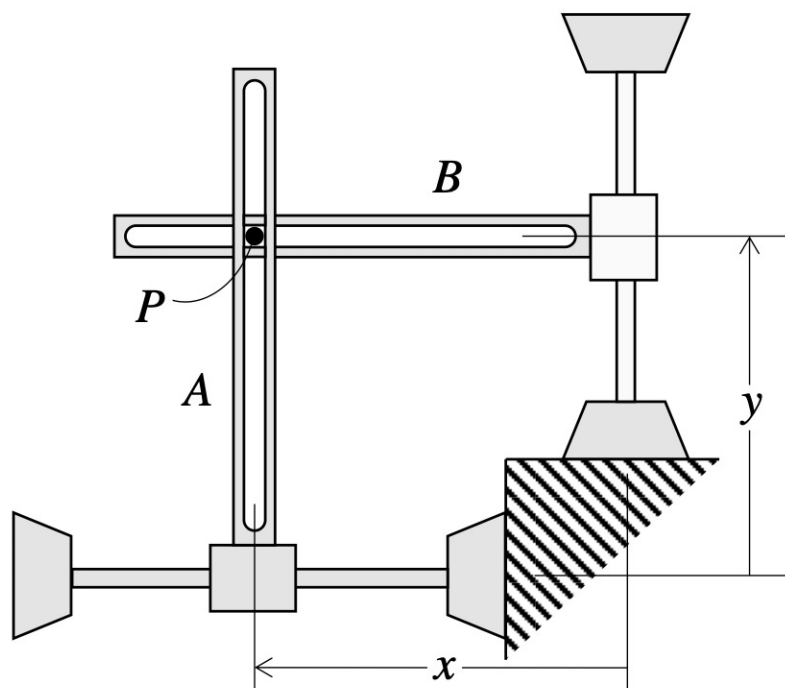
# Particle Kinematics Homework



**Homework H1.A**

**Given:** Pin  $P$  moves within slots cut into links  $A$  and  $B$ . The horizontal position of link  $A$  is given by  $x = 10 + t^2$ , whereas the vertical position of link  $B$  is given by  $y = 15 - t^3$ , where  $t$  is given in seconds and  $x$  and  $y$  are given in mm.

**Find:** Determine the velocity and acceleration of  $P$  at  $t = 5$  s.



**Problem H1.B**

**Given:** A particle P travels on a path described by the Cartesian coordinates of  $y = cx(b - x)$ , where  $x$  and  $y$  have the units of meters. The  $x$ -component of velocity,  $\dot{x}$ , for P is constant.

**Find:** For this problem:

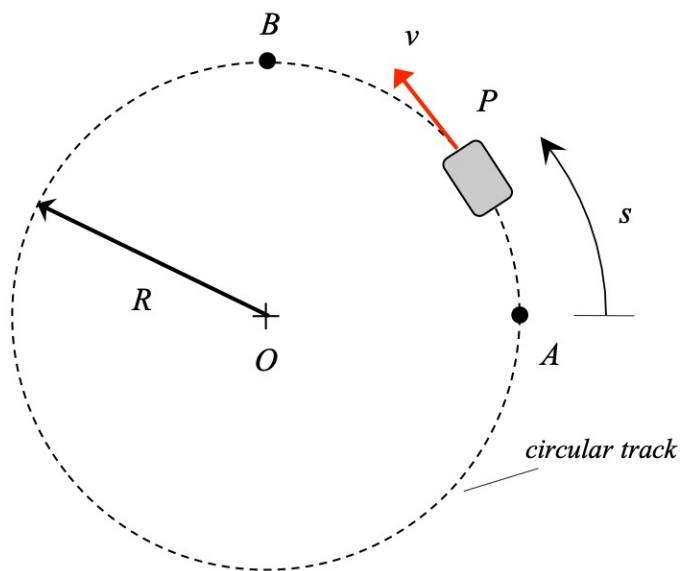
- (a) Make a sketch of the path of P over the range of  $0 < x < b$ .
- (b) Determine the Cartesian components of the velocity and acceleration of P at  $x = 0$ . Add a sketch of the velocity and acceleration vectors for P to your path drawn above.
- (c) Determine the Cartesian components of the velocity and acceleration of P at  $x = b/2$ . Add a sketch of the velocity and acceleration vectors for P to your path drawn above.

Use the following parameters in your analysis:  $b = 2$  m,  $c = 5$  and  $\dot{x} = 4$  m/s.

**Problem H1.C**

**Given:** An automobile  $P$  is traveling along on a circular track of radius  $R$ . A position  $A$  on the track, the automobile has a speed of  $v_A$ . At this position, the driver of automobile applies the brakes with the speed of the automobile changing with distance  $s$  traveled along the track according to the following equation:  $v(s) = v_A \cos(bs)$ , where  $s$  is given in meters.

**Find:** Determine the magnitude of the acceleration for the driver when the automobile reaches position  $B$  on the track where  $B$  is a quarter of the distance around the track from position  $A$ .



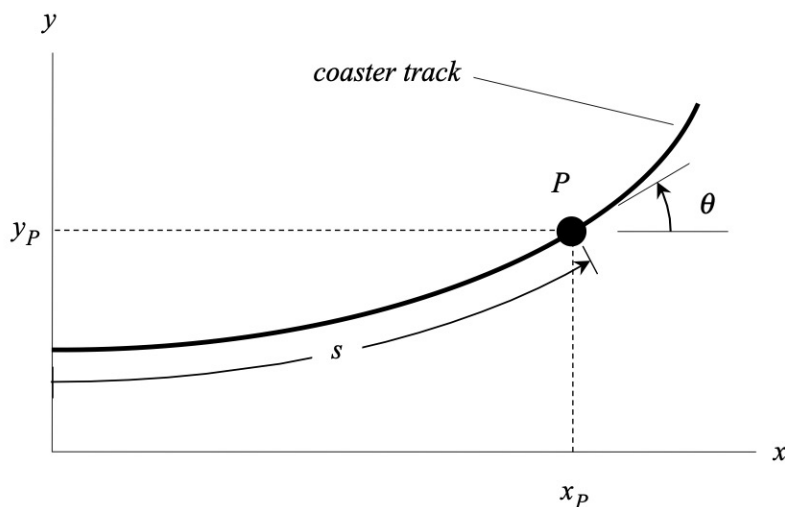
Use the following parameters in your analysis:  $b = 0.001/\text{m}$ ,  $R = 400 \text{ m}$  and  $v_A = 60 \text{ m/s}$ .

**Homework H1.D**

**Given:** Cart P travels on a rollercoaster track. Let  $s$  represent the distance traveled by P on this track, where  $s$  has units of feet. In terms of the distance  $s$ , the radius of curvature of the track, the angle of the tangent to the track and the speed of P are known to be  $\rho = 1/b s$ ,  $\theta = b s^2/2$  and  $v(s) = d - c s^2$ , respectively.

**Find:** For this problem:

- Determine the path variable components of velocity and acceleration of P as a function of  $s$ .
- Evaluate your results in (a) above for  $s = 100$  ft. Make a sketch of the velocity and acceleration vectors at this position.

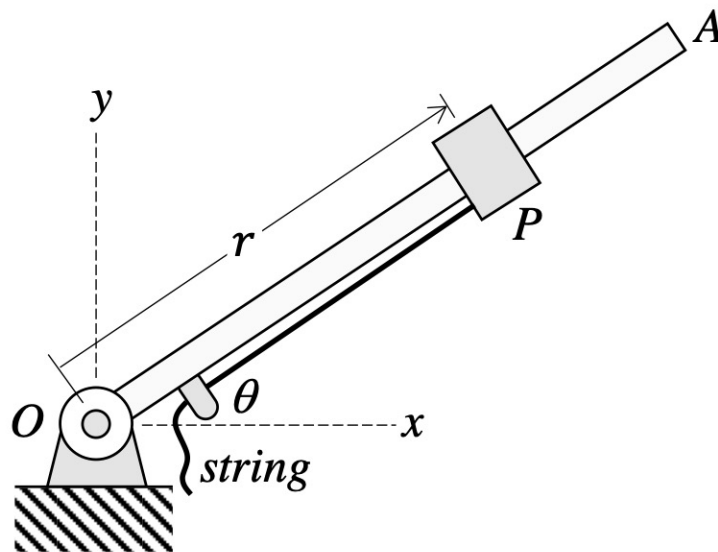


Use the following parameters in your analysis:  $b = 1 \times 10^{-4}/\text{ft}^2$ ,  $d = 150 \text{ ft/s}$  and  $c = 1 \times 10^{-2}/\text{ft} \cdot \text{s}$ .

**Problem H1.E**

**Given:** A string is used to pull in particle P in such a way that the radial position of P is given by  $r = 1 - 0.05t^2$ , while the angular orientation of arm OA is given by  $\theta = 0.25 + 0.1t$ , where  $r$ ,  $\theta$  and  $t$  are given in meters, radians and seconds, respectively.

**Find:** Determine the velocity and acceleration of P.



Use the following parameters in your analysis:  $t = 4$  s.

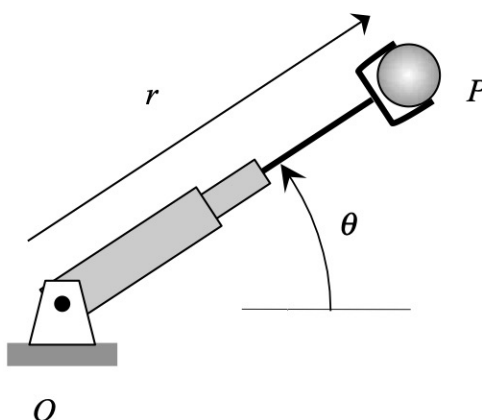
**Problem H1.F**

**Given:** A rotating and telescoping robotic arm is gripping a small sphere P in its end effector. The arm is rotating counterclockwise with a constant angular speed of  $\dot{\theta}$ . The arm is extending such that the radial distance from O to P is related to the rotation angle  $\theta$  by the following equation:

$$r(\theta) = R_0 + R_1 \cos 2\theta$$

where  $r$  and  $\theta$  are given in terms of meters and radians, respectively.

**Find:** Determine the velocity and acceleration of the sphere P. Write your answers as vectors in terms of the polar unit vectors  $\hat{e}_r$  and  $\hat{e}_\theta$ .



Use the following parameters in your analysis:  $R_0 = 2$  m,  $R_1 = 0.5$  m,  $\theta = \pi/2$  rad and  $\dot{\theta} = 2$  rad/s.



**Homework H1.G**

**Given:** Particle P travels within the  $x$ - $y$  plane along a path given by  $y(x) = x^2/2 - 10x$ , where  $x$  and  $y$  are given in feet. The  $x$ -component of the position for P is changing at a constant rate of  $\dot{x}$ .

**Find:** For this problem:

- (a) Make a sketch of the path of particle P.
- (b) Determine the velocity and acceleration of P.
- (c) Show the velocity and acceleration vectors of P in your sketch of P's path.
- (d) Determine the rate of change of speed of P.

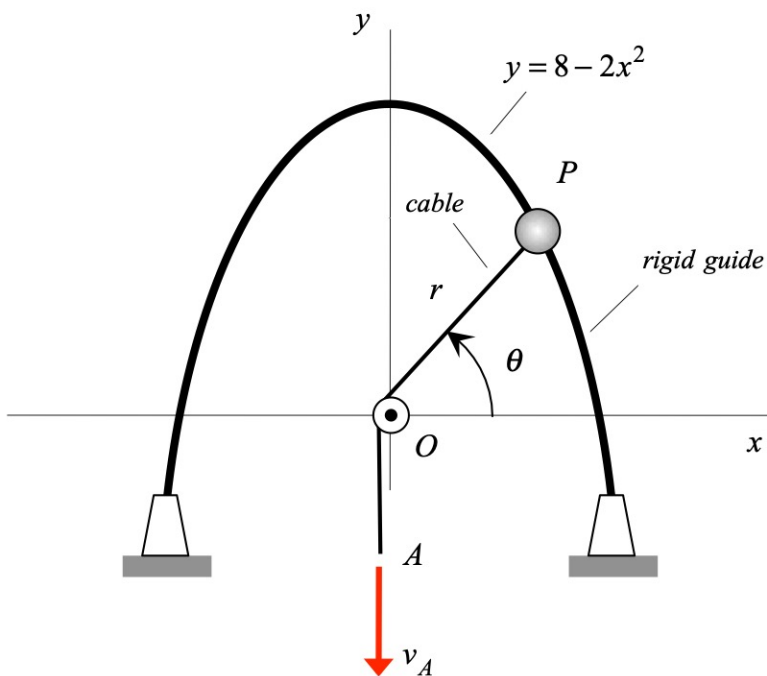
Use the following parameters in your analysis:  $\dot{x} = 6$  ft/s and  $x = 12$  ft.

**Problem H1.H**

**Given:** Particle P is able to slide along a rigid guide whose shape is given in terms of Cartesian components as:  $y(x) = 8 - 2x^2$ , where  $x$  and  $y$  are given in meters. A cable is attached to P with the cable being pulled over a small pulley at O and with the other end of the cable moving downward with a constant speed of  $v_A$ .

**Find:** When the particle P is at a position defined by  $y_P = 0$  m:

- Determine the velocity and acceleration of P in terms of their Cartesian components.
- Determine the speed and magnitude of acceleration of P.



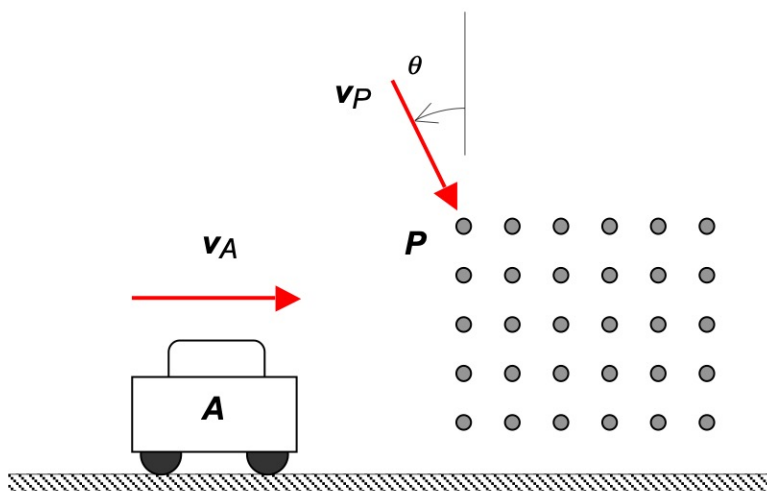
Use the following parameter in your analysis:  $v_A = 1.5$  m/s.

**Homework H1.1**

**Given:** Automobile A is traveling down a roadway with a speed of  $v_A$  when it encounters a hailstorm. Hailstone P is known to be falling with a speed of  $v_P$  at an angle of  $\theta$  forward of the automobile.

**Find:** For this problem:

- Write down, as a vector, the velocity of P as seen by a passenger in the automobile.
- At what angle with the vertical is this observed velocity of P?

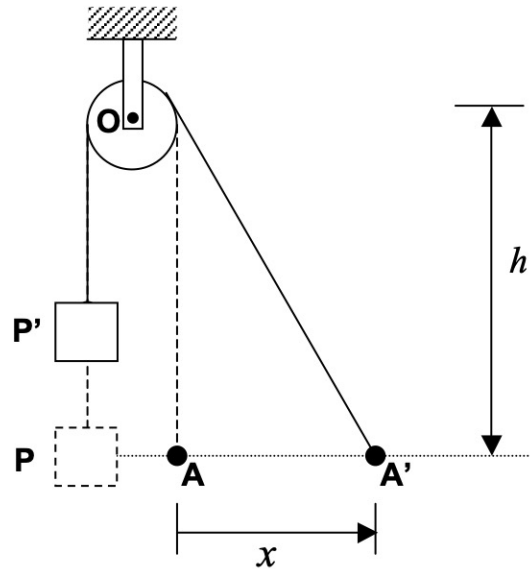


Use the following parameters in your analysis:  $v_A = 30$  m/s,  $v_P = 15$  m/s and  $\theta = 20^\circ$ .

**Homework H1.J**

**Given:** A worker is lifting a large crate, P, using a pulley shown in the diagram. He is holding the rope at point A and is walking to the right at a constant speed  $v_A$ .

**Find:** What is the speed of the crate when the worker has walked a distance  $x$ ?



Use the following parameters in your analysis:  $v_A = 5 \text{ m/s}$ ,  $x = 2 \text{ m}$  and  $h = 8 \text{ m}$ .

## Chapter 2

# Planar Rigid Body Kinematics Homework

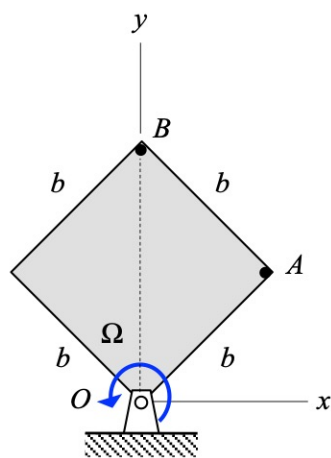
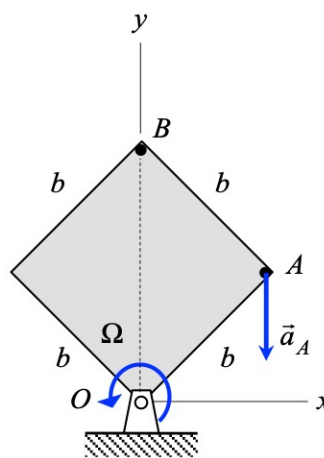


**Homework H2.A**

**Given:** A square plate (having side lengths of  $b = \sqrt{2}$  m) rotates with a counterclockwise sense at a rate of  $\Omega = 5$  rad/s about a shaft passing through corner O. At the position shown below, corner B is directly above the shaft O.

**Find:** Consider the following two parts of this problem:

- (a) For the first part, we are given that the rotation rate of the plate is changing at a rate of  $\dot{\Omega} = 10$  rad/s<sup>2</sup>. Determine the velocity and acceleration vectors for corners A and B of the plate. Make sketches of these vectors.
- (b) For the second part, we are not given information on  $\dot{\Omega}$ . Instead, we know the acceleration of corner A to be in the negative y-direction (the x-component is zero), as shown in the figure below. For this, determine the numerical value of  $\dot{\Omega}$  and of the acceleration vector for corner B. Make a sketch of the acceleration vector for corner B.

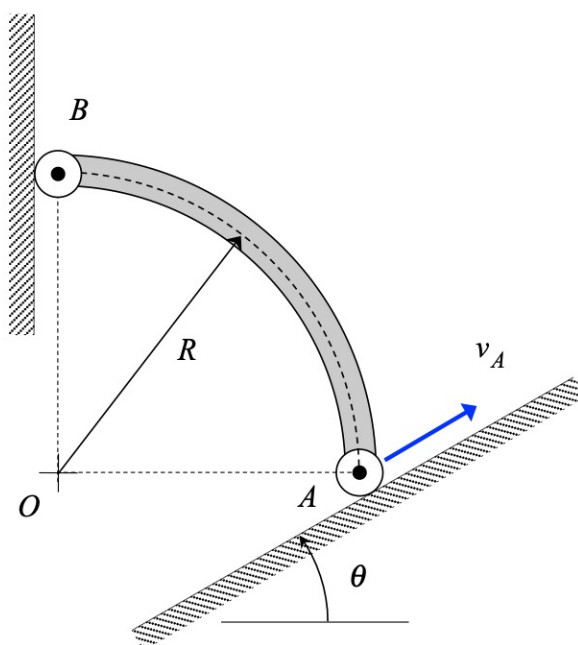
**Part (a)****Part (b)**

**Homework H2.B**

**Given:** Rigid body AB is shaped as quarter-circle arc with a radius of  $R$ . End B of the bar is constrained to move along a vertical wall, whereas end A moves along an incline at an angle of  $\theta = 53.13^\circ$  with respect to the horizontal. At the instant shown, the center O of the AB arc is directly below end B, and end A moves with a constant speed of  $v_A$ .

**Find:** For this problem:

- Determine the velocity and acceleration of end B of the bar. Express your answers as vectors and in terms of the parameters of  $v_A$  and  $R$ .
- Is the speed of B increasing, decreasing or constant?



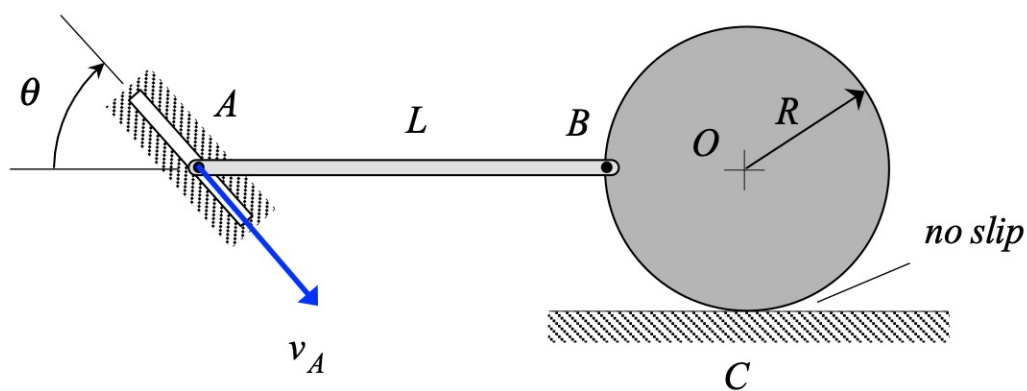


**Homework H2.C**

**Given:** Rigid bar AB is constrained to move along an angled slot at end A. A circular disk with an outer radius of  $R$  is able to roll without slipping on a rough, horizontal floor. Bar AB is pinned to disk at point B on the perimeter of the disk. Pin A is known to move with a constant speed of  $v_A$  in the slot. At the position shown, bar AB is horizontal

**Find:** For position shown:

- Determine the angular velocities of link AB and of the disk. Write your answers as vectors
- Determine the angular accelerations of link AB and of the disk. Write your answers as vectors



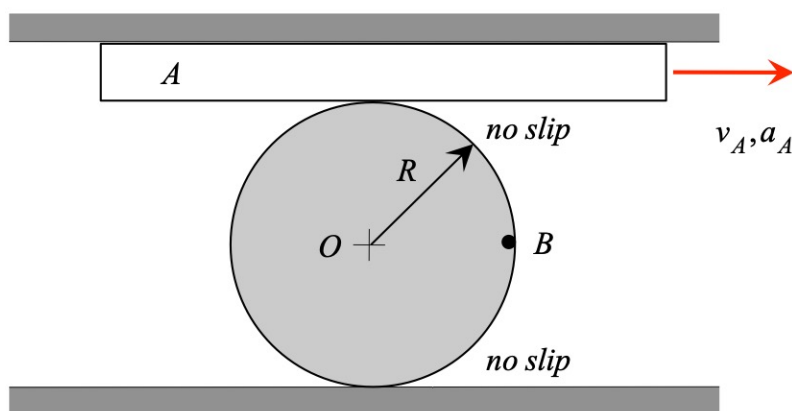
Use the following parameters in your analysis:  $R = 2$  ft,  $L = 8$  ft,  $v_A = 12$  ft/s and  $\theta = 36.87^\circ$ .

**Homework H2.D**

**Given:** A circular disk rolls without slipping on the horizontal surface below the disk. Block A is constrained to move along along a second horizontal surface above the block. As block A moves to the right with a speed of  $v_A$  and acceleration  $a_A$  it does not slip on the disk. At the position shown, point B on the perimeter of the disk is directly to the right of the center O of the disk.

**Find:** For this problem:

- Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors.
- Determine the velocity and acceleration vectors for point B for the position shown. Make a sketch of these vectors.



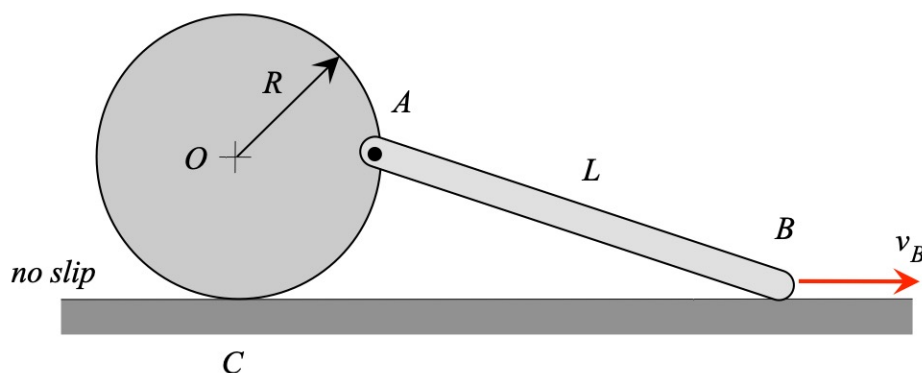
Use the following parameters in your analysis:  $R = 3$  ft,  $v_A = 12$  ft/s and  $a_A = 3$  ft/s<sup>2</sup>.

**Homework H2.E**

**Given:** The circular disk shown rolls without slipping on a straight horizontal surface. Bar AB is pinned to point A on the disk, with end B constrained to move along a smooth horizontal surface with a constant speed  $v_B$ . At the position shown, A is directly to the right of the center O of the disk.

**Find:** For this position:

- Determine the angular velocities of link AB and of the disk. Write your answers as vectors.
- Determine the angular accelerations of link AB and of the disk. Write your answers as vectors.

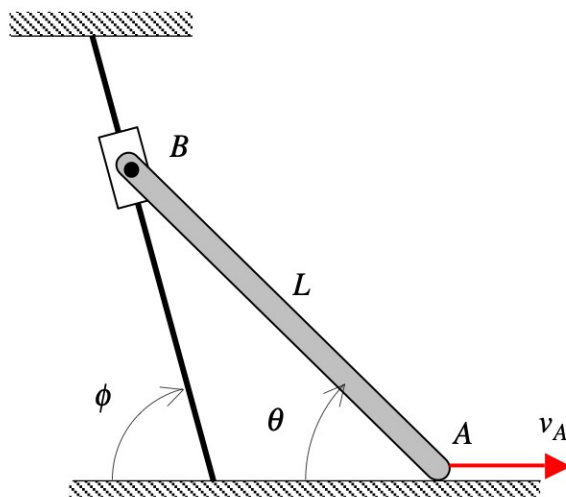


Use the following parameters in your analysis:  $R = 8$  in,  $L = 10$  in and  $v_B = 80$  in/s.

**Homework H2.F**

**Given:** Thin bar AB (having a length of  $L$ ) moves in a way that ends A and B slide along straight, fixed guides as shown in the figure. End A has a constant speed of  $v_A$  to the right as the bar moves.

**Find:** Determine the velocity and acceleration of end B.



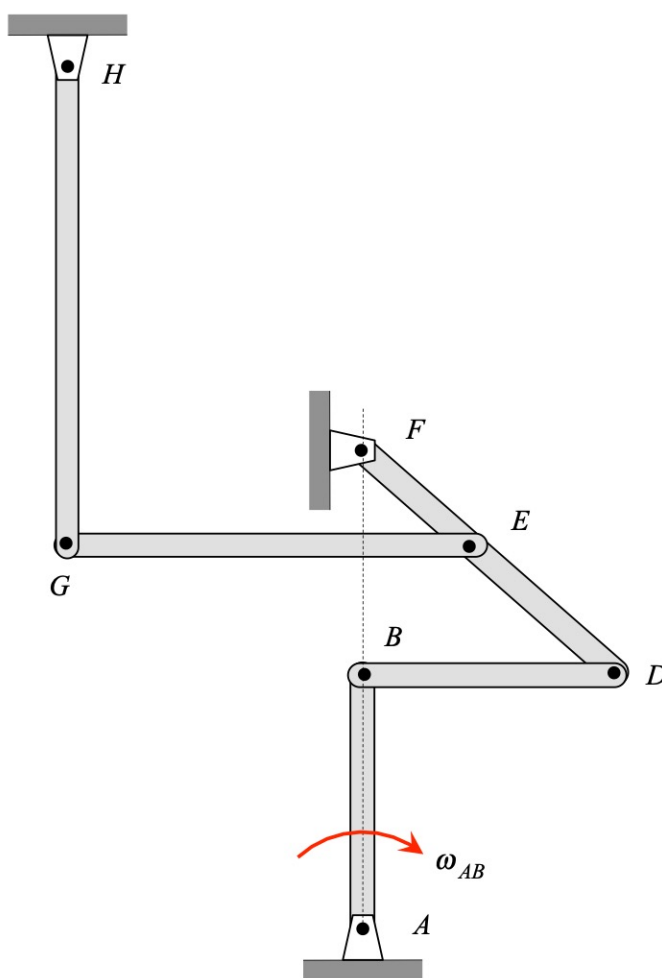
Use the following parameters in your analysis:  $L = 3$  ft,  $v_A = 6$  ft/s,  $\theta = 36.87^\circ$  and  $\phi = 45^\circ$ .

**Homework H2.G**

**Given:** The mechanism shown below is made up of links AB, BD, DF, EG and GH. Link AB is known to be rotating clockwise with an angular speed of  $\omega_{AB}$ .

**Find:** For this problem:

- Locate the instant centers for all links.
- Determine the sense of rotation (clockwise or counterclockwise) for all links.
- Use the locations of the instant centers found above to approximate the ratio of  $\omega_{GH}/\omega_{AB}$ . Assume that the drawing of the mechanism has been done to scale.



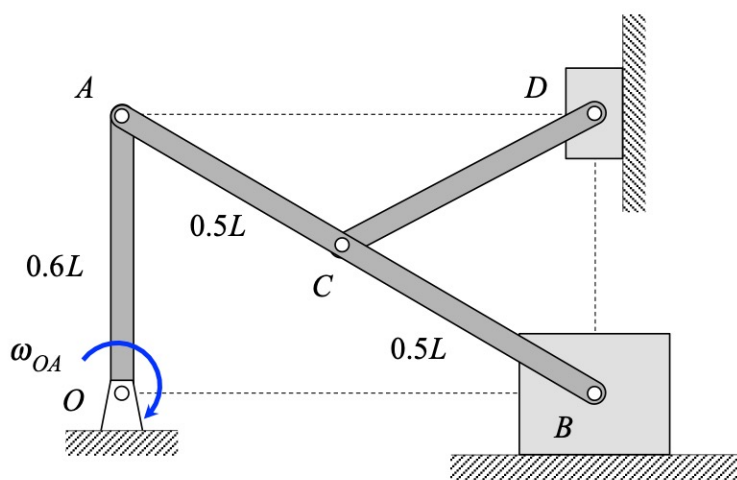
**Homework H2.H**

**Given:** The mechanism shown is made up of rigid links OA, AB and CD. Link OA has a pin joint at end O and is known to be rotating in the clockwise sense about O with a rotation rate of  $\omega_{OA}$ . AB is pinned to OA at end A and is pinned to a slider at B, with B moving along a horizontal guide. Link CD connects the center C of AB to a second slider at D through pin joints, with this slider constrained to move along a vertical guide. At the instant shown link OA is oriented vertically, B is directly below D and D is directly to the right of A.

**Find:** For this problem:

- Locate the instant center (IC) for link AB. Based on the location of this IC, what is the speed of pin C and the direction of travel for C, as well as the speed of slider B?
- Locate the IC for link CD. Based on this location, determine the speed of slider D.

**NOTE:** Please use only the instant center approach for this problem. Do not use vector analysis to find your answers.

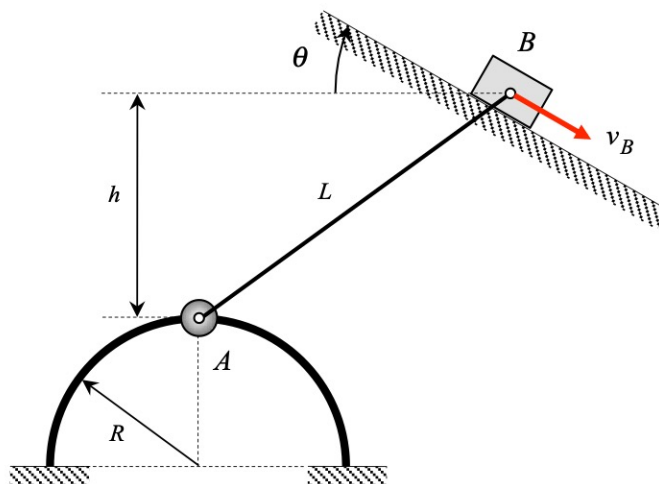


**Homework H2.1**

**Given:** Particle A is constrained to move on a semi-circular guide of radius  $R$ . Block B is constrained to move along an inclined ramp that is oriented at an angle of  $\theta$  relative to horizontal, as shown in the figure below. A and B are each pinned to ends of a rigid bar AB. At the instant shown, particle A is directly above the center of the semi-circular guide, block B is at a vertical distance of  $h$  above A and block B is traveling with a constant speed of  $v_B$ .

**Find:** For the position shown:

- Determine the angular velocity and angular acceleration of link AB. Write your answers as vectors.
- Determine the velocity and acceleration of particle A. Write your answers as vectors.
- Locate the instant center for link AB (assume that the figure provided has been drawn to scale). Does the location of this instant center agree with your results above for the direction of the velocity of particle A? Explain.



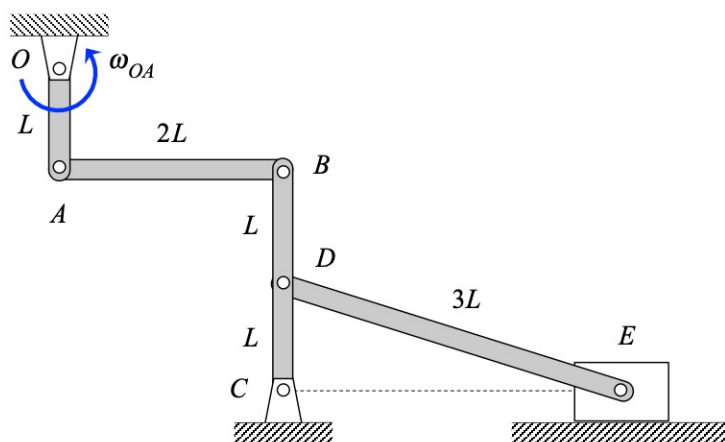
Use the following parameters in your analysis:  $\theta = 20^\circ$ ,  $v_B = 3000$  mm/s,  $L = 500$  mm,  $R = 100$  mm and  $h = 400$  mm.

**Homework H2.J**

**Given:** A mechanism is made up of rigid links OA, AB, BC and DE. A slider is pinned to end E of link DE and is constrained to move along a horizontal guide. For the position shown, link OA is rotating in the counterclockwise sense about O with a constant rotation rate of  $\omega_{OA}$ , with links OA and BC being vertically oriented and link AB being horizontally oriented.

**Find:** For the position shown:

- Use the instant center approach to determine the angular velocities of links AB, BC and DE, along with the speed of slider E.
- Use vector analysis to determine the angular accelerations of links AB, BC and DE, along with the acceleration of slider E. Is the speed of E increasing, decreasing or constant at this instant?

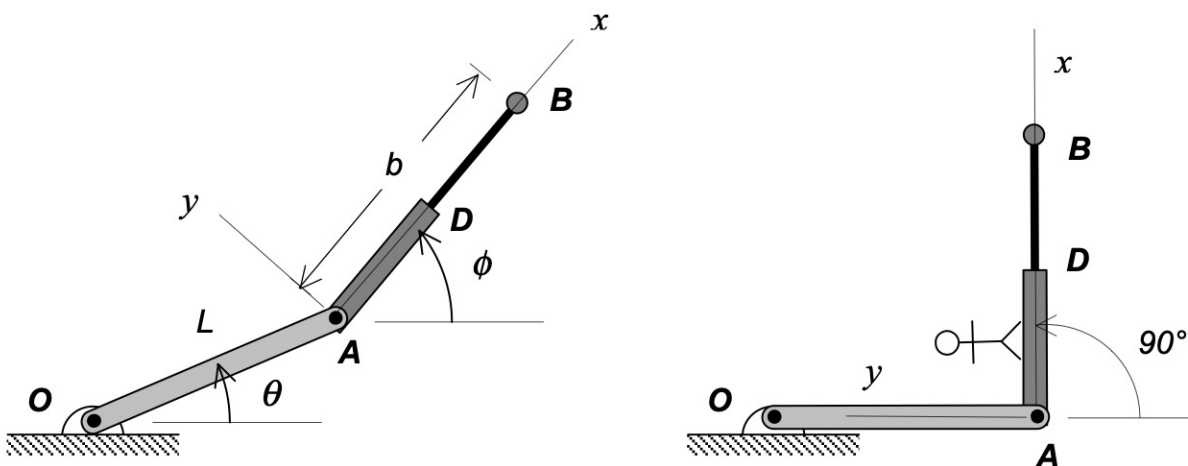




**Homework H3.A**

**Given:** A robotic manipulator is made up of two links OA and ADB as shown in the figure below left. Link OA has a fixed length of  $L$ , and the length link ADB is changing at a constant rate of  $\dot{b}$ .

**Find:** For the position shown below right with  $\theta = 0^\circ$  and  $\phi = 90^\circ$ , determine the acceleration of point B on the manipulator.



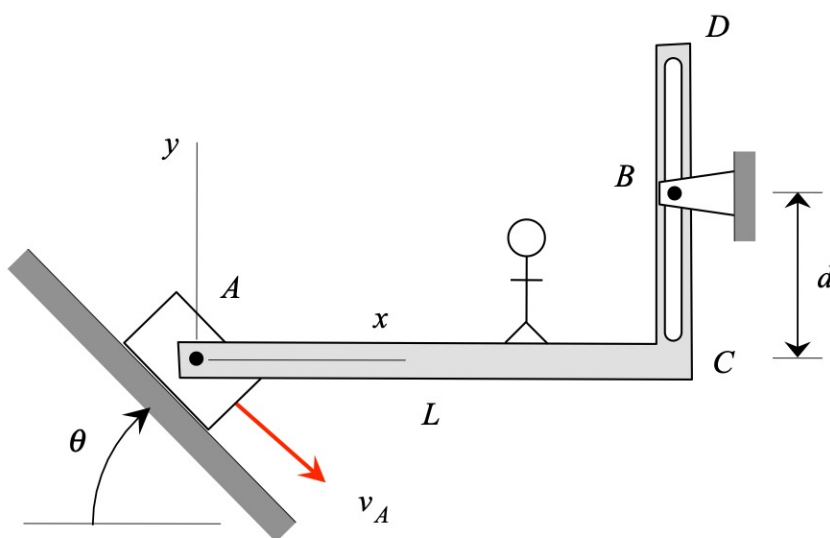
Use the following parameters in your analysis:  $b = 4$  ft,  $\dot{b} = 8$  ft/s = constant,  $\dot{\theta} = 4$  rad/s = constant,  $\dot{\phi} = 2$  rad/s = constant and  $L = 5$  ft.

**Homework H3.B**

**Given:** End A of bar ACD is constrained to move along a straight inclined surface. A straight slot is cut into section CD of bar ACD, as shown. A fixed pin B is allowed to slide relative to the slot. End A moves in the direction shown with a constant speed of  $v_A$ . At the position shown, section AC of ACD is horizontal and the slot in section CD of ACD is vertical. An observer and a set of  $xyz$  axes are attached to bar ACD.

**Find:** For this position, determine the angular velocity and angular acceleration of bar ACD. Write your answers as vectors.

**HINT:** Pin B is fixed; however, the observer on bar ACD observes motion of pin B. In what direction is this observed motion?

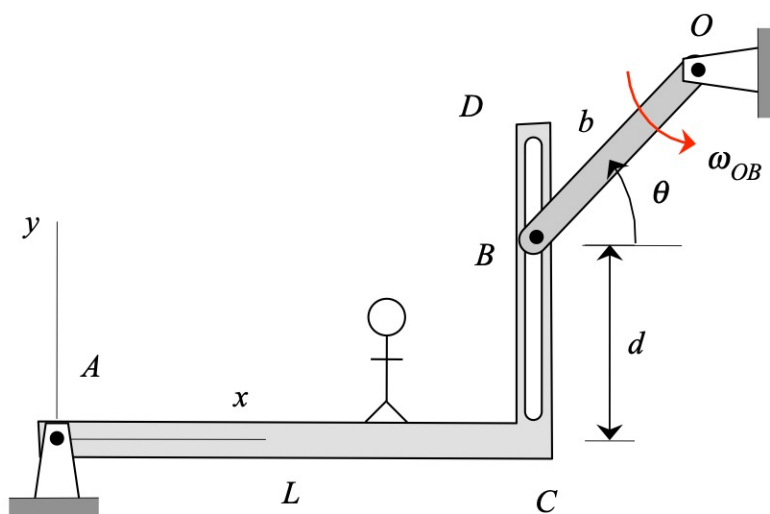


Use the following parameters in your analysis:  $\theta = 36.87^\circ$ ,  $L = 4$  ft,  $d = 3$  ft and  $v_A = 30$  ft/s.

**Homework H3.C**

**Given:** Bar ACD is pinned to ground at end A. A straight slot is cut into section CD of bar ACD, as shown. Pin B on link OB is allowed to slide relative to the slot. Link OB is rotating in the counterclockwise direction with a constant angular speed of  $\omega_{OB}$ . At the position shown, section AC of ACD is horizontal and the slot in section CD of ACD is vertical. An observer and a set of  $xyz$  axes are attached to bar ACD.

**Find:** For this position, determine the angular velocity and angular acceleration of bar ACD. Write your answers as vectors.



Use the following parameters in your analysis:  $\theta = 36.87^\circ$ ,  $L = 4$  ft,  $d = 3$  ft,  $b = 2$  ft,  $\omega_{OB} = 6$  rad/s.

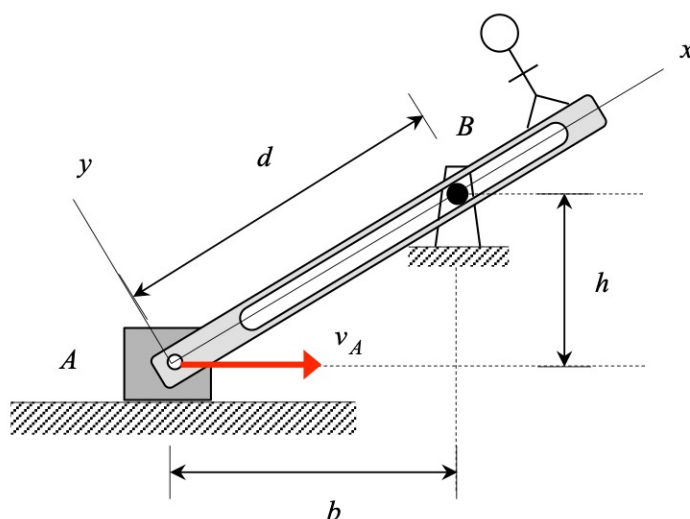
**Homework H3.D**

**Given:** Block A moves to the right with a constant speed  $v_A$ . A slotted arm is pinned to block A, with the slot in the arm being allowed to slide over a fixed pin at B.

**Find:** For this position,

- Determine the angular velocity of the arm and the value of  $\dot{d}$ .
- Determine the angular acceleration of the arm and the value of  $\ddot{d}$ .

**HINT:** Use an observer attached to the arm, and relate the kinematics of points A and B through the moving reference frame kinematics equations.

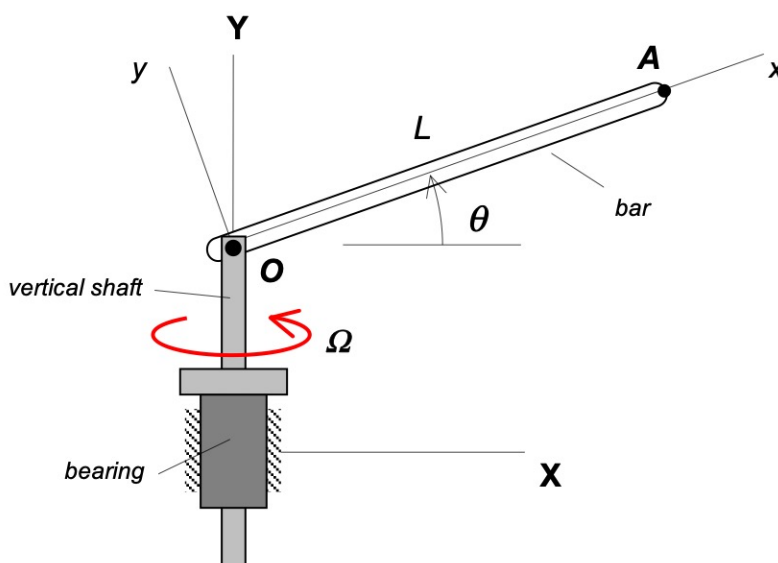


Use the following parameters in your analysis:  $b = 0.8$  ft,  $h = 0.6$  ft and  $v_A = 30$  ft/s.

**Homework H3.E**

**Given:** A shaft rotates about a fixed vertical axis at a constant rate of  $\Omega$ , as shown below. A straight bar OA, having a length of  $L$ , is pinned to point O on the shaft, with O being on the rotation axis of the shaft. At the instant when  $\theta = 0^\circ$ , bar OA is being raised at a rate of  $\dot{\theta}$  from the horizontal plane, with this rate changing at a rate of  $\ddot{\theta}$ . A set of  $xyz$  coordinate axes is attached to bar OA with its origin at O. A second set of coordinate axes,  $XYZ$ , are fixed to ground. At the instant when  $\theta = 0^\circ$ , the  $xyz$  and  $XYZ$  axes are aligned with each other.

**Find:** For the instant when  $\theta = 0^\circ$ , determine the acceleration of point A on the bar.

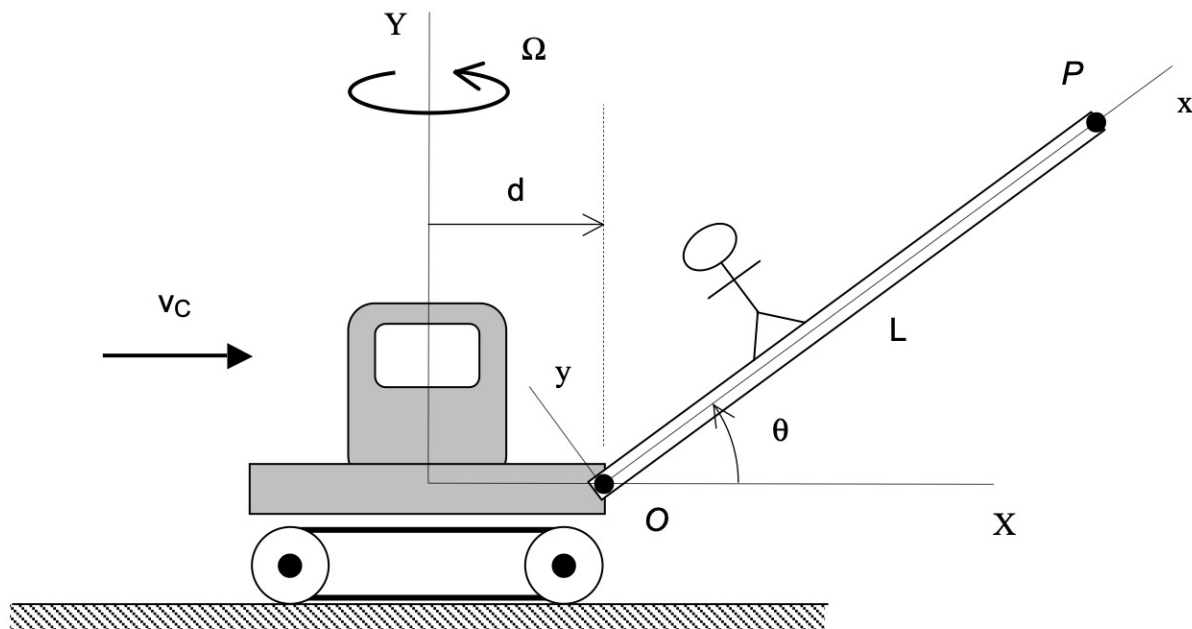


Use the following parameters in your analysis:  $\Omega = 6 \text{ rad/s}$ ,  $\dot{\theta} = 3 \text{ rad/s}$ ,  $\ddot{\theta} = -2 \text{ rad/s}^2$  and  $L = 3 \text{ m}$ .

## Homework H3.F

**Given:** A crane is moving to the right with a constant speed of  $v_C$  and is rotating about a vertical axis with a constant rate of  $\Omega$ . The boom is being raised at a rate of  $\dot{\theta}$  with  $\dot{\theta}$  changing at a rate of  $\ddot{\theta}$ .

**Find:** Determine the acceleration of end P of the boom.



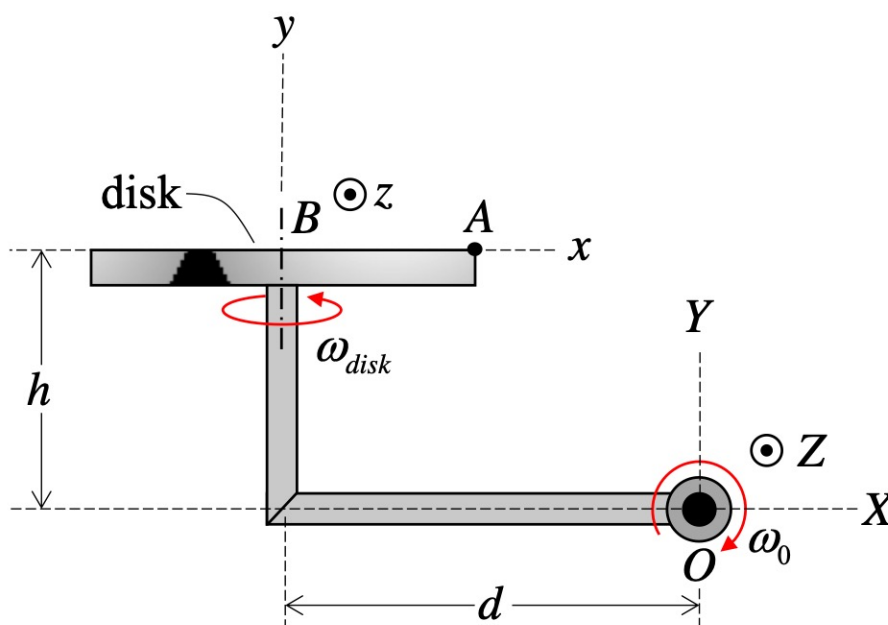
Use the following parameters in your analysis:  $v_C = 0.5$  ft/s,  $\Omega = 2$  rad/s,  $\theta = 36.87^\circ$ ,  $\dot{\theta} = 0.8$  rad/s,  $\ddot{\theta} = 1.5$  rad/s<sup>2</sup>,  $L = 25$  ft and  $d = 5$  ft.

**Homework H3.G**

**Given:** Arm  $OB$  rotates about a fixed axis with a constant rate of  $\omega_0$ . A disk of radius  $R$  rotates about its central axis with a constant rate of  $\omega_{disk}$  relative to the arm  $OB$ . The  $XYZ$  coordinate system is fixed with the  $Z$ -axis aligned with the fixed rotation axis of  $OB$ . The  $xyz$  coordinate system is attached to the disk with the  $y$ -axis aligned with the upper part of the arm for all time. For the position shown below, the  $xyz$  axes are aligned with the  $XYZ$  axes.

**Find:** For the position shown:

- Determine the angular acceleration of the disk. Write your answer as a vector in terms of its  $xyz$  components.
- Determine the acceleration of point  $A$  on the disk. Write your answer as a vector in terms of its  $xyz$  components.



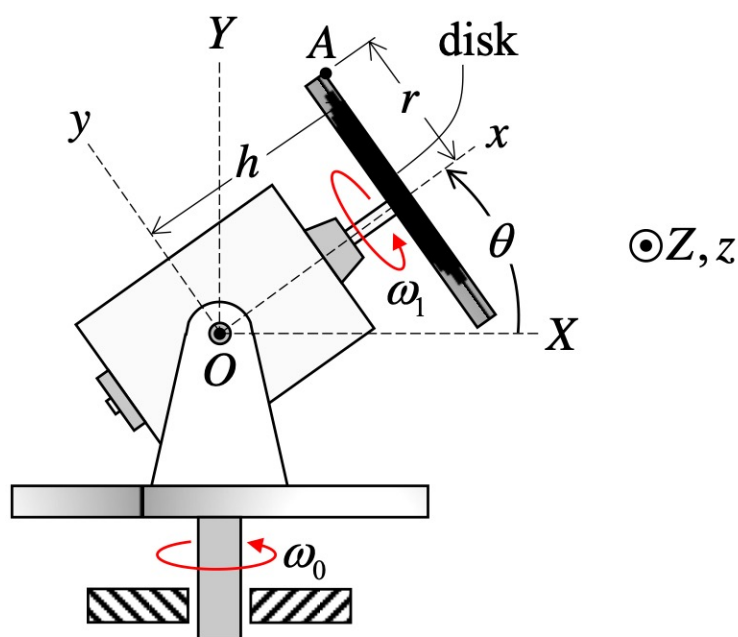
Use the following parameters in your analysis:  $\omega_0 = 5$  rad/s,  $\omega_{disk} = 4$  rad/s,  $d = 20$  cm,  $h = 15$  cm and  $R = 8$  cm.

## Homework H3.H

**Given:** A motor is attached to a platform that is rotating with a constant rate of  $\omega_0$  about a fixed vertical axis. The body of the motor pivots about a moving horizontal axis at a constant rate of  $\dot{\theta}$  with the shaft of the motor rotating at a constant rate of  $\omega_1$ .

**Find:** Determine:

- The angular acceleration of the disk attached to the shaft of the motor.
- The velocity of point A on the disk when A is at the top of the disk.



Use the following parameters in your analysis:  $\omega_0 = 2 \text{ rad/s}$ ,  $\theta = 30^\circ$ ,  $\dot{\theta} = 0.5 \text{ rad/s}$ ,  $\omega_1 = 60 \text{ rad/s}$ ,  $h = 0.15 \text{ m}$ , and  $r = 0.1 \text{ m}$ .

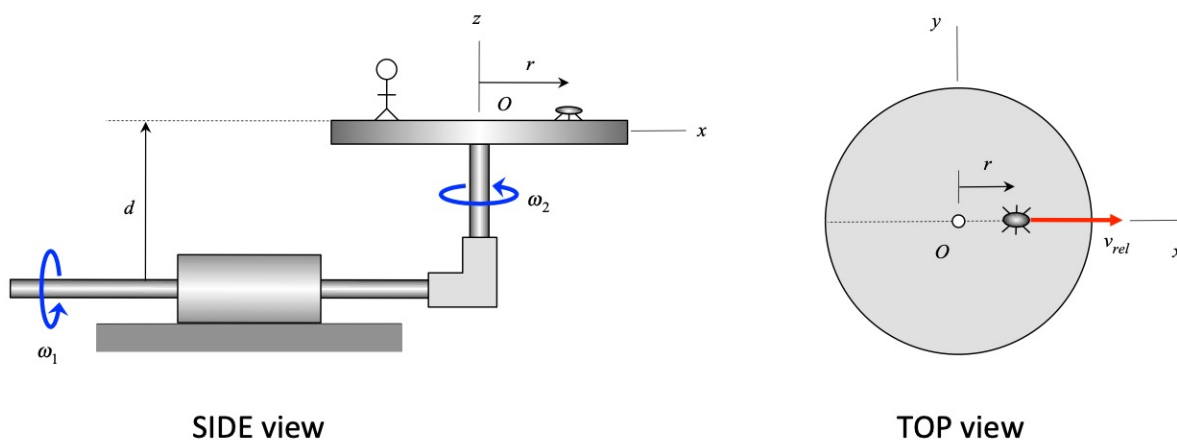


**Homework H3.1**

**Given:** A shaft rotates with a constant rate of  $\omega_1$  about a stationary horizontal axis. An L-shaped coupler at the end of the shaft is connected to a shaft passing through the center  $O$  of a circular disk. The disk rotates with a constant rate of  $\omega_2$  relative to the horizontal shaft. An insect is walking along a radial line of the disk with a constant speed of  $v_{rel}$  relative to the disk.

**Find:** For this problem:

- Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors.
- Determine the acceleration of the insect. Write your answer as a vector.

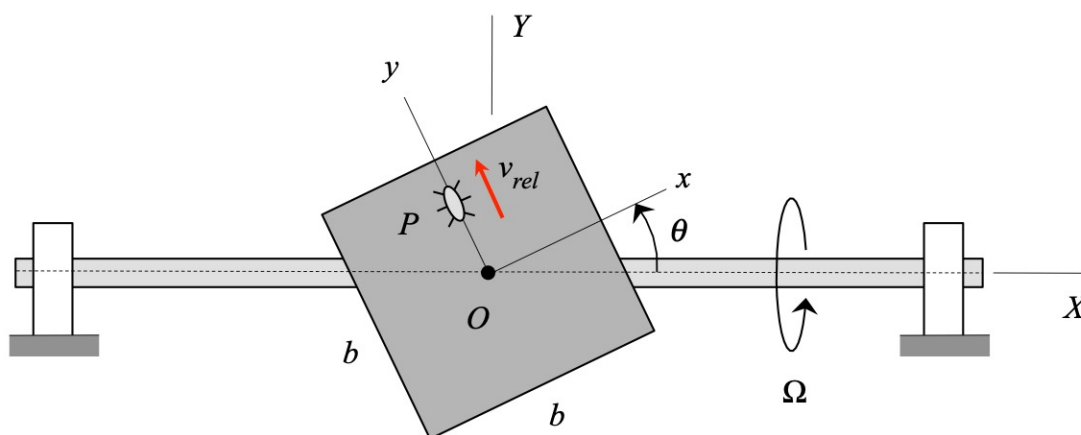


Use the following parameters in your analysis:  $\omega_1 = 6$  rad/s,  $\omega_2 = 3$  rad/s,  $r = 5$  in,  $d = 6$  in and  $v_{rel} = 6$  in/s.

**Homework H3.J**

**Given:** A shaft is rotating about the fixed  $X$ -axis at a constant rate of  $\Omega$ . A square plate is pinned at its center  $O$  to the centerline of the shaft and is rotating relative to the shaft about  $O$  at a constant rate of  $\dot{\theta}$ . A set of  $xyz$  axes are attached to the plate with its origin at  $O$ . An insect on the plate is walking along the  $y$ -axis with a constant speed of  $v_{rel}$  relative to the plate.

**Find:** Determine the velocity and acceleration of the insect when the insect has reached the edge of the plate. The insect reaches the edge of the plate when  $\theta = 0^\circ$ .



Use the following parameters in your analysis:  $b = 8$  in,  $v_{rel} = 15$  in/s,  $\Omega = 4$  rad/s and  $\dot{\theta} = 6$  rad/s.

## Chapter 4

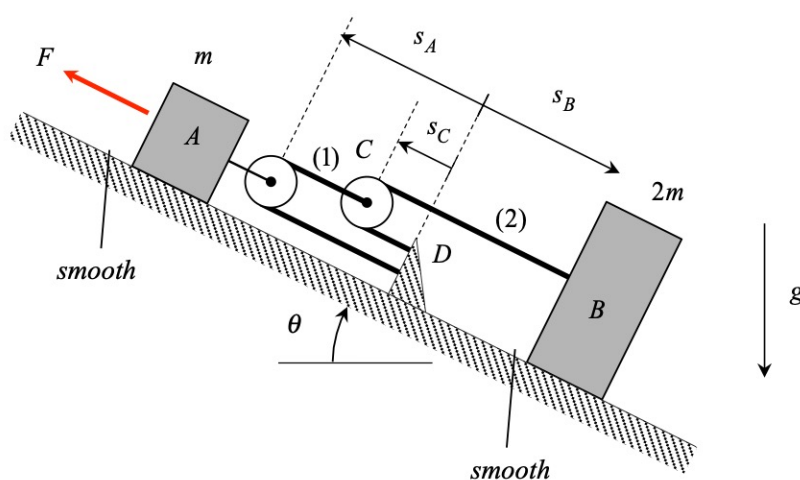
# Particle Kinetics Homework



**Homework H4.A**

**Given:** Blocks A and B (having masses of  $m$  and  $2m$ , respectively) are constrained to move along a smooth inclined surface. Cable (1) is connected to fixed ground at D and to the center of pulley C, as shown, with cable (1) being wrapped around a pulley connected to block A. A second cable (2) is connected between the fixed ground at D and block B. The pulleys are to be assumed to be of negligible mass, and the cables are assumed to be inextensible and not allowed to go slack. The sections of the cables not wrapped around pulleys are parallel to the incline on which blocks A and B move. A force  $F$  acts along the direction of the incline on block A.

**Find:** For this problem, determine the accelerations of blocks A and B.

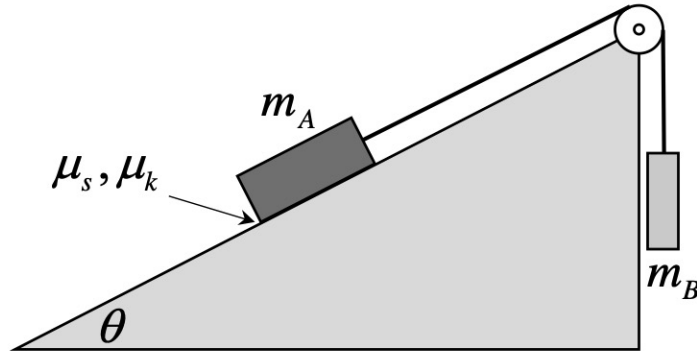


**Homework H4.B**

**Given:** The system shown is released from rest.

**Find:** On release, determine:

- (a) The acceleration of each block;
- (b) The tension in the cable.

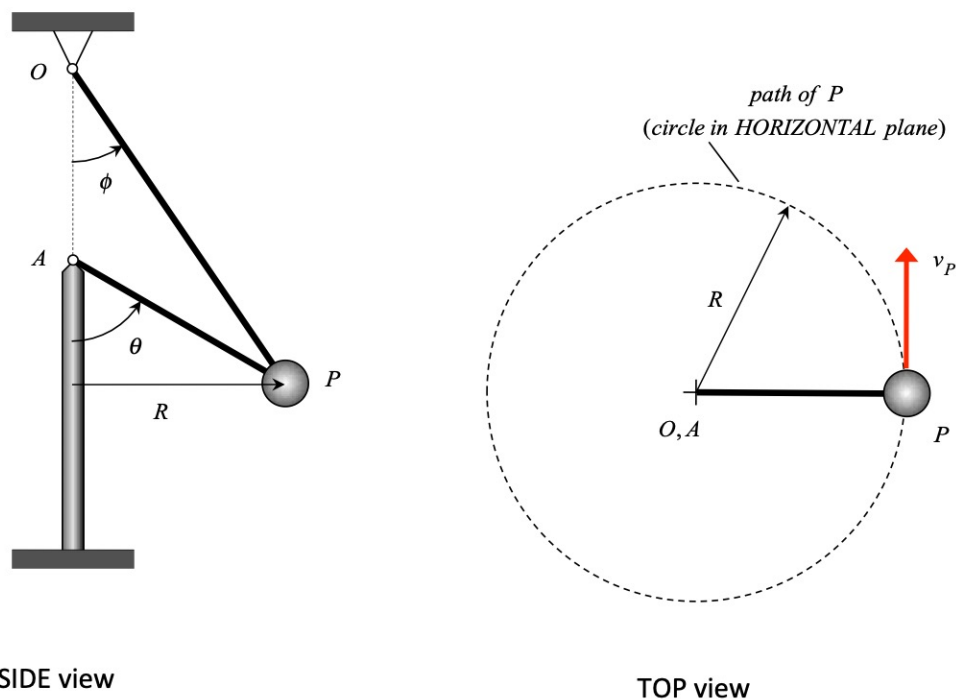


Use the following parameters in your analysis:  $\mu_s = 0.30$ ,  $\mu_k = 0.2$ ,  $m_A = 3$  kg,  $m_B = 4$  kg, and  $\theta = 36.87^\circ$ .

**Homework H4.C**

**Given:** Two wires, OP and AP, connect particle P (having a mass of  $m$ ) to fixed points O and A, respectively, where OA is a vertical line. The particle rotates about axis OA such that P has a constant speed of  $v_P$  and with the two wires remaining taut as particle moves on a circular path of radius  $R$ . Let  $\phi$  and  $\theta$  be the angles that wires OP and OA, respectively, make with the vertical.

**Find:** Determine the range of values for  $v_P$  for which wires OP and OA remain taut.

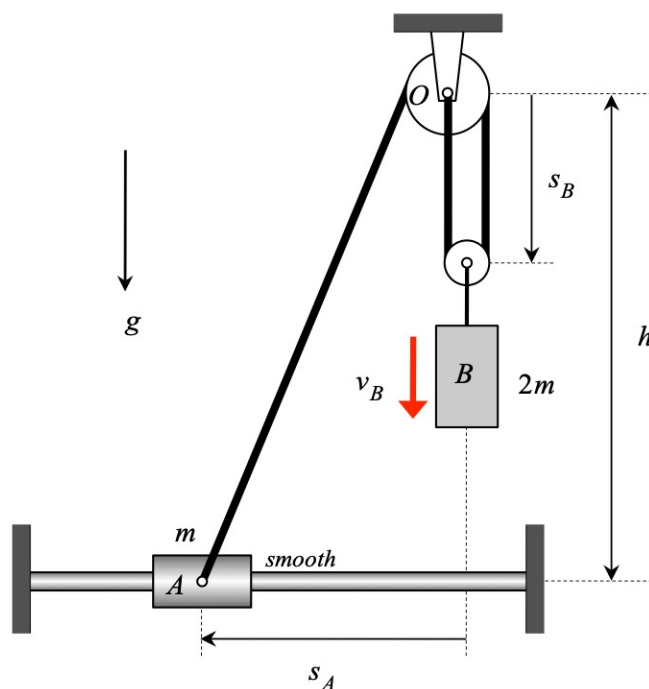


Use the following parameters in your analysis:  $R = 2$  m,  $\theta = 60^\circ$ ,  $\phi = 30^\circ$  and  $m = 6$  kg.

## Homework H4.D

**Given:** Block A (having a mass of  $m$ ) is connected to block B (with a mass of  $2m$ ) through the cable-pulley system shown. The system is released with block B moving downward with a speed of  $v_B$  and with block A displaced to the left of the path of B by an amount of  $s_A$  and moving to the right along its horizontal guide. Consider all surfaces to be smooth. Assume that the cable remains taut during this motion and that the radii of the pulleys are small.

**Find:** Determine the acceleration of blocks A and B for this position.



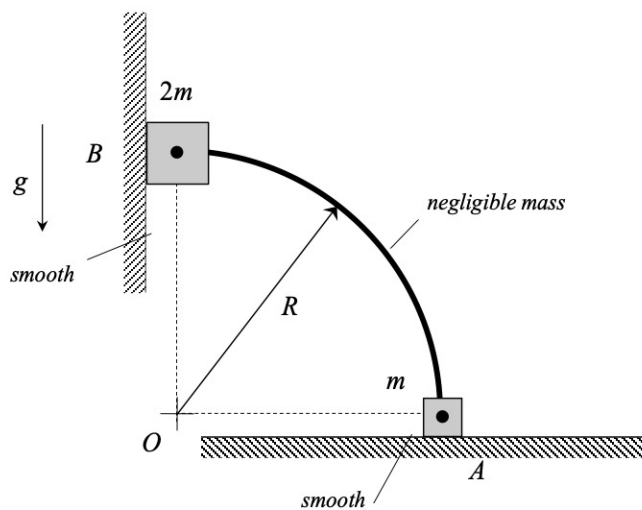
Use the following parameters in your analysis:  $m = 20$  kg,  $s_A = 0.5$  m,  $h = 2$  m and  $v_B = 12$  m/s.



**Homework H4.E**

**Given:** Blocks A and B (having masses of  $m$  and  $2m$ , respectively) are constrained to move along the smooth surfaces shown in the figure below. Member AB, in the shape of a quarter-circle arc, connects blocks A and B, with AB having a mass that is negligible compared to the masses of A and B. At the position shown, when the center O of the circular arc AB is directly below block B, the system is released from rest.

**Find:** For this position, determine the acceleration of blocks A and B on release.



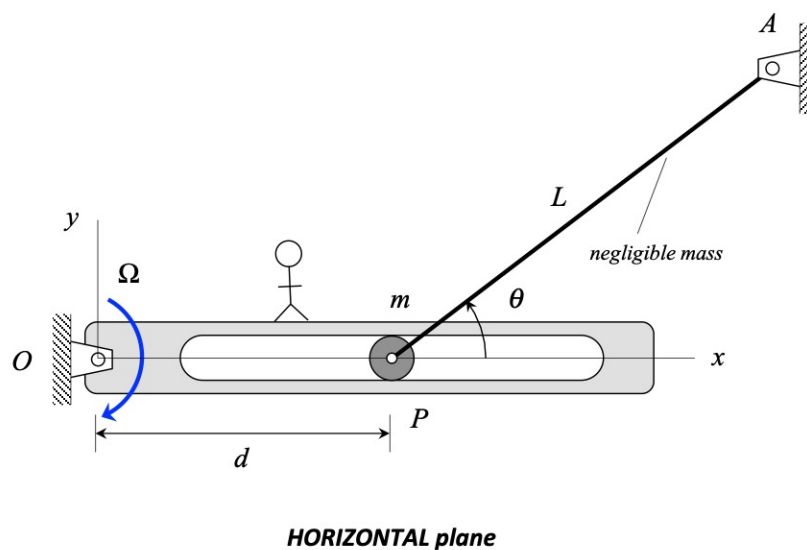
**HINT:** Note that AB is a two-force member. What does this say about the direction of the reaction forces on blocks A and B due to member AB?

**Homework H4.F**

**Given:** A slotted arm is rotating about end  $O$  with a constant rate of  $\Omega$ . Particle  $P$  (of mass  $m$ ) is attached to link  $AP$ , with the mass of  $AP$  assumed to be negligible compared to the mass of  $P$ .  $P$  is constrained to move within a smooth, straight slot. At the position of interest,  $AP$  is at an angle of  $\theta$  measured counterclockwise from the slotted arm. Note that  $AP$  is a two-force member. The mechanism moves in a horizontal plane.

**Find:** For this position,

- Determine the velocity and acceleration of  $P$ . Write your answers as vectors, in terms of their  $xy$  coordinates. It is suggested that you use the moving reference frame equations in your analysis for the velocity and acceleration of  $P$ . Use an observer attached to the slotted arm.
- Determine the normal force acting on  $P$  and the force acting on  $P$  by link  $AP$ .

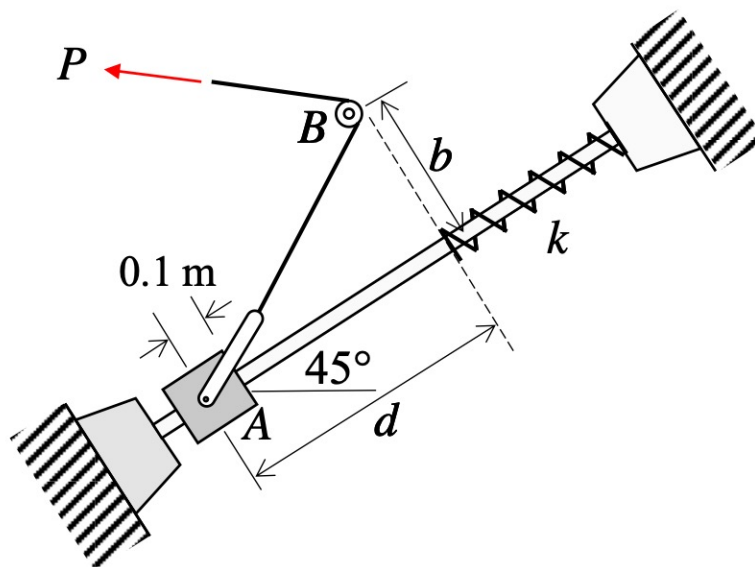


Use the following parameters in your analysis:  $m = 30 \text{ kg}$ ,  $\theta = 36.87^\circ$ ,  $d = 0.5 \text{ m}$ ,  $L = 1.5 \text{ m}$  and  $\Omega = 6 \text{ rad/s}$ .

## Homework H4.G

**Given:** A constant force  $P$  acts at the free end of the cable as block A (having mass  $m$ ) is pulled up the smooth rod. The system starts out from rest.

**Find:** Determine the stiffness  $k$  of the spring corresponding to a maximum spring compression of  $\Delta_{max}$ .

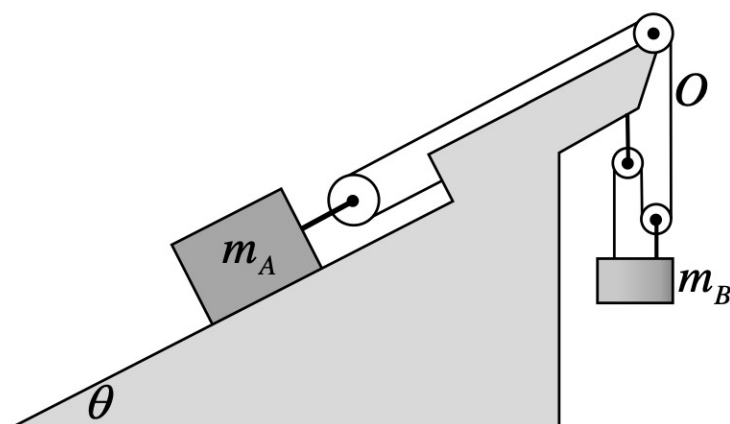


Use the following parameters in your analysis:  $m = 28\text{ kg}$ ,  $P = 500\text{ N}$ ,  $b = 1\text{ m}$ ,  $d = 1.2\text{ m}$  and  $\Delta_{max} = 0.02\text{ m}$ .

**Homework H4.H**

**Given:** The system shown is released from rest. Assume all surfaces to be smooth and that the mass of the pulleys is negligible.

**Find:** Determine the speed of both masses after B has moved a distance of  $d$ .

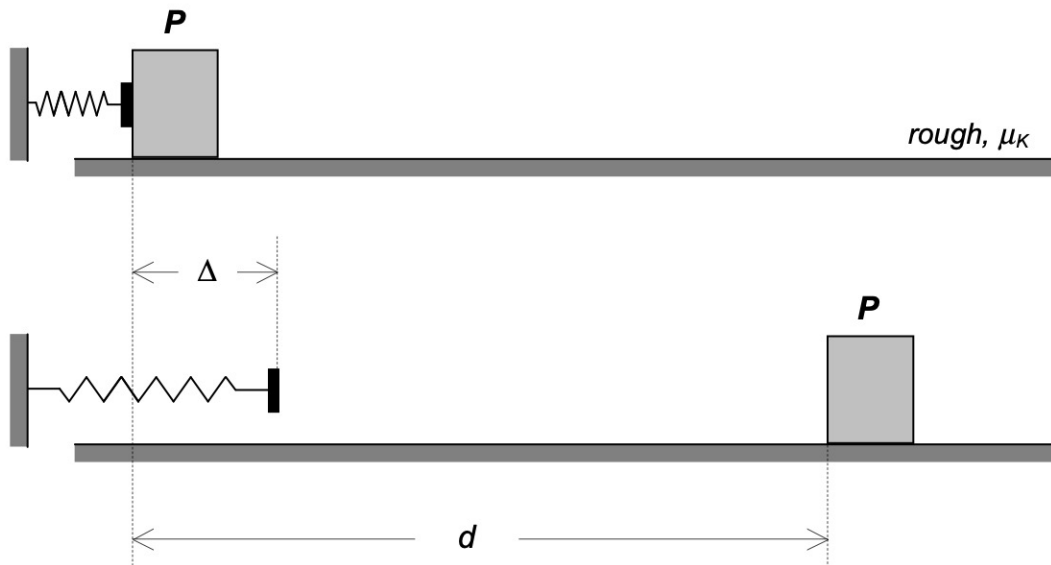


Use the following parameters in your analysis:  $\theta = 20^\circ$ ,  $m_A = 6$  kg,  $m_B = 14$  kg and  $d = 0.4$  m.

**Homework H4.1**

**Given:** Particle P, having a mass of  $m$ , is pressed against a spring (having a stiffness of  $k$ ) with the spring being compressed by an amount of  $\Delta$ . Upon release from rest, the particle travels along a rough horizontal surface for which the kinetic coefficient of friction is known to be  $\mu_k$ .

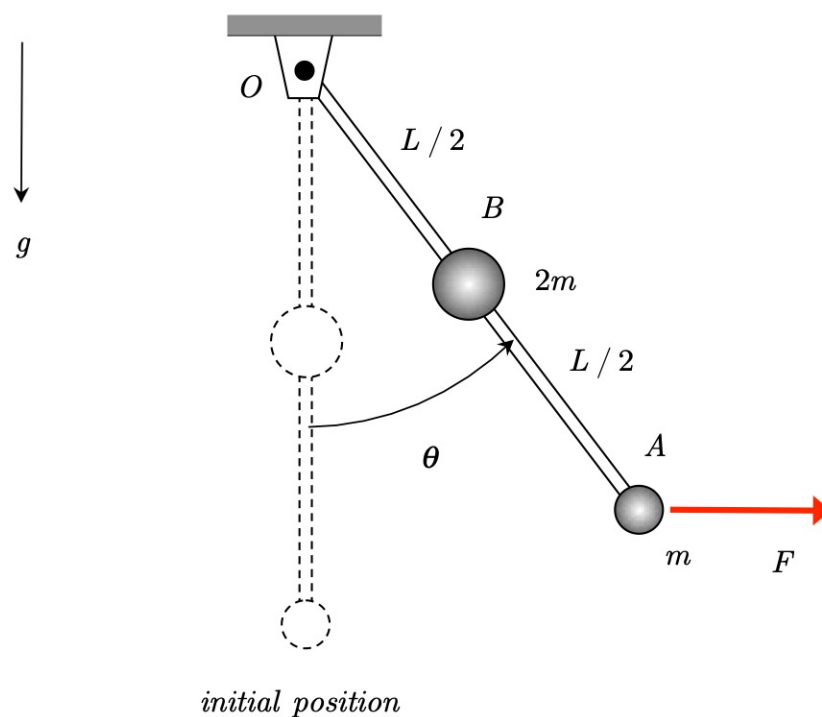
**Find:** Determine the total distance  $d$  traveled by P before coming to rest. Assume that  $d > \Delta$ . Leave your answer in symbolic form.



**Homework H4.J**

**Given:** Particles A and B (having masses of  $m$  and  $2m$ , respectively) are attached to a lightweight rigid bar as shown in the figure. A constant horizontal force  $F$  acts on particle A. At the initial state of  $\theta = 0^\circ$ , the system is at rest.

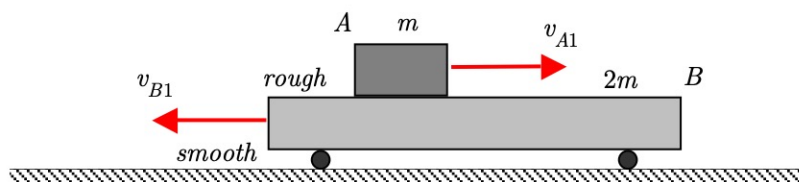
**Find:** Determine the angular speed of the bar as a function of  $\theta$  and in terms of the parameters of the problem.



**Homework H4.K**

**Given:** Block B (of mass  $2m$ ) is able to slide along a smooth horizontal surface. Block A (of mass  $m$ ) is able to slide along the rough top surface of block B, as shown in the figure. Initially, A is traveling to the right with a speed of  $v_{A1}$ , and block B is traveling to the left with a speed of  $v_{B1}$ .

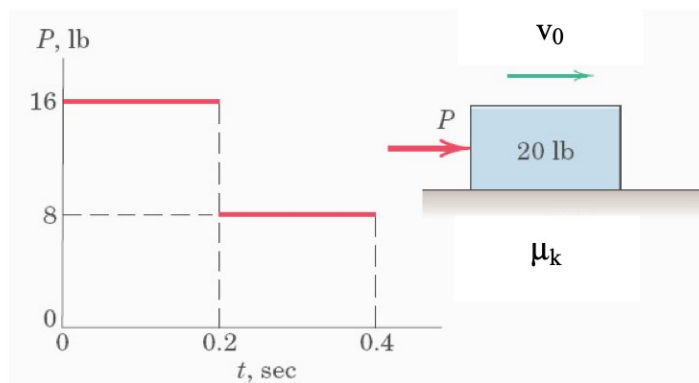
**Find:** Determine the velocity of block B when block A has to come rest relative to block B.



**Homework H4.L**

**Given:** A block is sliding to the right on a rough, horizontal surface with a speed of  $v_0$  when a force  $P$  is applied.

**Find:** Determine the speed of the block at time  $t$ .



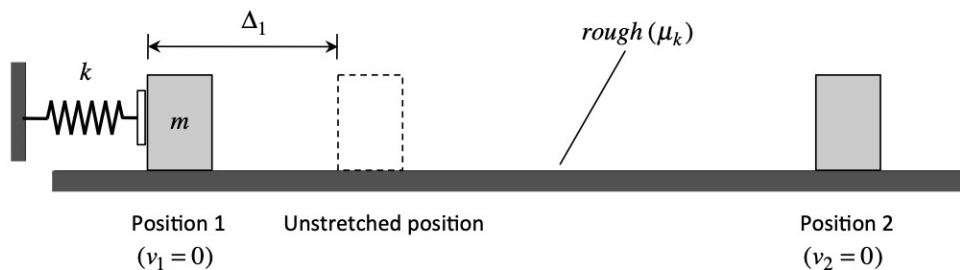
Use the following parameters in your analysis:  $v_0 = 2$  ft/s,  $\mu_k = 0.3$  and  $t = 0.4$  s.



**Homework H4.M**

**Given:** A block of mass  $m$  is pressed against a spring of stiffness  $k$  to a spring compression of  $\Delta_1$ . On release from rest, the block is allowed to slide along a rough horizontal surface with the coefficient of kinetic friction between the block and the surface being  $\mu_k$ . After losing contact with the spring, the block is known to slide for an elapsed time of  $\Delta t$  to Position 2 where the block comes to rest.

**Find:** Determine the coefficient of kinetic friction between the block and the surface,  $\mu_k$ .

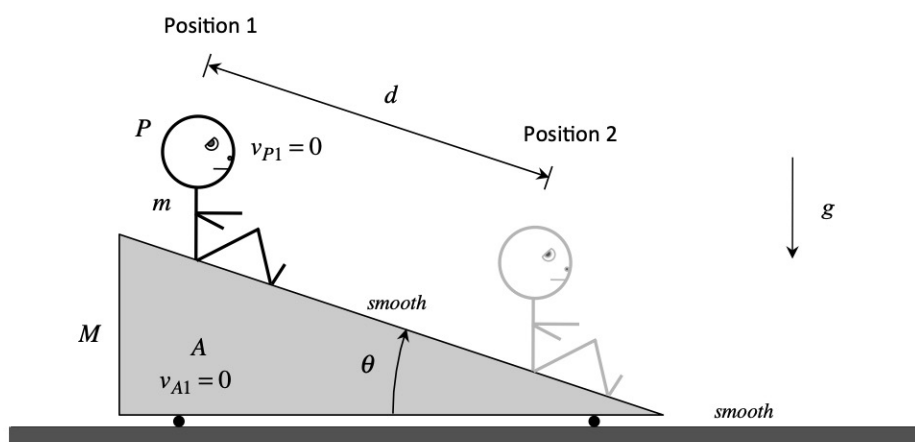


Use the following parameters in your analysis:  $m = 15$  kg,  $k = 800$  N/m,  $\Delta_1 = 0.03$  m and  $\Delta t = 3$  s.

## Homework H4.N

**Given:** A wedge-shaped block of mass  $M$  is able to slide freely on a horizontal surface. A person of mass  $m$  climbs onto the inclined surface of the wedge. With both the person and cart initially at rest, the person slides down the incline of the wedge. Consider the person to move as a particle as she moves down the inclined surface. Also, consider all surfaces to be smooth.

**Find:** Determine the velocity of the person and of the cart after the person has slid down a distance  $d$  along the surface of the inclined wedge. Write your answers as vectors.



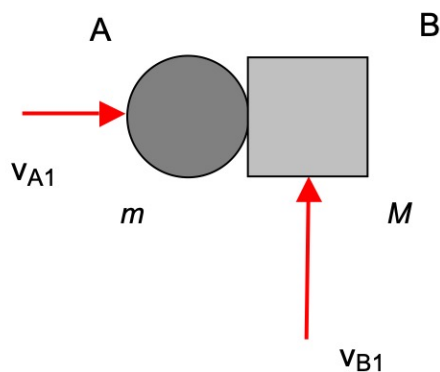
Use the following parameters in your analysis:  $mg = 140$  lb,  $Mg = 200$  lb,  $\theta = 36.87^\circ$  and  $d = 8$  ft.

**Homework H4.O**

**Given:** Blocks A and B (having masses of  $m$  and  $M$ , respectively) are initially traveling in directions perpendicular to each other with speeds of  $v_{A1}$  and  $v_{B1}$ , respectively, as shown below in the figure. After impacting each other, A is traveling to the RIGHT with a speed of  $v_{A2}$ , and B travels with a speed of  $v_{B2}$  (the direction of motion for B after impact is not known). Consider all surfaces to be smooth.

**Find:** For this problem:

- Determine the mass  $M$  of block B;
- Determine the coefficient of restitution  $e$  for the impact of A and B.



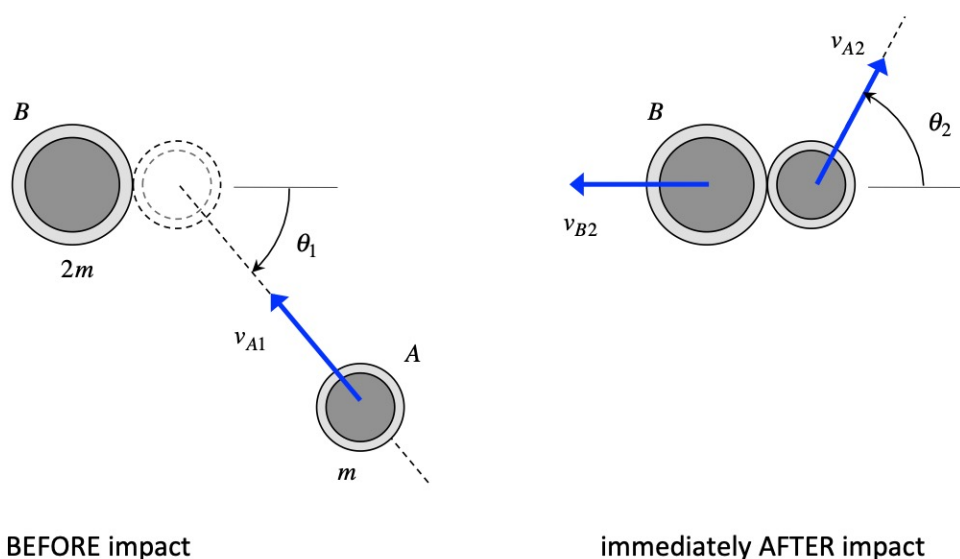
Use the following parameters in your analysis:  $m = 3$  kg,  $v_{A1} = 4$  m/s,  $v_{B1} = 4$  m/s,  $v_{A2} = 2$  m/s and  $v_{B2} = 5$  m/s.

## Homework H4.P

**Given:** Disks A and B (of masses  $m$  and  $2m$ , respectively) are able to slide freely on a smooth horizontal surface. Disk A moves toward the stationary disk B with a speed of  $v_{A1}$  in the direction shown in the figure. After impact, disk B moves immediately to the left, whereas disk A moves in a direction given by the angle  $\theta_2$ . The coefficient of restitution for the impact between the two disk is  $e$  and the contacting surfaces between A and B during impact is smooth.

**Find:** For this problem:

- Derive an expression for the angle  $\theta_2$  in terms of  $e$ .
- Knowing that  $0 \leq e \leq 1$ , use your answer from (a) to determine the range of possible angles  $\theta_2$ .



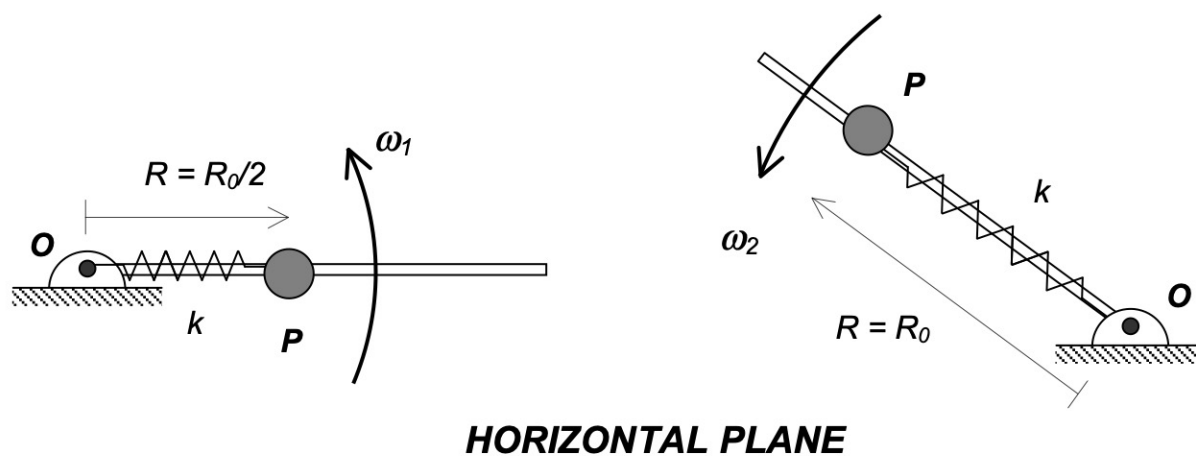
Use the following parameters in your analysis:  $\theta_1 = 30^\circ$ .

**Homework H4.Q**

**Given:** A particle P, having a mass of  $m$ , is free to slide on a smooth, lightweight bar. The bar is free to rotate in a horizontal plane about a vertical shaft passing through O. A spring, having a stiffness  $k$  and unstretched length  $R_0$ , is connected between P and O. The spring is compressed to half of its unstretched length and released when the bar has a rotational speed of  $\omega_1$ , as shown in the figure below left.

**Find:** For the position when  $R = R_0$  (shown in the figure below right):

- Determine the rotation rate  $\omega_2$  of the bar;
- Determine the value of  $\dot{R}$ .

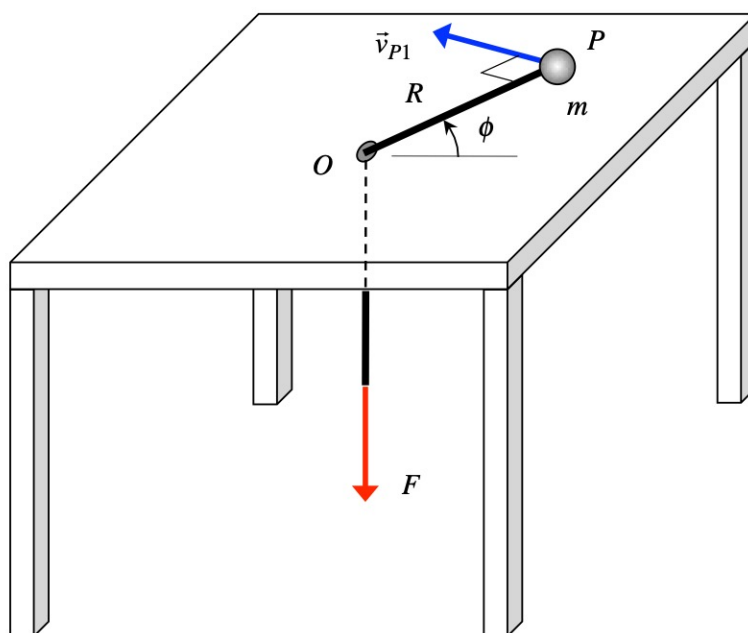


Use the following parameters in your analysis:  $m = 20$  kg,  $k = 600$  N/m,  $R_0 = 2$  m and  $\omega_1 = 5$  rad/s.

**Homework H4.R**

**Given:** Particle P, having a mass of  $m$ , is able to slide on the smooth, horizontal top of a table. A flexible cable is attached to P, with the cable being fed through a hole in the table at O. A constant force  $F$  acts on the other end of the cable. The system is released with P being at a radial distance  $R = R_1$  from O, and with P having a velocity perpendicular to OP with a speed of  $v_{P1}$ .

**Find:** Determine the numerical values for  $\dot{R}$  and  $\dot{\phi}$  when P has moved to a position for which  $R = R_2$ .

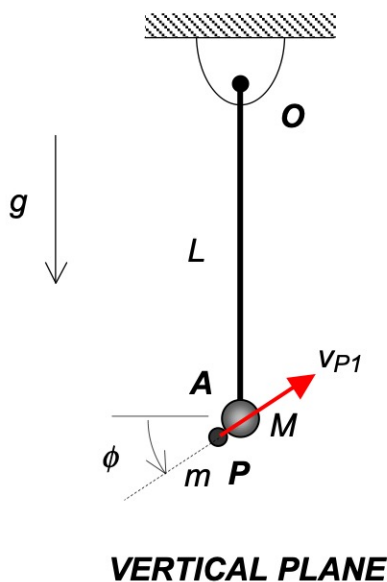


Use the following parameters in your analysis:  $m = 2$  kg,  $R_1 = 1.5$  m,  $R_2 = 0.5$  m,  $v_{P1} = 5$  m/s and  $F = 236$  N.

**Homework H4.S**

**Given:** Rigid arm OA (having length  $L$  and having negligible mass) is pinned to ground at end O. A particle of mass  $M$  is attached to end A of OA. At instant "1", a pellet P (having a mass of  $m$ ) strikes the stationary particle A with a speed of  $v_{P1}$  in the direction shown below in the figure. At the end of a short time interval impact, P sticks to A.

**Find:** Determine the angular speed of arm OA immediately after P sticks to A.



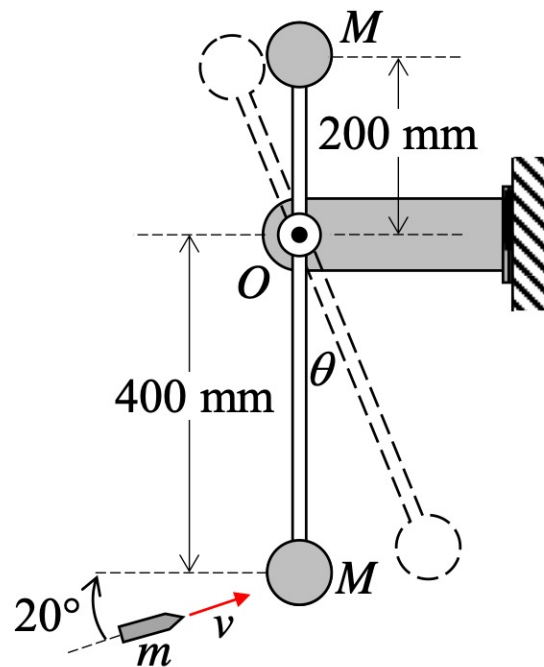
Use the following parameters in your analysis:  $\phi = 30^\circ$ ,  $L = 5$  ft,  $mg = 4$  lb,  $Mg = 8$  lb and  $v_{P1} = 150$  ft/s.

**Homework H4.T**

**Given:** A bullet strikes the lower particle of a stationary pendulum with a speed of  $v$ . After impact, the bullet sticks to the particle.

**Find:** Determine:

- The angular speed of the pendulum immediately after impact;
- The maximum rotation angle through which the pendulum swings after impact.



Use the following parameters in your analysis:  $v = 150$  m/s,  $m = 40$  g and  $M = 4$  kg.



## Chapter 5

# Rigid Body Kinetics Homework

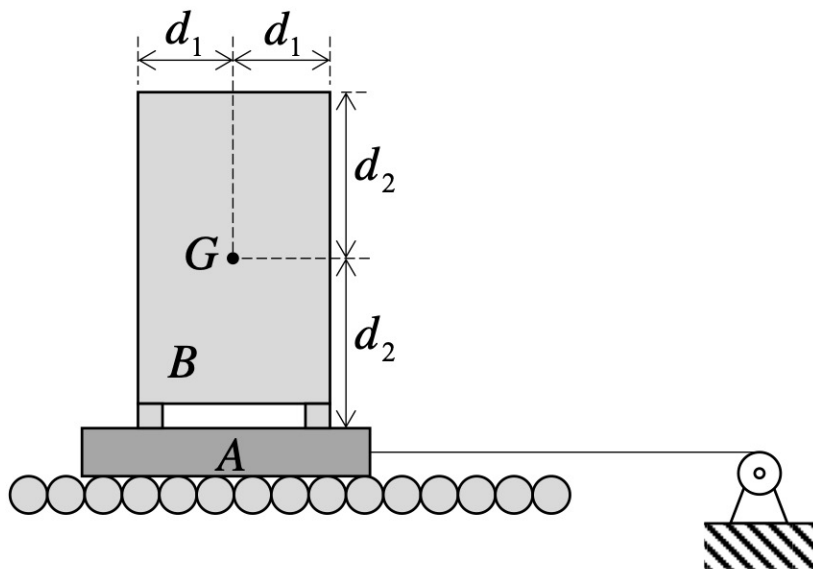


**Homework H5.A**

**Given:** A pallet/winch/roller system is used to move crate B of mass  $M$ , where the pallet A has a mass of  $m$ .

**Find:**

- Determine the maximum winch force that can be applied to the pallet such that the crate does not tip; and
- Determine the coefficient of static friction required between the crate and pallet to ensure that tipping occurs prior to slip.



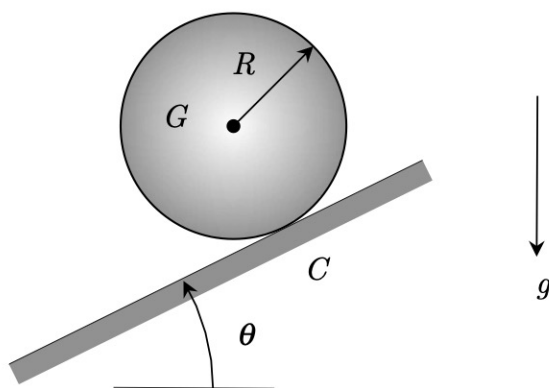
Use the following parameters in your analysis:  $m = 15$  kg,  $M = 80$  kg,  $d_1 = 0.3$  m and  $d_2 = 0.6$  m.

**Homework H5.B**

**Given:** A homogeneous sphere of mass  $m$  and radius  $R$  is released from rest while in contact with a rough inclined surface.

**Find:** For this problem:

- If the sphere is able to roll without slipping on the inclined surface, determine the acceleration of its center of mass  $G$ .
- If the sphere slips on the inclined surface (coefficient of kinetic friction  $\mu_k = 0.2$ ), determine the acceleration of its center of mass  $G$ .



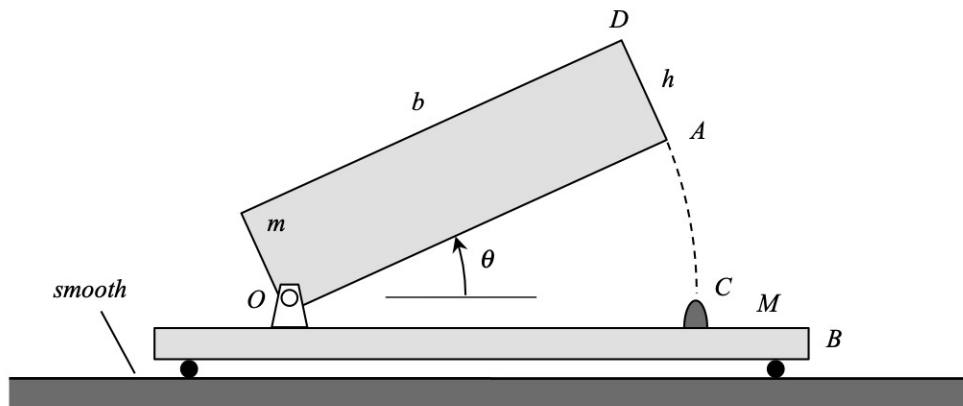
Use the following parameters in your analysis:  $m = 10$  kg,  $R = 0.2$  m and  $\theta = 60^\circ$ .

**Homework H5.C**

**Given:** A homogeneous plate of mass  $m$  is attached to cart B by a pin joint at corner O of the plate. The cart (having a mass of  $M$ ) is free to move along a smooth horizontal surface. The plate and cart are released from rest with the plate at angle of  $\theta$ .

**Find:**

- Determine the angular acceleration of the plate on release.
- Determine the acceleration of the cart on release.

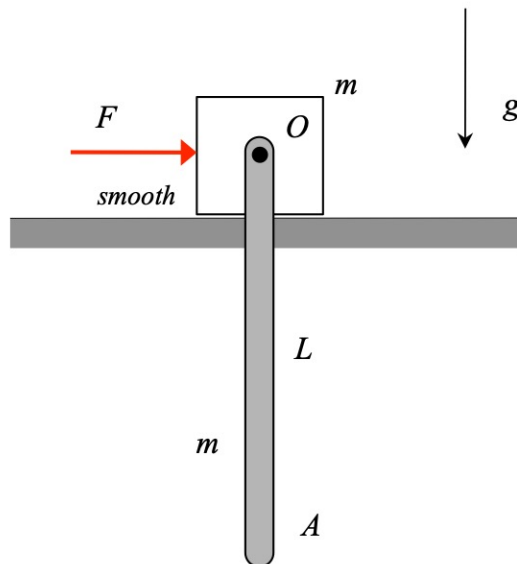


Use the following parameters in your analysis:  $m = 15$  kg,  $M = 20$  kg,  $h = 0.2$  m,  $b = 0.8$  m, and  $\theta = 30^\circ$ .

**Homework H5.D**

**Given:** A thin homogeneous bar OA (of mass  $m$  and length  $L$ ) is pinned to a block of mass  $m$ . The block is able to slide on a smooth horizontal surface. A horizontal force  $F$  acts on the block. The system is released from rest with the bar hanging vertically.

**Find:** Determine the angular acceleration of bar OA on release.



Use the following parameters in your analysis:  $m = 30$  kg,  $L = 2$  m and  $F = 200$  N.

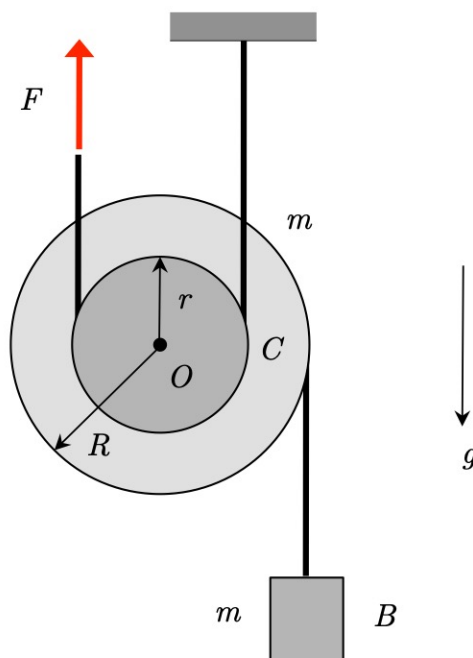
**Homework H5.E**

**Given:** An inextensible cable is wrapped around the inner radius of a solid, stepped drum (having a mass of  $m$ , inner radius  $r$ , outer radius  $R$  and radius of gyration about its center  $O$  of  $k_O$ ). One end of the cable is attached to ground, with a vertical force  $F$  acting at the free end of the cable. A second inextensible cable is wrapped around the outer radius of the drum with block B (of mass  $m$ ) attached at its free end. Neither cable slips on the drum as the system moves. The system is released from rest.

**Find:** On release:

- Determine the angular acceleration of the drum.
- Determine the acceleration of block B.

**HINT:** Since the drum does not slip on the cables, point C is the instant center for the drum.



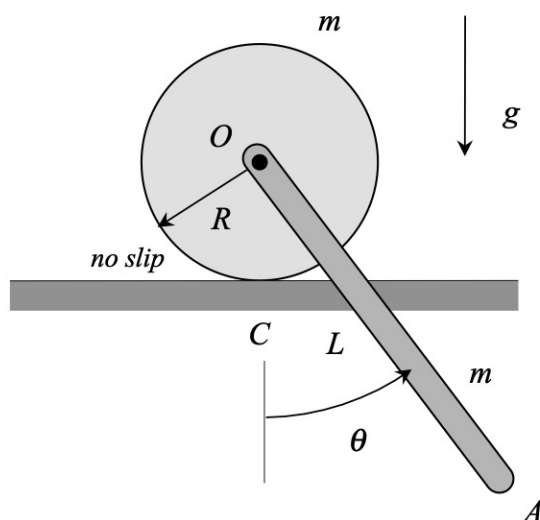
Use the following parameters in your analysis:  $m = 8$  kg,  $R = 0.5$  m,  $r = 0.25$  m,  $k_O = 0.3$  m and  $F = 20$  N.

**Homework H5.F**

**Given:** A thin homogeneous bar OA (of mass  $m$  and length  $L$ ) is pinned to a homogeneous disk (of mass  $m$  and radius  $R$ ) at the disk's center  $O$ . The disk is able to roll without slipping on a rough horizontal surface. The system is released from rest with the bar at an angle of  $\theta$  measured counterclockwise from vertical.

**Find:** For this problem:

- Determine the angular acceleration of the disk on release.
- Determine the angular acceleration of bar OA on release.



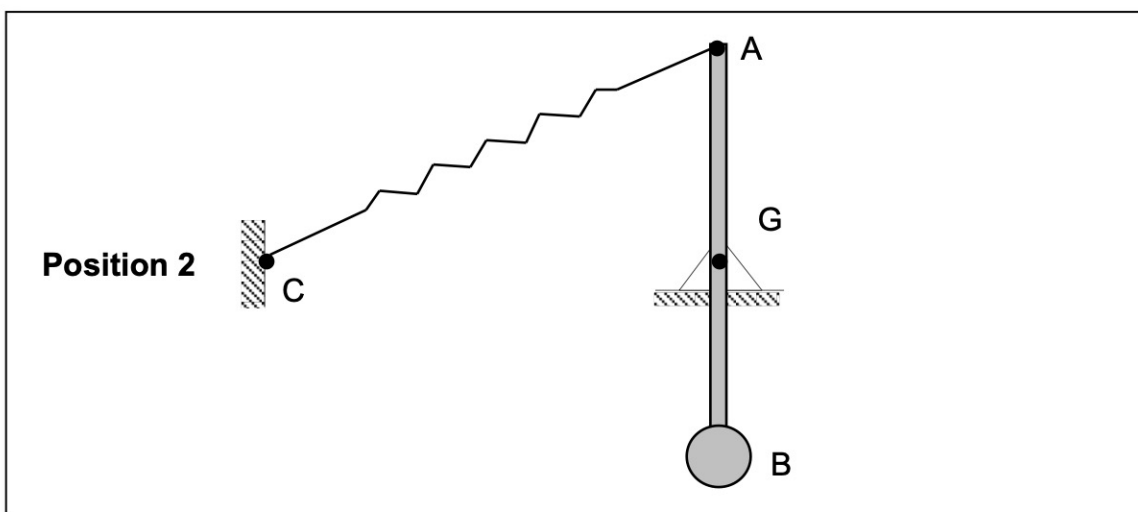
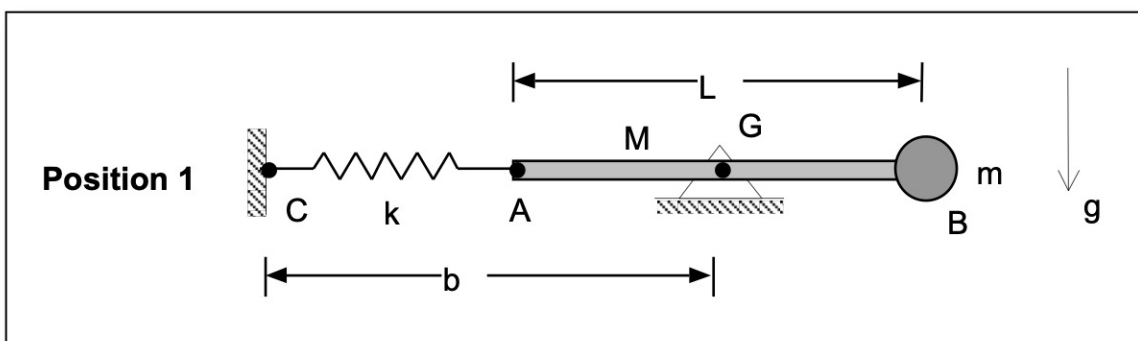
Use the following parameters in your analysis:  $m = 30$  kg,  $R = 0.5$  m,  $L = 2$  m and  $\theta = 60^\circ$ .



## Homework H5.G

**Given:** A thin, homogeneous bar having a mass of  $M$  and length  $L$  is pinned to ground at its mass center  $G$ . Particle  $B$ , having a mass of  $m$ , is rigidly attached to the right end of the bar. A spring, having a stiffness of  $k$ , is attached between end  $A$  of the bar and pin  $C$  on a wall. The pin  $G$  is a distance of  $b$  from the wall. When the bar is horizontal (Position 1 shown below), the spring is unstretched.

**Find:** If the bar is released from rest in Position 1 above, find the angular velocity of the bar in Position 2 when the bar is in a vertical position.

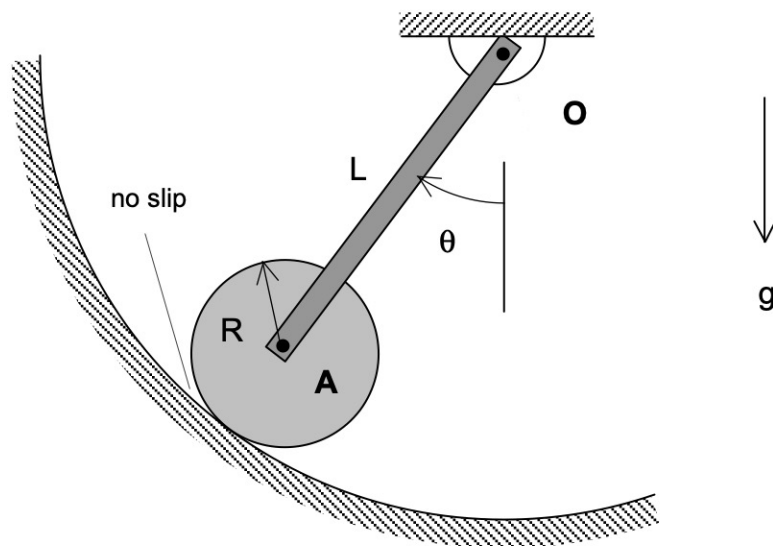


Use the following parameters in your analysis:  $M = 20$  kg,  $m = 25$  kg,  $k = 100$  N/m,  $L = 3$  m and  $b = 2.5$  m.

**Homework H5.H**

**Given:** A thin homogeneous bar of length  $L$  and mass  $m$  is pinned to ground at point  $O$ . A homogeneous disk with a mass of  $M$  and radius  $R$  is PINNED to end  $A$  of the bar. The disk rolls without slipping on the inside of a circular surface. The system is released from rest with  $\theta = 90^\circ$ .

**Find:** Find the angular velocity of the bar when  $\theta = 0^\circ$ .

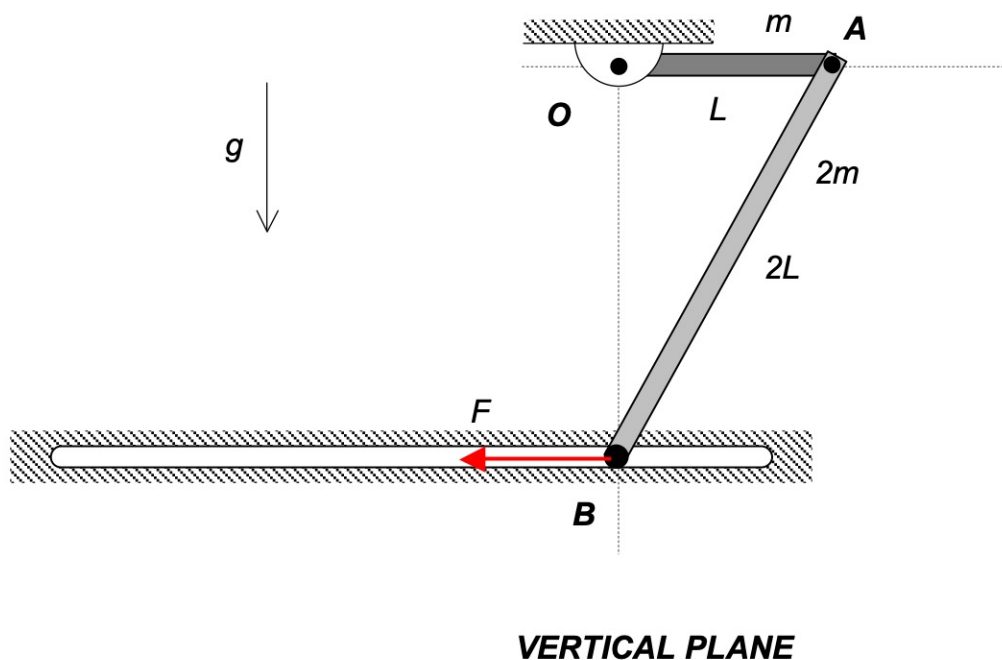


Use the following parameters in your analysis:  $L = 2$  m,  $R = 0.6$  m,  $m = 30$  kg and  $M = 100$  kg.

**Homework H5.1**

**Given:** A thin homogeneous bar OA having a length of  $L$  and mass  $m$  is pinned to ground at O. A second thin homogeneous bar AB (having a length of  $2L$  and mass  $2m$ ) is pinned to bar OA at A, and end B of the bar is constrained to move within a smooth, horizontal track. A constant force  $F$  acts horizontally at end B. The bar is released from rest with link OA being horizontal and with pin B being directly below pin O, as shown in the figure.

**Find:** Determine the speed of end B of bar AB when the system has reached a position of link OA being vertical.



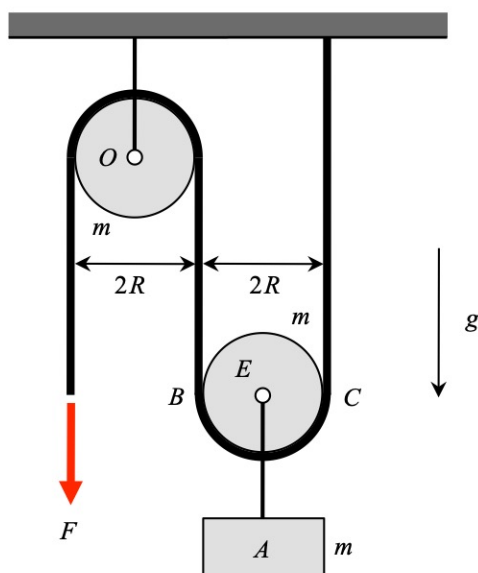
Use the following parameters in your analysis:  $F = 600$  lb,  $L = 2$  ft and  $mg = 200$  lb.

## Homework H5.J

**Given:** A homogeneous disk of mass  $m$  and outer radius  $R$  is supported by the cable-pulley system shown. The pulley (having a mass of  $m$  and with an outer radius of  $R$ ) is supported by a smooth shaft at its center  $O$ . Block A (with a mass of  $m$ ) is supported at the center  $E$  of the disk. A constant force  $F$  is applied to the free end of the cable. The system is released from rest. Assume the pulley and disk do not slip on the cable.

**Find:**

- Determine the direction of motion of block A on release; and
- Determine the speed of block A after A has moved through a distance of  $s_A$ .



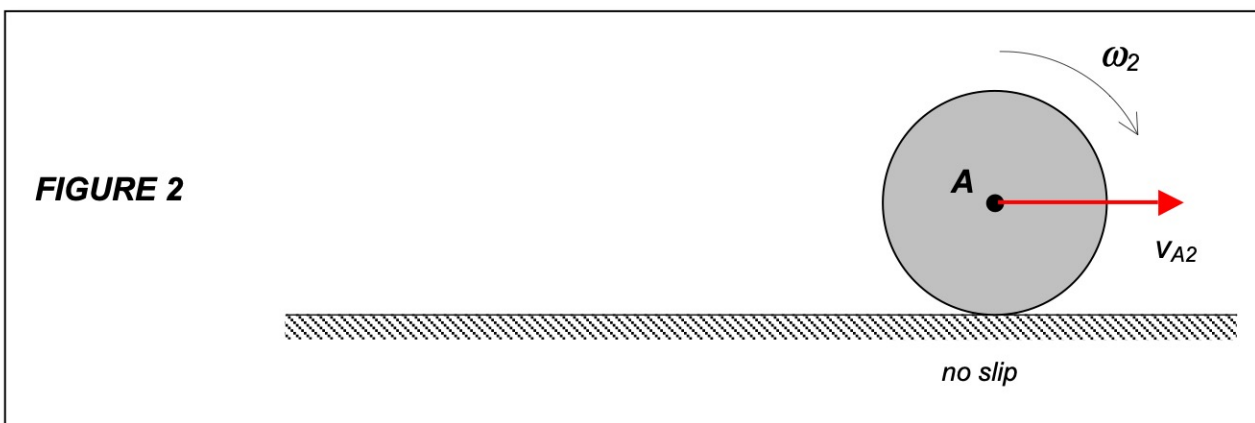
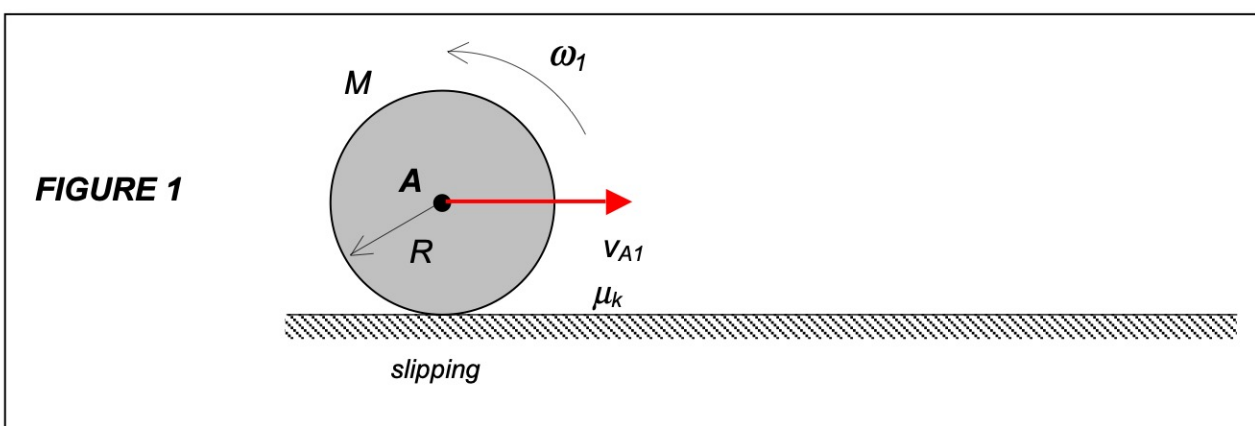
Use the following parameters in your analysis:  $m = 15$  kg,  $R = 0.25$  m,  $s_A = 0.6$  m and  $F = 300$  N.

**Homework H5.K**

**Given:** A homogeneous disk (with mass  $M$  and outer radius of  $R$ ) is placed on a rough surface. When placed on this surface, the center of the disk  $A$  is moving to the right with a speed of  $v_{A1}$  and has a counterclockwise rotation rate of  $\omega_1$ , as shown in Figure 1 below. In Figure 2 below is shown the instant at which the disk ceases to slip as it continues to move on the horizontal surface.

**Find:** For this problem:

- Determine the speed of  $A$ ,  $v_{A2}$ , at the instant in Figure 2 when the disk ceases to slip on the horizontal surface; and
- Determine the elapsed time during the motion as the disk moves from the position in Figure 1 to the position in Figure 2.

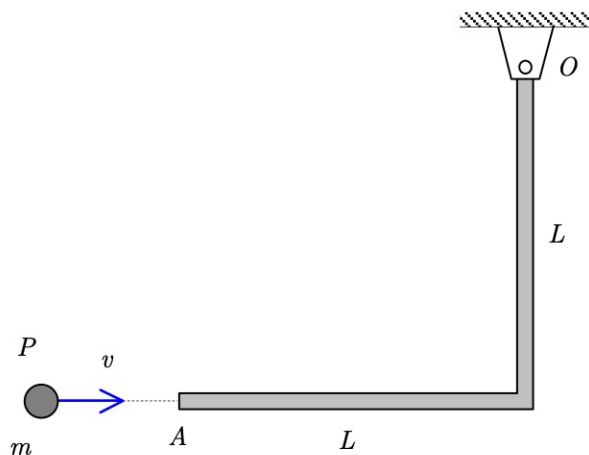


Use the following parameters in your analysis:  $M = 50$  kg,  $R = 0.5$  m,  $\mu_k = 0.3$ ,  $v_{A1} = 5$  m/s and  $\omega_1 = 8$  rad/s.

**Homework H5.L**

**Given:** Particle P (of mass  $m$ ) strikes end A of a stationary homogeneous L-shaped bar (of mass  $M$ ) with a speed of  $v$ . The coefficient of restitution for the impact of P with end A of the bar is known to be  $e$ .

**Find:** Determine the angular speed of bar OA immediately after the impact.

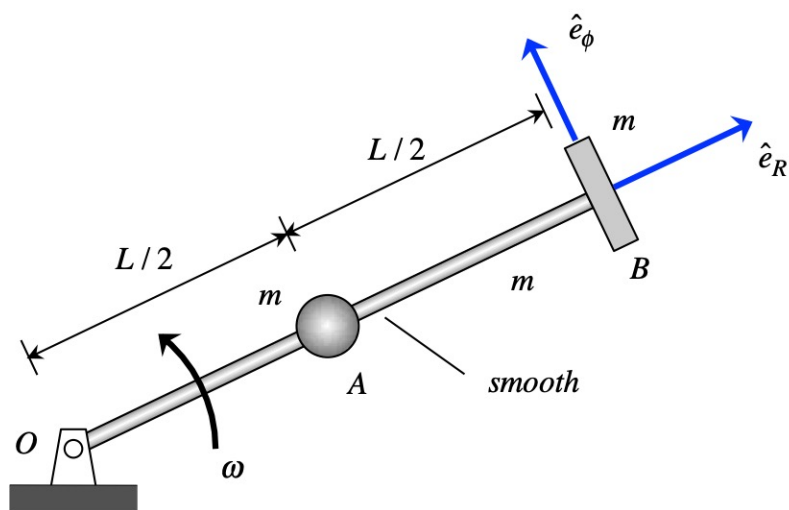
***HORIZONTAL PLANE***

Use the following parameters in your analysis:  $M = 30$  kg,  $m = 15$  kg,  $L = 2$  m,  $v = 40$  m/s and  $e = 0$ .

**Homework H5.M**

**Given:** Particle A, of mass  $m$ , is able to slide on a smooth homogeneous bar of length  $L$  and mass  $m$ . The bar is pinned to ground at end O and particle B (of mass  $m$ ) is rigidly attached to the other end. The bar is given an initial rotation rate of  $\omega_1$  when the A is at the midpoint of the bar, after which A slides outward on the bar. Eventually particle A impacts particle B, an impact having a coefficient of restitution of  $e$ .

**Find:** Determine the velocity of A immediately after impact. Express your answer in terms of its  $R$ - $\phi$  components.



HORIZONTAL plane

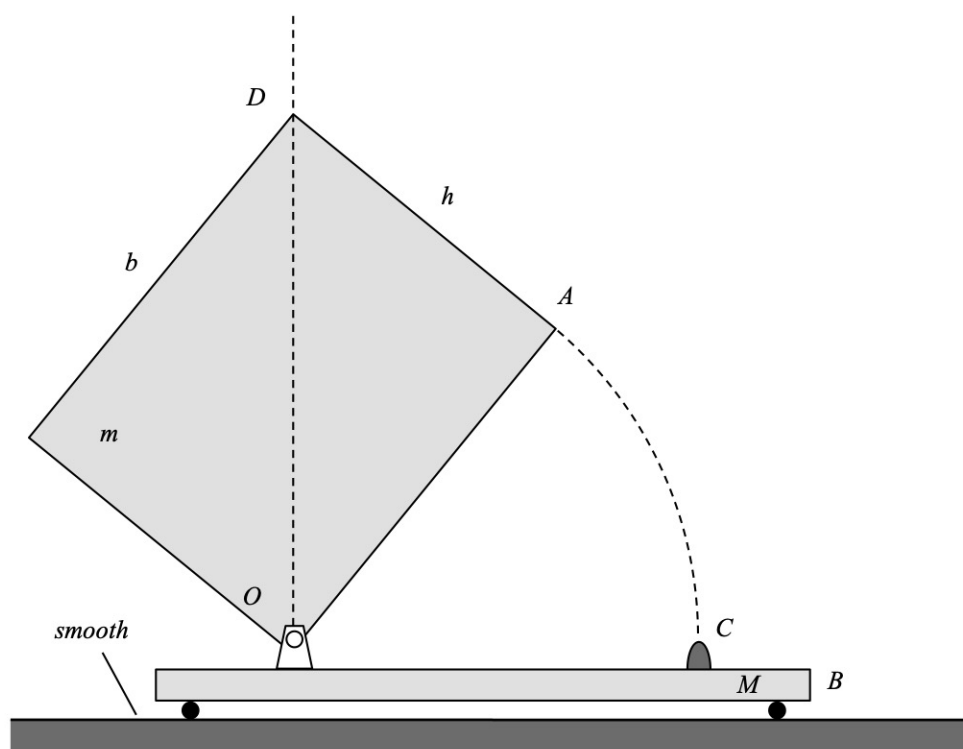
Use the following parameters in your analysis:  $\omega_1 = 6 \text{ rad/s}$ ,  $e = 0.4$ ,  $m = 10 \text{ kg}$  and  $L = 0.5 \text{ m}$ .

## Homework H5.N

**Given:** A homogeneous rectangular plate of mass  $m$  is pinned to cart B at corner O, where cart B is constrained to move along a smooth horizontal surface. The system is released from rest with corner D displaced slightly to the right of a vertical line passing through the pin at O. As a result, the plate eventually impacts bumper C on the cart, with the coefficient of restitution between the plate and the bumper being  $e$ .

**Find:** For this problem:

- Determine the velocity of the center of mass of the plate immediately before the plate contacts the bumper C. Write your answer as a vector.
- Determine the velocity of the center of mass of the plate immediately after the plate contacts the bumper C. Write your answer as a vector.



Use the following parameters in your analysis:  $m = 20$  kg,  $M = 25$  kg,  $b = 2$  m,  $h = 1$  m and  $e = 0$ .

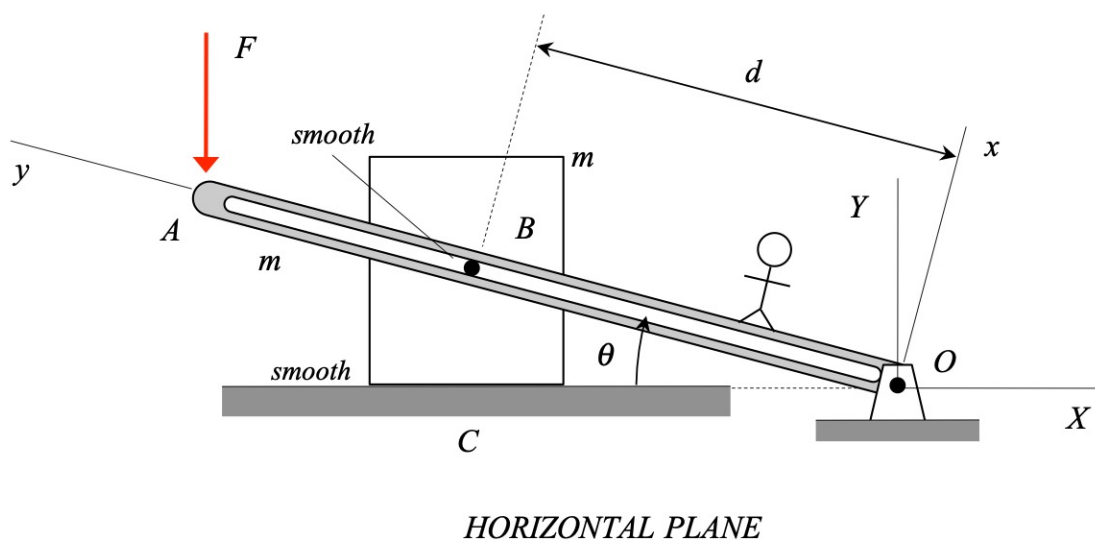


**Homework H5.O**

**Given:** A block of mass  $m$  is able to slide on a smooth, flat surface. Slotted arm OA (of mass  $m$ , length  $L$  and radius of gyration about O of  $k_O$ ) is pinned to ground at O, with a small pin B on the block being constrained to move within the smooth slot of OA. A force  $F$  acts at end A of arm OA in a direction that is perpendicular to the surface on which the disk rolls. The system is released from rest at the instant shown. Note that the system moves in a horizontal plane.

**Find:** Determine the acceleration of the block.

**HINT:** For the kinematics portion of your solution it is suggested that you use the moving reference frame kinematics equations with an observer attached to arm OA.

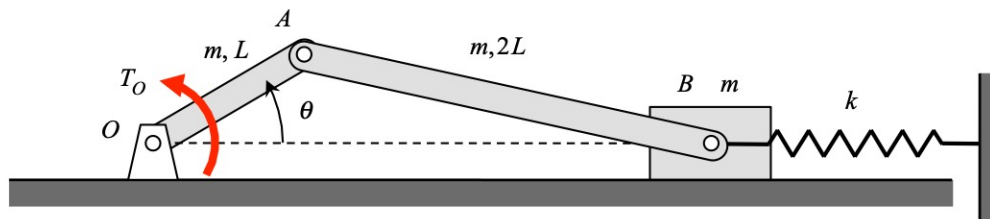


Use the following parameters in your analysis:  $m = 20$  kg,  $d = 1.5$  m,  $k_O = 0.9$  m,  $L = 2$  m,  $\theta = 30^\circ$  and  $F = 100$  N.

**Homework H5.P**

**Given:** A mechanism is made up of links OA and AB, along with the slider B. A spring of stiffness  $k$  is attached between the slider B and ground, with the spring being unstretched when  $\theta = 90^\circ$ . A constant torque  $T_O$  is applied to link OA at the shaft passing through OA at point O. The mechanism is released from rest when  $\theta = 0^\circ$ . The mechanism moves in a horizontal plane.

**Find:** Determine the speed of slider B when  $\theta = 90^\circ$ .



Use the following parameters in your analysis:  $m = 10$  kg,  $L = 0.2$  m and  $k = 300$  N.

## Chapter 6

# Vibrations Homework

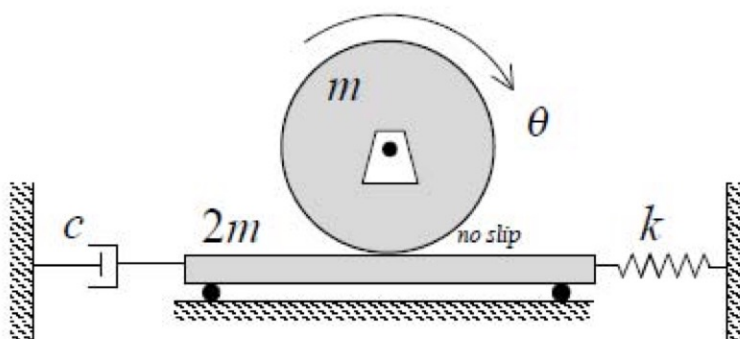


**Homework H6.A**

**Given:** A homogeneous drum (of mass  $m$  and radius  $R$ ) is pinned to ground at its center. The drum rolls without slipping on a block of mass  $2m$ , with the block, in turn, being able to slide on a smooth horizontal surface. A dashpot and a spring are attached between the block and ground, as shown in the figure. Let  $\theta$  represent the rotation of the drum, and  $\theta = 0$  represent the state where the spring is unstretched.

**Find:** For this problem:

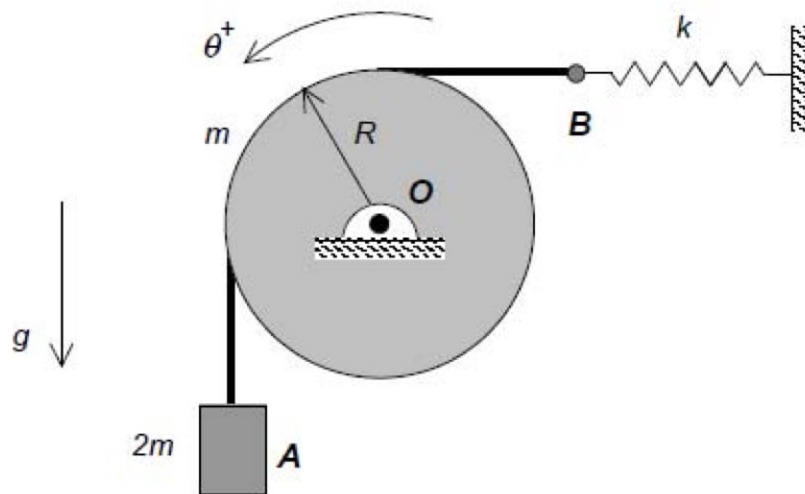
- Draw individual free body diagrams of the drum and block; and
- Derive the single differential equation of motion (EOM) for the system in terms of the coordinate  $\theta$ , its time derivatives, and, at most, the following parameters:  $m$ ,  $c$ ,  $R$ , and  $k$ .



**Homework H6.B**

**Given:** A homogeneous circular disk (having a mass of  $m$  and outer radius of  $R$ ) is pinned to ground with a smooth pin at  $O$ . An inextensible cable is wrapped over the outer surface of the disk. One end of the cable supports block A (of mass  $2m$ ), and the other end of the cable B is connected to ground with a spring of stiffness  $k$ , as shown below. Assume that the cable does not slip on the disk and that the cable remains taut for all motion. Let  $\theta$  describe the rotation of the disk, and let the spring be unstretched when  $\theta = 0$ .

**Find:** For this problem, derive the differential equation of motion for the system in terms of the  $\theta$  coordinate.

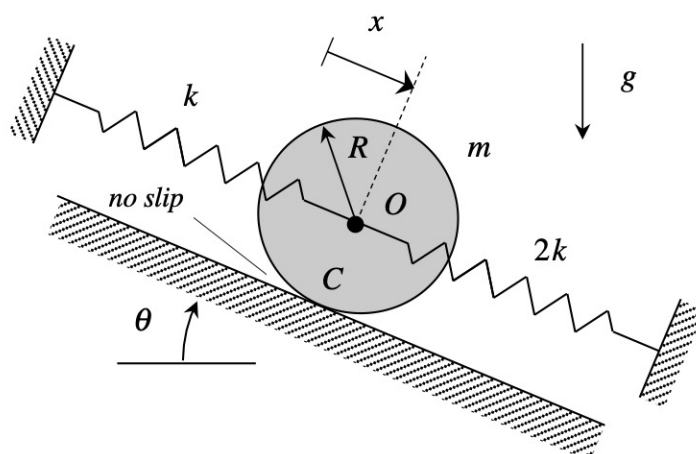


**Homework H6.C**

**Given:** A homogeneous disk of mass  $m$  and outer radius  $R$  is able to roll without slipping on a rough, inclined surface. The center of the disk  $O$  is attached to ground with two springs of stiffnesses  $k$  and  $2k$ , as shown in the figure. Let  $x$  represent the motion of  $O$  along the incline as the disk rolls, where  $x = 0$  when the springs are unstretched.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$ ;
- From the EOM, determine the static displacement of  $O$ ,  $x_{st}$ ;
- Rewrite the EOM of the system in terms of the variable  $z = x - x_{st}$ , where  $z$  represents the position of  $O$  relative to its static equilibrium position; and,
- Determine the natural frequency of the system in terms of, at most, the given parameters of the problem:  $m$ ,  $k$  and  $R$ .

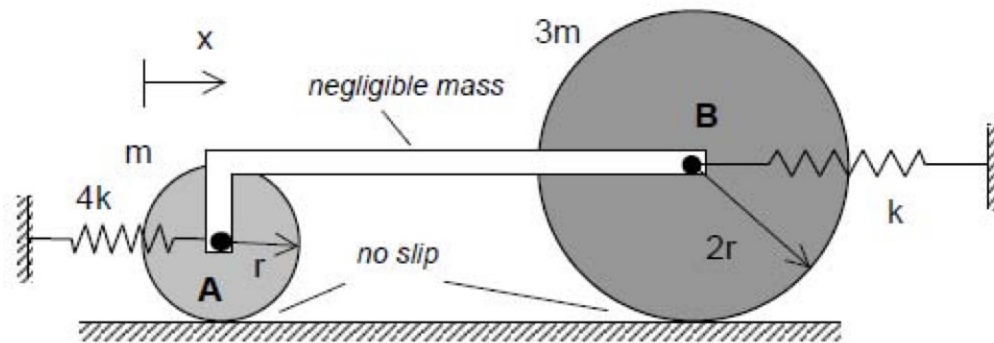


## Homework H6.D

**Given:** Two homogeneous wheels, having masses of  $m$  and  $3m$  and outer radii of  $r$  and  $2r$ , respectively, are connected by a rigid, L-shaped bar, where the mass of the bar is negligible compared to the mass of the wheels. The two wheels roll without slipping on a rough, horizontal surface. Two springs, having stiffness of  $4k$  and  $k$ , connect points A and B, respectively, to ground, where A and B are the centers of the two wheels. The coordinate  $x$  gives the position of Point A measured from the position at which the two springs are unstretched, with  $x$  being measured positive to the right (as shown below).

**Find:** For this problem:

- Derive the single differential equation of motion (EOM) for the system in terms of the coordinate  $x$ ; and
- Determine the natural frequency of free oscillation for the system.



Use the following parameters in your analysis:  $m = 20$  kg,  $k = 500$  N/m, and  $r = 0.5$  m.

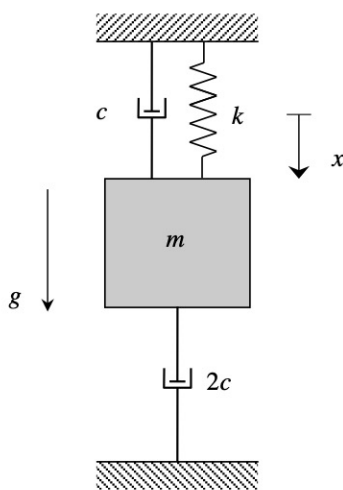


**Homework H6.E**

**Given:** A block of mass  $m$  is attached to a grounded spring (of stiffness  $k$ ) and two dashpots (having damping coefficients  $c$  and  $2c$ ), as shown in the figure. Let  $x$  represent the motion of the block, with  $x = 0$  when the spring is unstretched.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$ ;
- Determine the static equilibrium position of the block,  $x_{st}$ ;
- Rewrite the EOM of the system in terms of the variable  $z = x - x_{st}$ , where  $z$  represents the position of the block relative to its static equilibrium position; and,
- Determine undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped natural frequency  $\omega_d$  for the system in terms of, at most, the parameters of the problem:  $m$ ,  $c$  and  $k$ .

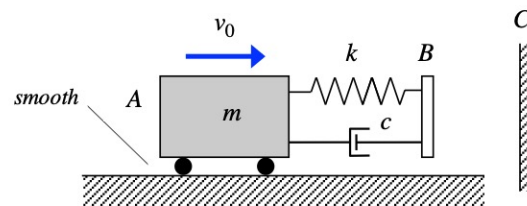


**Homework H6.F**

**Given:** A block of mass  $m$  is attached to a spring of stiffness  $k$  and a dashpot with a damping coefficient  $c$ , with the opposite ends of the spring and dashpot joined to connector B, where the mass of B can be considered to be negligible. A is initially traveling to the right with a speed of  $v_0$  when it strikes a stationary wall at C. B immediately sticks to C after impact. Let  $x$  represent the motion of A after B has stuck to the wall, with  $x$  being measured positively to the right.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$  for motion occurring after B sticks to C;
- Determine the undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped natural frequency  $\omega_d$  for the system; and,
- Determine the maximum compression of the spring after B impacts and sticks to C.

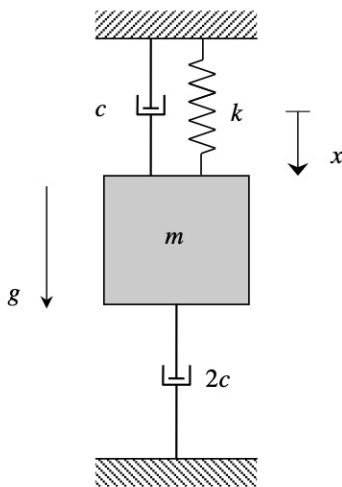


**Homework H6.G**

**Given:** A block of mass  $m$  is attached to a grounded spring (of stiffness  $k$ ) and two dashpots (having damping coefficients  $c$  and  $2c$ ), as shown in the figure. Let  $x$  represent the motion of the block, with  $x = 0$  when the spring is unstretched.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$ ;
- Determine the static equilibrium position of the block,  $x_{st}$ ;
- Rewrite the EOM of the system in terms of the variable  $z = x - x_{st}$ , where  $z$  represents the position of the block relative to its static equilibrium position; and,
- Determine undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped natural frequency  $\omega_d$  for the system in terms of, at most, the parameters of the problem:  $m$ ,  $c$  and  $k$ .

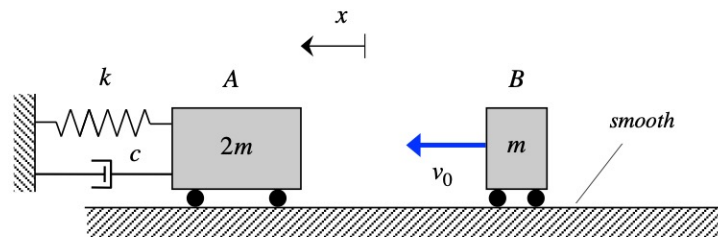


**Homework H6.H**

**Given:** Particle A (having a mass of  $2m$ ) is attached to a grounded spring of stiffness  $k$  and a dashpot with a damping coefficient  $c$ . Let  $x$  represent the motion of A, with  $x = 0$  when the spring is unstretched. At an instant when A is at rest and with the spring being unstretched/uncompressed (that is, when  $x = 0$ ), particle B (of mass  $m$ ), traveling with a speed of  $v_0$ , strikes A. On the impact with A, block B immediately sticks to block A.

**Find:** For this problem:

- Determine the speed of A immediately after B sticks to it.
- Derive the dynamical equation of motion (EOM) of A+B in terms of the coordinate  $x$  describing the motion after the two blocks stick together;
- Determine the undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped natural frequency  $\omega_d$  for the system;
- Determine the response  $x(t)$  of the system after A and B stick together.



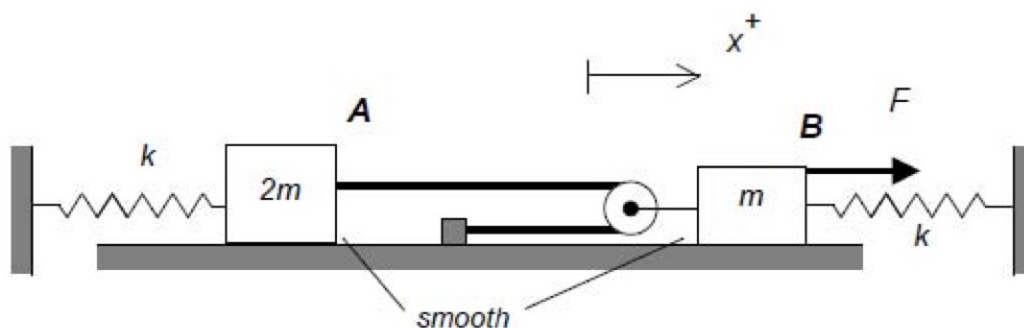
Use the following parameters in your analysis:  $m = 10$  kg,  $k = 3000$  N/m and  $c = 360$  kg/s.

**Homework H6.1**

**Given:** Blocks A and B (having masses of  $m$  and  $2m$ , respectively) are connected by a cable-pulley system as shown below. Two springs, each of stiffness  $k$ , are attached between blocks A and B and ground, as shown below. A horizontal force  $F$  is applied to B. The mass of the pulley is negligible, and the cable remains taut during all motion. Let  $x$  describe the position of B, and let  $x = 0$  correspond to the state at which the springs are unstretched.

**Find:** For this problem:

- Draw a free body diagram for each block; and
- Derive the differential equation of motion for the system in terms of the coordinate  $x$ .

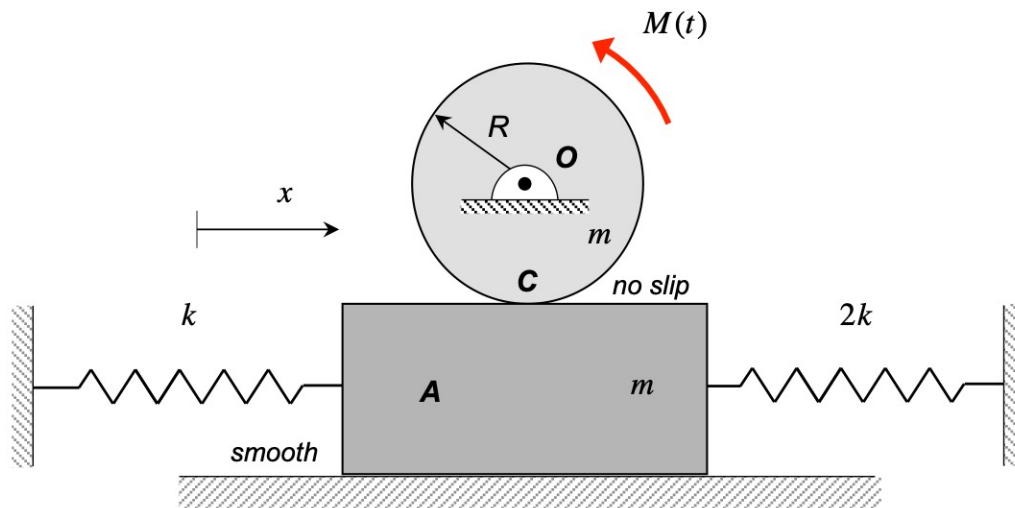


**Homework H6.J**

**Given:** A homogeneous disk of mass  $m$  and with an outer radius of  $R$  is pinned to ground at its center  $O$ . Block A (of mass  $m$ ) is connected to grounded springs of stiffnesses  $k$  and  $2k$ , as shown, as it slides on a smooth horizontal surface. The disk and block A are in contact as the system moves, with the disk not slipping on the surface of block A. A torque  $M(t)$  acts on the disk. Let  $x$  measure the position of block A, with  $x$  measured positively to the right and with  $x = 0$  corresponding to the springs being unstretched.

**Find:** For this problem:

- Derive the differential equation of motion (EOM) for the system in terms of the coordinate  $x$ ;
- Determine the natural frequency of the system; and,
- Determine the particular solution of the EOM corresponding to an input torque of  $M(t) = M_0 \sin \Omega t$ .

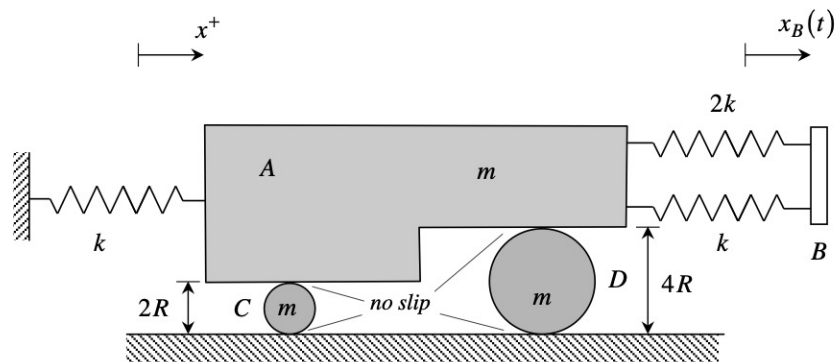


**Homework H6.K**

**Given:** Cart A (of mass  $m$ ) is supported by two rollers C and D (each of mass  $m$ , and having outer radii of  $R$  and  $2R$ , respectively). The rollers are known to roll without slipping with respect to both the ground and cart A. A spring of stiffness  $k$  is attached between the left side of the cart and ground. Two additional springs, of stiffnesses  $k$  and  $2k$ , are attached between the right side of cart A and a moveable base B. The base B is given a prescribed motion of  $x_B(t) = b \cos \Omega t$ . Let  $x$  represent the motion of cart A, where  $x = x_B = 0$  when all three of the springs are unstretched.

**Find:** For this problem:

- Derive the differential equation of motion (EOM) for the structure in terms of the coordinate  $x$ ;
- Identify the natural frequency  $\omega_n$  of oscillations for the system;
- Derive the particular solution for the EOM,  $x_P(t)$ ; and,
- What is the amplitude of the motion for the cart when  $\Omega = 2\omega_n$ ? Is the cart moving in-phase or out-of-phase with the base B at this frequency of excitation?

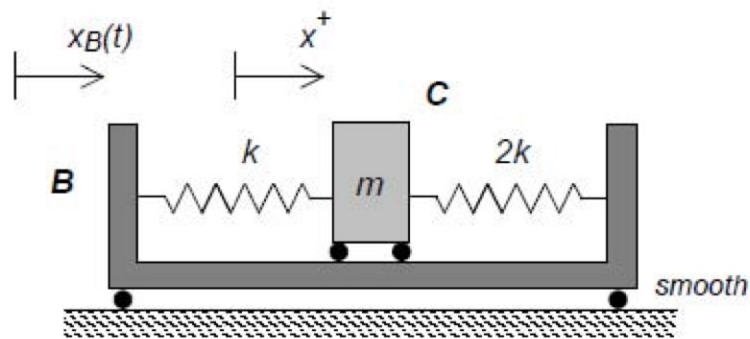


**Homework H6.L**

**Given:** Block C, having a mass of  $m$ , is attached to a moveable base B by two springs, as shown in the figure below. Base B is given a prescribed horizontal motion of  $x_B(t) = b \sin \omega t$ . Let  $x$  represent the motion of C (measured positively to the right), and let the springs be unstretched when  $x = x_B = 0$  m.

**Find:** For this problem:

- Draw a free body diagram of block C;
- Derive the differential equation of motion for the system in terms of the coordinate  $x$ ;
- Derive the particular solution  $x_p(t) = A \sin \omega t$  for the previously-obtained equation of motion;
- Make a plot of the amplitude of  $x_p$  (i.e.  $|A|$ ) versus the excitation frequency  $\omega$ ; and
- Determine the two positive values of the excitation frequency  $\omega$  for which the amplitude of  $x_p$  is exactly twice the amplitude of the base motion (that is, when  $|A| = 2b$ ).



Use the following parameters in your analysis:  $m = 36$  kg,  $k = 5808$  N/m, and  $b = 0.02$  m.

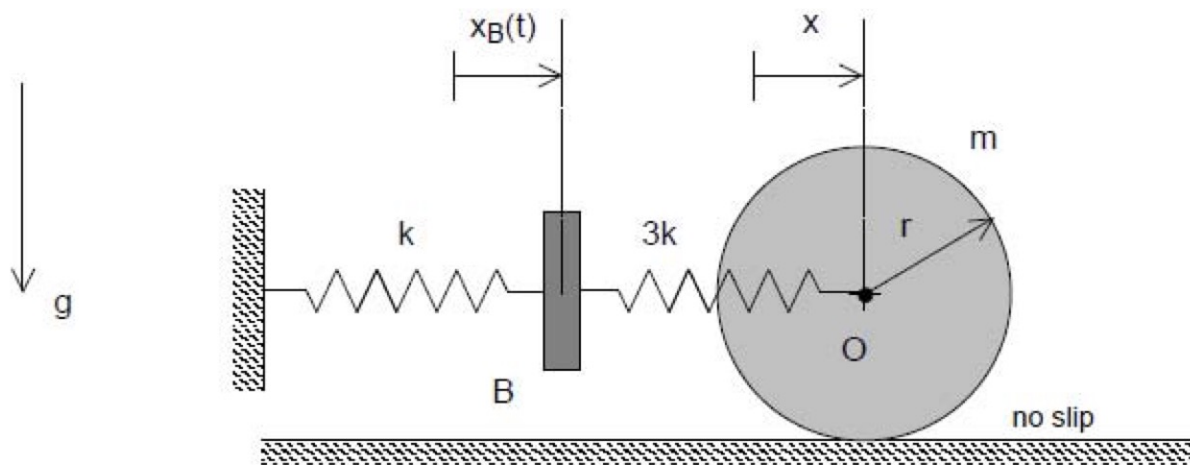


**Homework H6.M**

**Given:** A homogeneous disk (of mass  $m$  and outer radius  $R$ ) rolls without slipping on a rough, horizontal surface. A spring (of stiffness  $3k$ ) is attached between the center  $O$  of the disk and a moveable base  $B$ . A second spring (of stiffness  $k$ ) is attached between point  $B$  and ground. Base  $B$  is given a prescribed motion of  $x_B(t) = b \sin \Omega t$ . The coordinates  $x$  and  $x_B$  are both zero when the springs are unstretched.

**Find:** For this problem:

- Derive the differential equation of motion for the disk in terms of the coordinate  $x$ ;
- Determine the numerical value for the natural frequency of this system;
- Determine the numerical value of  $X$ , if the particular solution of the system is written as  $x_p(t) = X \sin \Omega t$ ; and
- Determine if the disk is moving in-phase or out-of-phase with the base  $B$ .



Use the following parameters in your analysis:  $m = 80$  kg,  $k = 640$  N/m,  $r = 0.25$  m,  $b = 0.16$  m, and  $\Omega = 10$  rad/s.

**Homework H6.N**

**Given:** A homogeneous disk of mass  $m$  and with an outer radius of  $R$  rolls without slipping on a rough horizontal surface. Bar A (of mass  $m$ ) is attached to a grounded spring of stiffness  $2k$  on its right end. The left end of A is attached to a spring (of stiffness  $k$ ) connected to a moveable base B. Bar A is supported by the top of the disk, with A not slipping on the disk as the system moves. The base B is given a prescribed motion of  $x_B(t) = b \sin \Omega t$ . Let  $x$  measure the position of A from its position when the springs are unstretched; i.e.,  $x = 0$  when the springs are unstretched.

**Find:** For this problem:

- Derive the differential equation of motion (EOM) for the system in terms of the coordinate  $x$ ;
- Determine the natural frequency  $\omega_n$  of the system;
- Determine the particular solution of the EOM; and,
- If  $\Omega = 0.5 \omega_n$ , does A move in phase or out of phase with B?

