Chapter 1

Particle Kinematics Homework

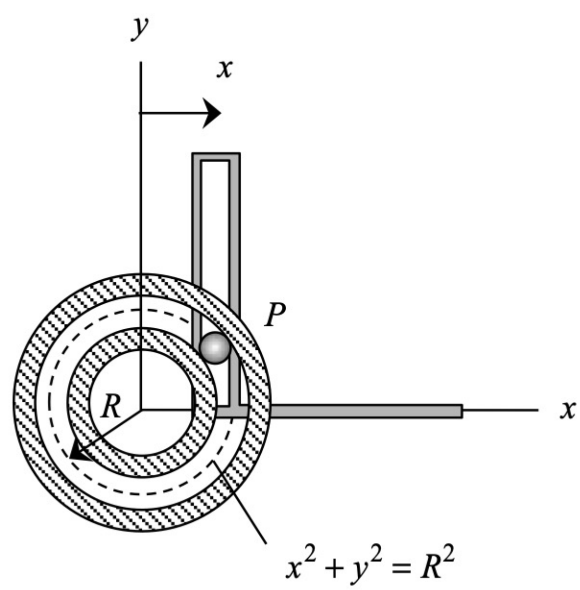
Homework H1.A

Given: Particle P is constrained to move along a fixed circular path of radius R, with the Cartesian components of locations on the circular path related by $x^2 + y^2 = R^2$, where x, y and R given in mm. In addition, P is also constrained to move within a vertical slot whose horizontal position is governed by $x = b sin\omega t$, where t is time in seconds.

Find: For the time $t = 0$:

(a) show the Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ in a sketch.

(b) determine the velocity $\vec{v}_P$ and acceleration $\vec{a}_P$ of P. Write your answers as vectors in terms of their Cartesian components. Include these vectors in your sketch.



Use the following parameters in your work: $R = 50$ mm, $b = 40$ mm and $\omega = 5\pi$ rad/s.

Homework H1.B

Given: Particle P moves along a path described in terms of Cartesian coordinates of $x = he^{bt}$ and $y = ce^{-bt}$ with x and y given in feet, and t is given in seconds.

Find: For this problem:

(a) show that the path taken by P is a hyperbola in the xy-plane.

(b) show the Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ in a sketch.

(c) determine the velocity $\vec{v}_P$ and acceleration $\vec{a}_P$ of P at $t = 2$ s. Write your answers as vectors in terms of their Cartesian components. Include these vectors in your sketch.

Use the following parameters in your work: $b = 0.5/\text{s}$, $h = 1$ ft and $c = 16$ ft.

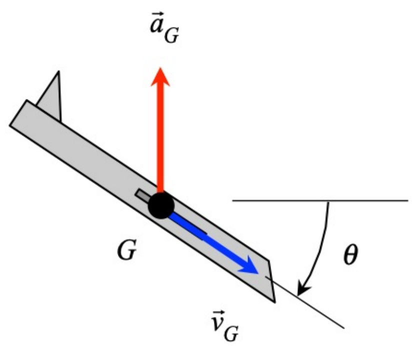
Homework H1.C

Given: At one instant in time, an aircraft is traveling along a path in a direction defined by θ below the horizontal with the center of mass G of the aircraft having a speed of $|\vec{v}_G|$. G is also known to have an acceleration that is pointing vertically upward with a magnitude of $|\vec{a}_G|$.

Find: For this given instant in time:

(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with $\vec{v}_G$ and $\vec{a}_G$, in a sketch.

(b) determine the rate of change of speed of G and the radius of curvature of G.



Use the following parameters in your work: $\theta = 36.87°$, $|\vec{v}_G| = 900$ km/hr and $|\vec{a}_G| = 30$ m/s^2.

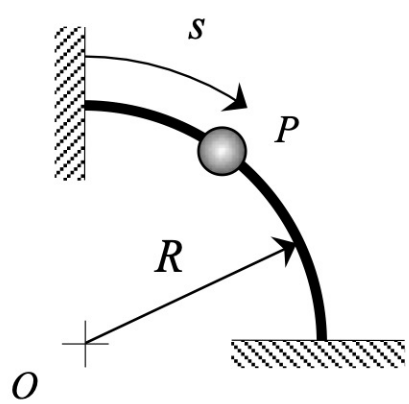
Homework H1.D

Given: Particle P moves along a circular guide having a radius of R. The distance s traveled by P is given as a function of time as: $s = bte^{-ct}$, where s is in millimeters and t is in seconds.

Find: For instant in time of t = 0.5 s:

(a) show the position of P and the path unit vectors $\hat{e}_t$ and $\hat{e}_n$ in a sketch.

(b) determine the velocity and acceleration of P. Express your answers as vectors in terms of the path unit vectors. Show these two vectors in your sketch.



Use the following parameters in your work: $R = 200$ mm, $b = 400$ mm and $c = 0.5$/s.

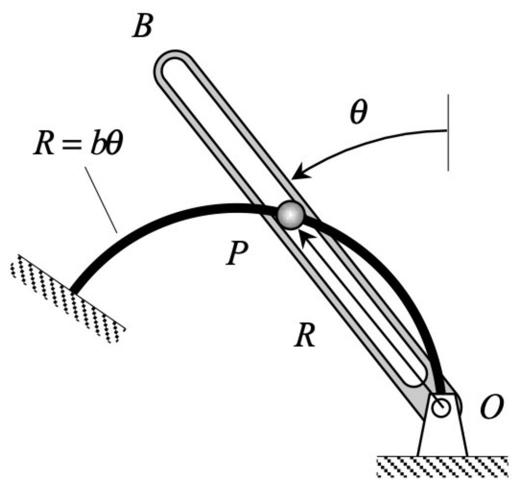
Homework H1.E

Given: Particle P is constrained to move along a fixed, curved guide whose shape is governed by the following equation in polar variables: $R = b\theta$, where R is in inches and θ is in radians. P is also constrained to move within a straight slot cut into arm OB, with OB pinned to ground at O, and whose orientation is at the angle of θ measured counterclockwise from the vertical. Arm OB is known to be rotating in the counterclockwise sense with a constant rate of $\dot{\theta} = \omega$.

Find: For position of $\theta = 30°$:

(a) show the position of P and the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$ in a sketch.

(b) determine the velocity and acceleration of P. Express your answers as vectors in terms of the polar unit vectors. Show these two vectors in your sketch.



Use the following parameters in your work: $b = 4$ in and $\omega = 16$ rad/s.

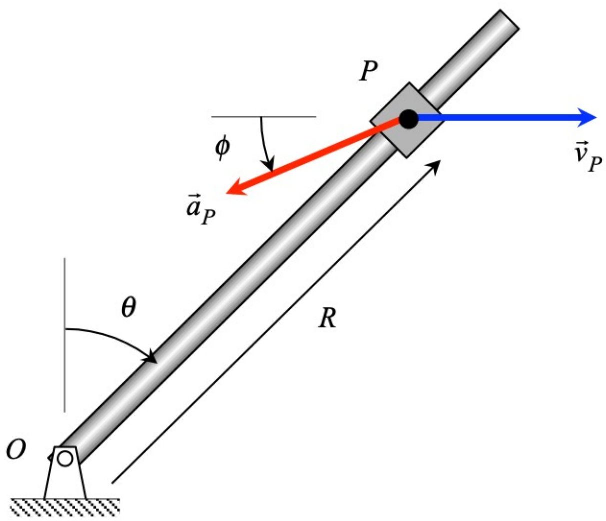
Homework H1.F

Given: Particle P is able to slide along an arm that is rotating about end O. At the instant shown, the arm is at an angle of θ measured clockwise from the vertical, the velocity of P is known to be horizontal, and the acceleration of P is in a direction defined by the angle ϕ from the horizontal, all as shown in the figure.

Find: For position of $\theta = 30°$:

(a) show the position of P and the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$, along with $\vec{v}_P$ and $\vec{a}_P$, in a sketch.

(b) determine numerical values for $\dot{R}$, $\ddot{R}$, $\dot{\theta}$ and $\ddot{\theta}$.



Use the following parameters in your work: $R = 2$m, $|\vec{v}_P| = 8$ m/s and $|\vec{a}_P| = 20$ m/s^2.

Problem H1.G

Given: Particle P moves along a hyperbolic path described in Cartesian coordinates as: $xy = b$, where x and y are given in feet. It is known that P moves in such a way that $\dot{x} = c =$ constant.

Find: For the position of $x = 2$ ft:

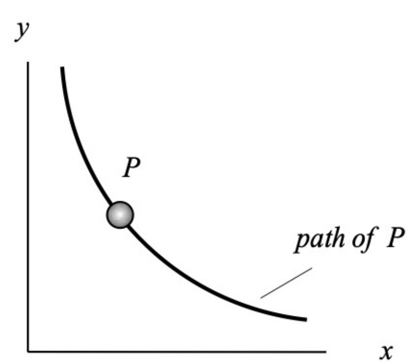
(a) determine the velocity and acceleration of P. Write your answers as vectors in terms of their Cartesian components.

(b) determine the Cartesian components of the unit path vector $\hat{e}_t$.

(c) show the position of P and the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with $\vec{v}_P$ and $\vec{a}_P$, in a sketch.

(d) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and the radius of curvature ρ_P for the path of P.

(e) is the speed of P increasing or decreasing? Explain.



Use the following parameters in your analysis: $b = 6$ ft^2 and $c = 30$ ft/s.

Problem 1H1.H

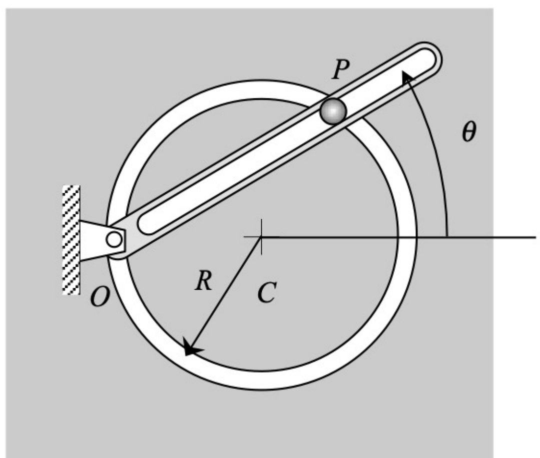
Given: Particle P is constrained to move within a circular slot with a radius of R and center at point C. P is also constrained to move within the straight slot cut in an arm, with the arm rotating about end O with a constant rate of $\dot{\theta} = \omega$. O is located within the circular slot immediately to the left of C.

Find: For the position of $\theta = 45°$:

(a) show the path unit vectors $\hat{e}_t$ and $\hat{e}_n$, along with polar unit vectors $\hat{e}_r$ and $\hat{e}_\theta$, in a sketch. Note that the polar variable r is measured from point O to P, thus defining the direction for $\hat{e}_r$.

(b) determine numerical values for the rate of change of speed $\dot{v}_P$ of P and for $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$.

(c) is the speed of P increasing or decreasing? Explain.



Use the following parameters in your analysis: $R = 8$ in and $\omega = 4$ rad/s.

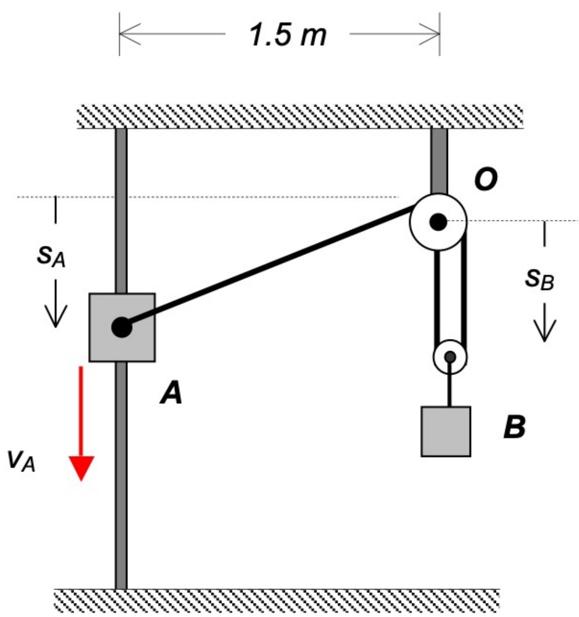
Homework H1.I

Given: Blocks A and B are connected by the pulley-system shown below. Block A moves downward with a constant speed of v_A on a vertical guide. Assume the radii of the pulleys to be small.

Find: For this problem:

(a) Determine the speed of block B when $s_A = 0$ m.

(b) Determine the speed of block B when $s_A = 4$ m.



Use the following parameters in your analysis: $v_A = 25$ m/s.

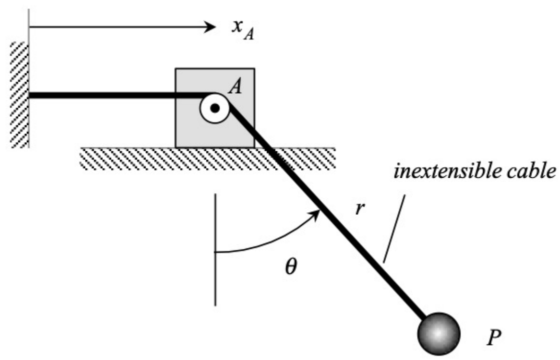
Problem H1.J

Given: Block A moves to the right with a constant speed of $\dot{x}_A$. An inextensible cable is wrapped around a small pulley on block A, with the left end of the cable attached to a fixed wall and the other end of the cable is attached to particle P. At a given instant in time, it is known that the time derivatives of the rotation angle for the cable are given by $\dot{\theta}$ and $\ddot{\theta}$. Assume that the cable remains taut at all time.

Find: For this instant:

(a) Determine the velocity vector of P.

(b) Determine the acceleration vector of P.



Use the following parameters in your analysis: $\theta = 0$, $\dot{x}_A = 10$ ft/s, $r = 2$ ft, $\dot{\theta} = 3$ rad/s and $\ddot{\theta} = 0$.

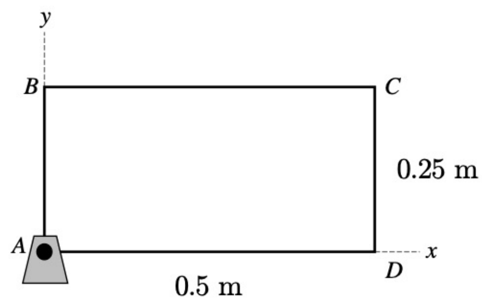
Chapter 2

Planar Rigid Body Kinematics Homework

Homework H2.A

Given: The rectangular plate ABCD freely rotates about a pivot at point A. At the instant shown, the acceleration of point C is given by $\vec{a}_C = a_x\hat{\imath} + a_y\hat{\jmath}$.

Find: Determine the angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$ of the plate.

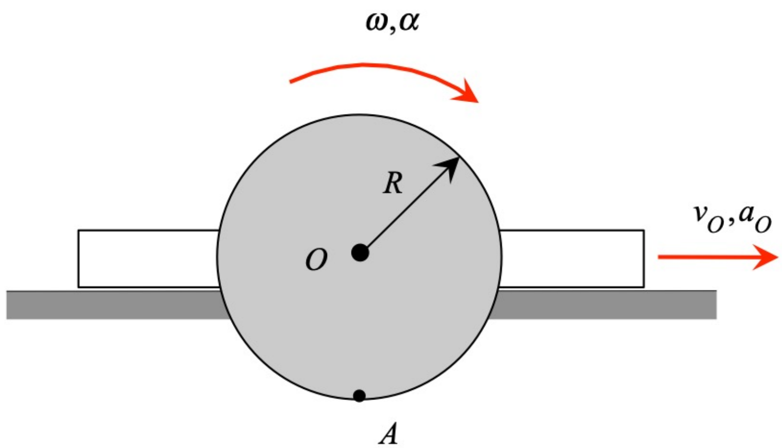


Use the following parameters in your analysis: a_x = 0 and a_y = -20 m/s^2.

Homework H2.B

Given: A circular disk is pinned to a block at its center O, with the block being constrained to move along a horizontal surface. The angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$ of the disk are in the directions shown in the figure. The block is moving the right with a speed of v_O and an acceleration of a_O. At the position shown, point A on the perimeter of the disk is directly below O.

Find: For this position, determine the velocity and acceleration of point A. Express your answers as vectors.

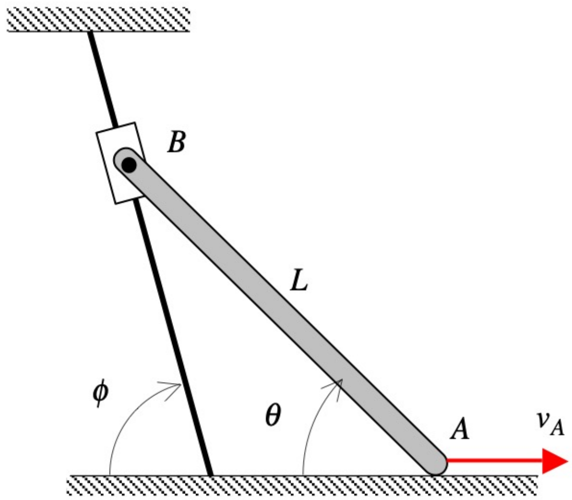


Use the following parameters in your analysis: $R = 0.75$ m, $\omega = 4$ rad/s, $\alpha = 2$ rad/s^2, $v_O = 3$ m/s and $a_O = 4$ m/s^2.

Homework H2.C

Given: Thin bar AB (having a length of L) moves in a way that ends A and B slide along straight, fixed guides as shown in the figure. End A has a constant speed of v_A to the right as the bar moves.

Find: Determine the velocity and acceleration of end B.



Use the following parameters in your analysis: $L = 2$ ft, $v_A = 4$ ft/s, $\theta = 36.87°$ and $\phi = 45°$.

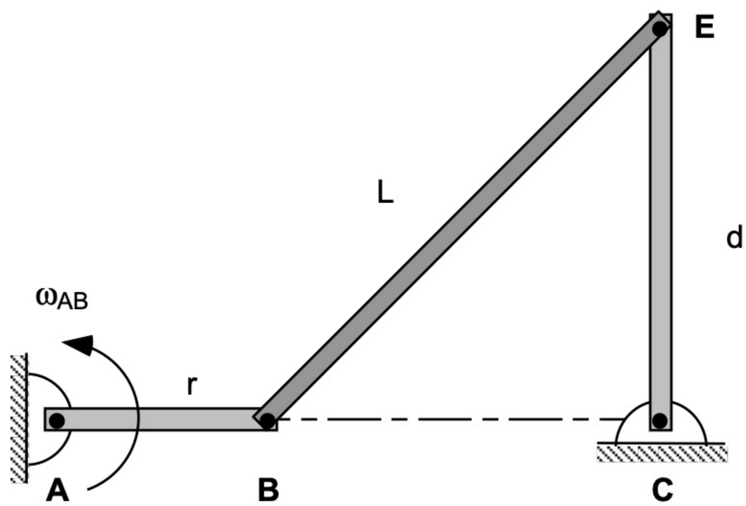
Homework H2.D

Given: The mechanism shown below is made up of links AB, BE and CE. Links AB and CE are pinned to ground at pins A and C, respectively. Link BE is pinned to links AB and CE at pins B and E, respectively. Link AB is rotating CCW at a constant rate of ω_{AB}. In the position shown link AB is horizontal, and link CE is vertical.

Find: For this position:

(a) Determine the angular velocity for links BE and CE.

(b) Determine the angular acceleration for links BE and CE.



Use the following parameters in your analysis: $r = 0.2$ ft, $L = 0.5$ ft, $d = 0.4$ ft and $\omega_{AB} = 4$ rad/s.

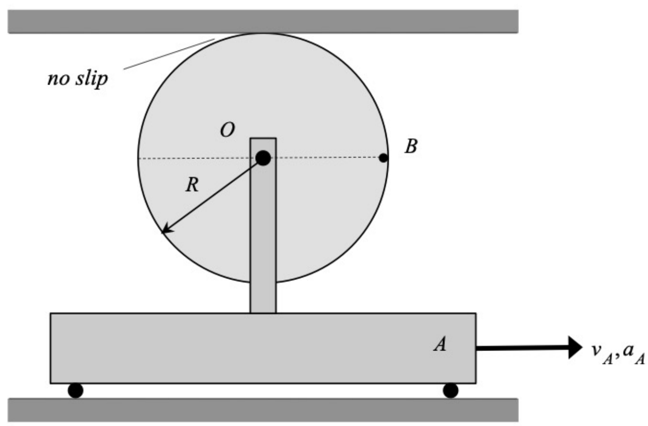
Homework H2.E

Given: Cart A moves to the right with a speed of v_A and an acceleration a_A. A disk of radius R is attached to a shaft on the cart at the center O of the disk. The disk is in contact with a horizontal surface at its top surface; as the cart moves, the disk rolls without slipping on this horizontal surface.

Find: For this problem:

(a) Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors.

(b) Determine the acceleration of point B on the circumference of the disk and with B being immediately to right of O at the instant shown.

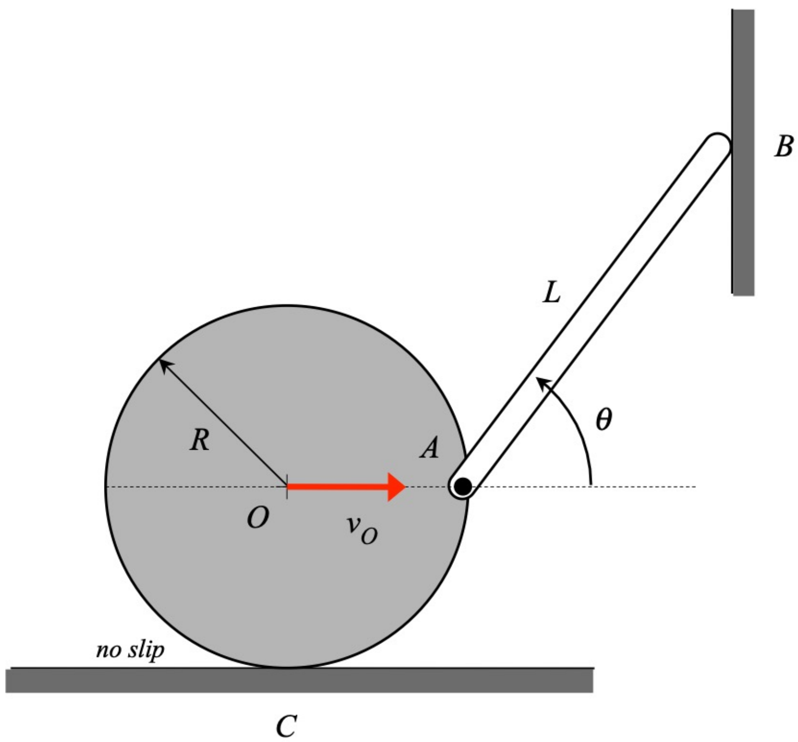


Use the following parameters in your analysis: $R = 2$ ft, $v_A = 4$ ft/s and $a_A = 5$ ft/s^2.

Homework H2.F

Given: A wheel of radius R rolls without slipping on a horizontal surface with its center O traveling with a constant speed of v_O. Bar AB is pinned to the outer perimeter of the wheel at end A, and end B of the bar is constrained to slide along a vertical wall. At the position shown, pin A is directly to the right of O.

Find: For the position show, determine the angular velocity of link AB and the angular acceleration of link AB.



Use the following parameters in your analysis: $\theta = 53.13°$, $L = 1.5$ ft, $R = 0.5$ ft and $v_O = 10$ ft/s.

Homework H2.G

This problem has three parts. In each part, you are asked to use the instant center approach in answering the questions related to the problems. In all cases, the figures are drawn to scale. Please use a straight edge when making your drawings.

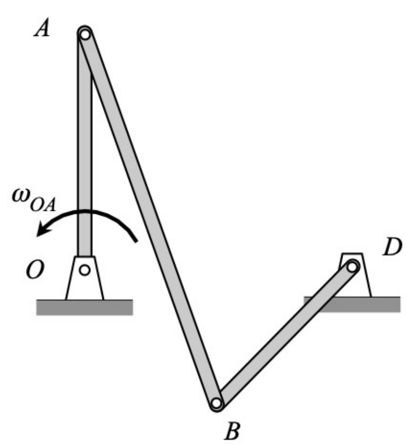
PART A

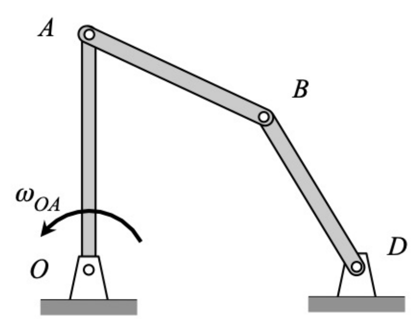
In the mechanisms shown below, link OA is rotating in the counterclockwise sense. For the position shown of EACH mechanism:

(a) Determine the location of the instant center for link AB.

(b) Determine the directions of rotation for links AB and BD. Justify your answers in words.

(c) Which is larger: $|\omega_{OA}|$ or $|\omega_{AB}|$? Justify your answers in words.



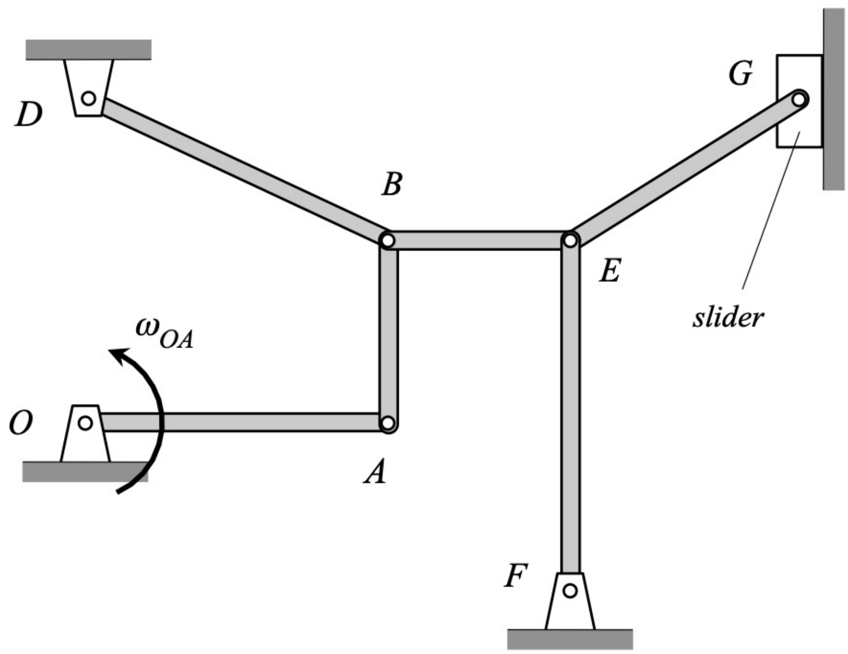


PART B

In the mechanism shown below, link OA is rotating in the counterclockwise sense.

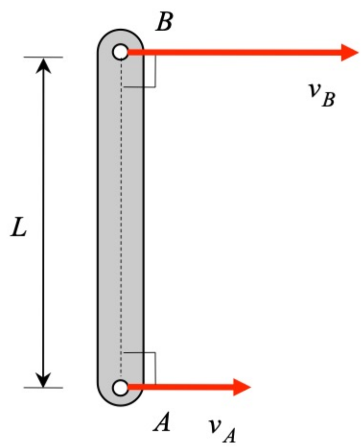
(a) Determine the locations of the instant centers for links AB, BE and EG.

(b) Determine the directions of rotation for links AB, BE and EG. Justify your answers in words.



PART C

Link AB, having a length of $L = 5$ in, is part of a planar mechanism. At the instant shown, the velocities of points A and B are known to be both perpendicular to a line connecting A and B, with $v_B = 3v_A = 30$ in/s. Determine the location of the instant center for link AB.



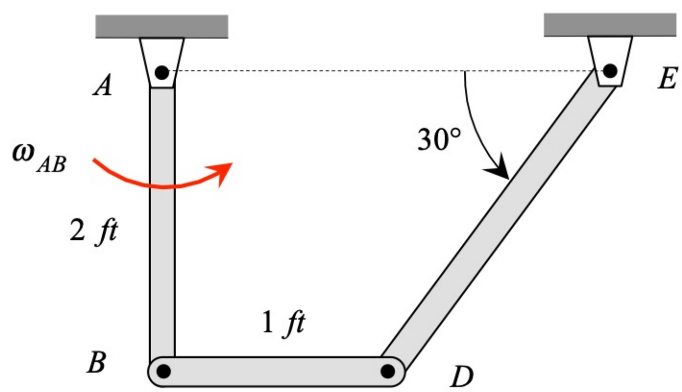
Homework H2.H

Given: The mechanism shown below is made up on links AB, BD and DE. Link AB is known to be rotating counterclockwise with an angular speed of ω_{AB}. At the instant shown, link BD is horizontal.

Find: For this problem:

(a) Locate the instant center for link BD.

(b) Using the instant center approach, determine the angular speeds of links BD and DE (in terms of ω_{AB}). Also, determine the sense of rotation (clockwise or counterclockwise) for links BD and DE.



Homework H2.I

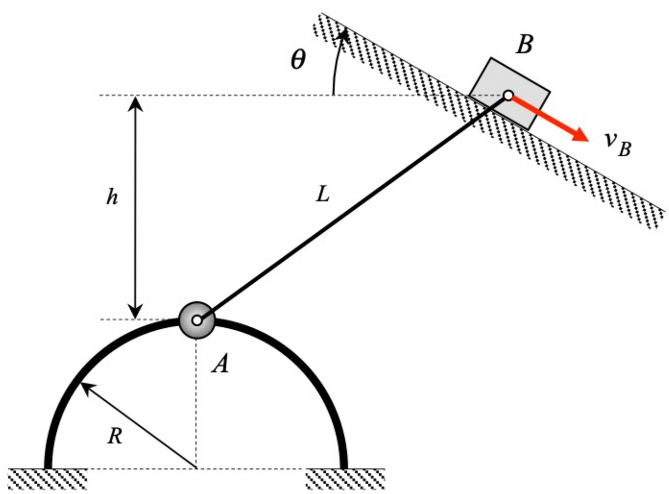
Given: Particle A is constrained to move on a semi-circular guide of radius R. Block B is constrained to move along an inclined ramp that is oriented at an angle of θ relative to horizontal, as shown in the figure below. A and B are each pinned to ends of a rigid bar AB. At the instant shown, particle A is directly above the center of the semi-circular guide, block B is at a vertical distance of h above A and block B is traveling with a constant speed of v_B.

Find: For the position shown:

(a) Determine the angular velocity and angular acceleration of link AB. Write your answers as vectors.

(b) Determine the velocity and acceleration of particle A. Write your answers as vectors.

(c) Locate the instant center for link AB (assume that the figure provided has been drawn to scale). Does the location of this instant center agree with your results above for the direction of the velocity of particle A? Explain.



Use the following parameters in your analysis: $v_B = 4000$ mm/s, $L = 200$ mm, $R = 150$ mm and $h = 120$ mm.

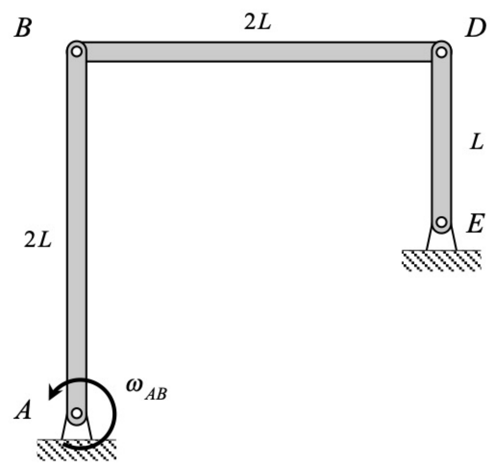
Homework H2.J

Given: Rigid links AB, BD and DE are joined together to form the mechanism shown. Link AB is known to be rotating at a constant rate of ω_{AB}. At the instant shown, links AB and DE are vertical, and link BD is horizontal.

Find: For the position shown:

(a) Where is the instant center for link BD? Based on this result, what are the angular velocities of links BD and DE? Write your answers as vectors.

(b) Determine the angular acceleration of links BD and DE. Write your answers as vectors.



Chapter 3

Moving Reference Frame Kinematics Homework

Homework H3.A

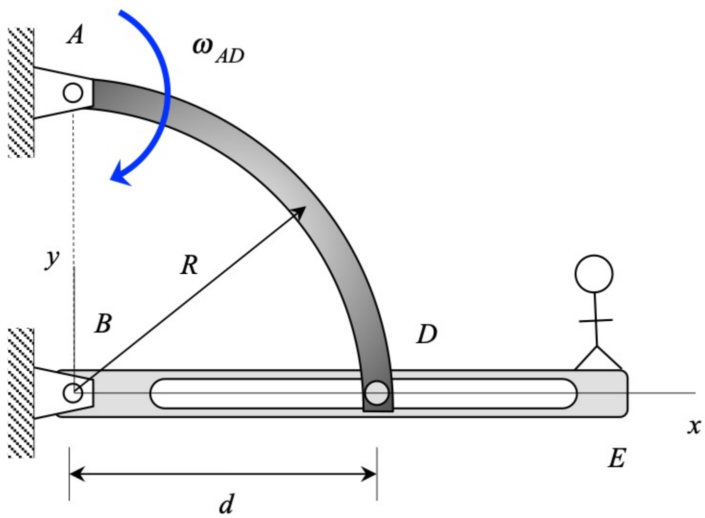
Given: Arm AD is made from a quarter circular arc bar of radius R and is pinned to fixed ground at end A. Slotted arm BE is pinned to fixed ground at end B with pin B located directly below pin A, as shown. A pin at end D of the curved arm is allowed to slide within the slot of arm BE. At the position shown, arm BE is horizontal, and arm AD is rotating CW with a constant rate of ω_{AD}.

Find: For this position,

(a) Determine the angular velocity of arm BE and the value of $\dot{d}$.

(b) Determine the angular acceleration of arm BE and the value of $\ddot{d}$.

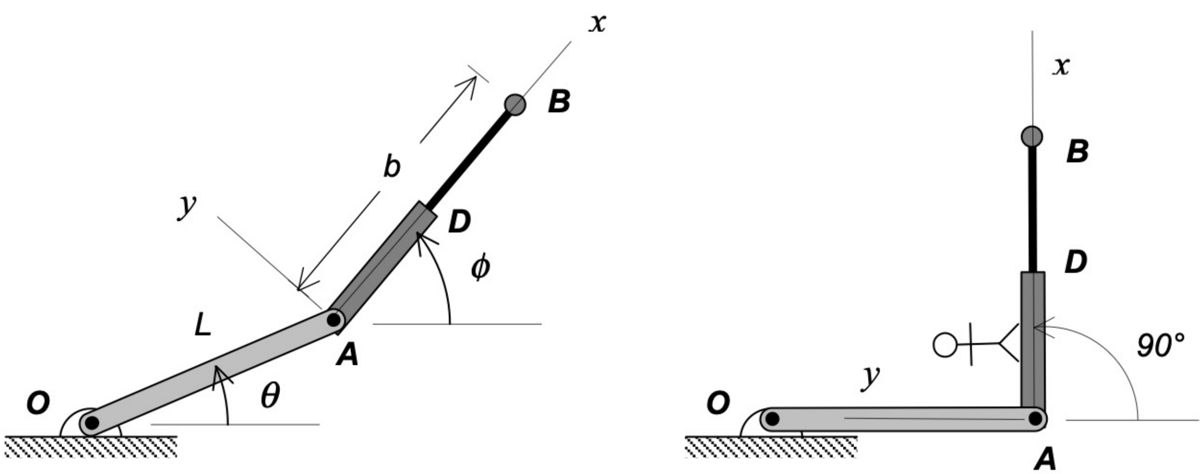
HINT: Use an observer attached to the slotted arm BE, and relate the kinematics of points B and D through the moving reference frame kinematics equations.



Homework H3.B

Given: A robotic manipulator is made up of two links OA and ADB as shown in the figure below left. Link OA has a fixed length of L, and the length link ADB is changing at a constant rate of $\dot{b}$.

Find: For the position shown below right with $\theta = 0°$ and $\phi = 90°$, determine the acceleration of point B on the manipulator.



Use the following parameters in your analysis: $b = 3$ ft, $\dot{b} = 6$ ft/s $=$ constant, $\dot{\theta} = 2$ rad/s $=$ constant, $\dot{\phi} = 3$ rad/s $=$ constant and $L = 4$ ft.

Homework H3.C

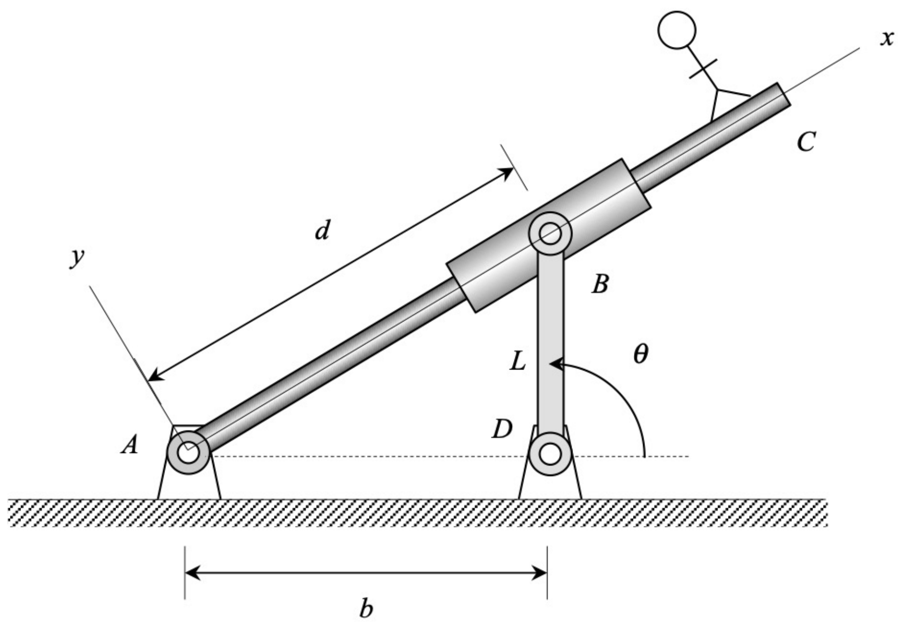
Given: Rod AC is pinned to ground at end A. Collar B is able to slide along the length of rod AC. Arm BD is connected between B and the fixed ground at D. The position of BD is represented by the angle θ measured CCW from the horizontal. At the position shown, $\theta = 90°$ with $\dot{\theta}$ being constant.

Find: For this position,

(a) Determine the angular velocity of AC and the value of $\dot{d}$.

(b) Determine the angular acceleration of AC and the value of $\ddot{d}$.

HINT: Use an observer attached to AC, and relate the kinematics of points A and B through the moving reference frame kinematics equations.



Use the following parameters in your analysis: $L = 30$ mm, $b = 40$ mm and $\dot{\theta} = 4$ rad/s.

Homework H3.D

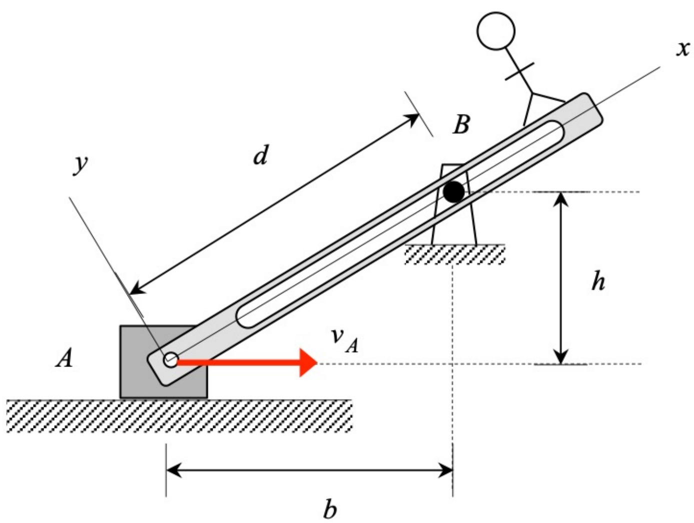
Given: Block A moves to the right with a constant speed v_A. A slotted arm is pinned to block A, with the slot in the arm being allowed to slide over a fixed pin at B.

Find: For this position,

(a) Determine the angular velocity of the arm and the value of $\dot{d}$.

(b) Determine the angular acceleration of the arm and the value of $\ddot{d}$.

HINT: Use an observer attached to the arm, and relate the kinematics of points A and B through the moving reference frame kinematics equations.



Use the following parameters in your analysis: $b = 0.8$ ft, $h = 0.6$ ft and $v_A = 15$ ft/s.

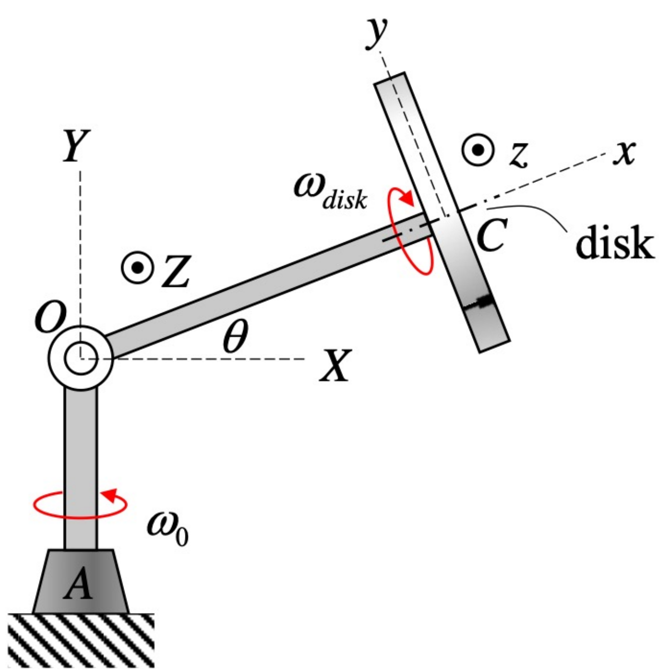
Homework H3.E

Given: A disk and shaft OC are mounted in a clevis that is attached to a fixed vertical axis such that $\omega_0 = 0$ rad/s. The shaft and disk rotate with respect to the clevis with a rate of ω_{disk} in the direction shown below, with the angle θ increasing at a constant rate of $\dot{\theta}$. The XYZ coordinate system is fixed with the Y-axis aligned with the fixed vertical direction. The xyz coordinate system is attached to the disk with the x-axis aligned with OC for all time. For the position shown below, the z- and Z-axes are aligned.

Find: For the position shown:

(a) Determine the angular velocity of the disk. Write your answer as a vector in terms of its xyz components.

(b) Determine the angular acceleration of the disk. Write your answer as a vector in terms of its xyz components.



Use the following parameters in your analysis: $\theta = 0$, $\dot{\theta} = 1.5$ rad/s, $\omega_{disk} = 2$ rad/s and $\dot{\omega}_{disk} = 5$ rad/s^2.

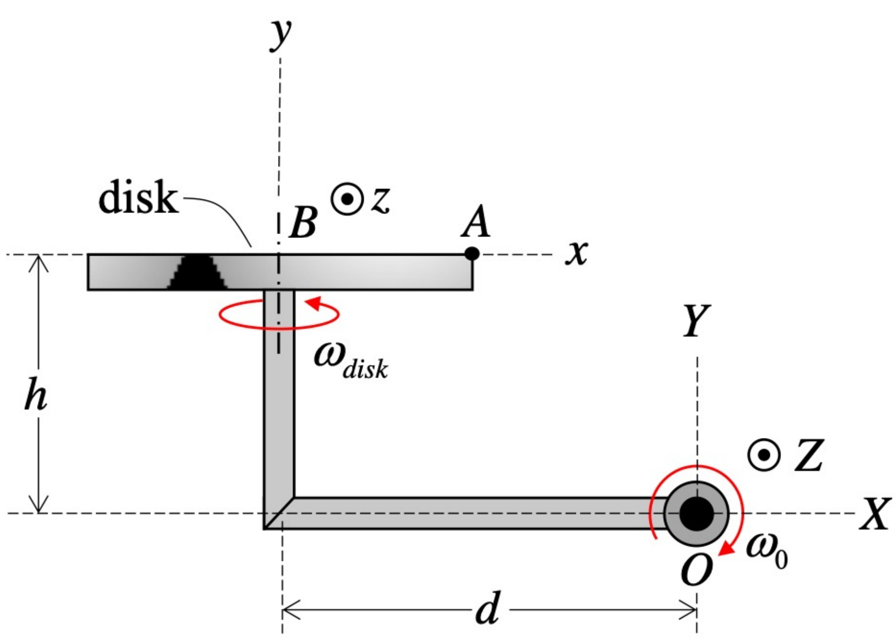
Homework H3.F

Given: Arm OB rotates about a fixed axis with a constant rate of ω_0. A disk of radius R rotates about its central axis with a constant rate of ω_{disk} relative to the arm OB. The XYZ coordinate system is fixed with the Z-axis aligned with the fixed rotation axis of OB. The xyz coordinate system is attached to the disk with the y-axis aligned with the upper part of the arm for all time. For the position shown below, the xyz axes are aligned with the XYZ axes.

Find: For the position shown:

(a) Determine the angular acceleration of the disk. Write your answer as a vector in terms of its xyz components.

(b) Determine the acceleration of point A on the disk. Write your answer as a vector in terms of its xyz components.

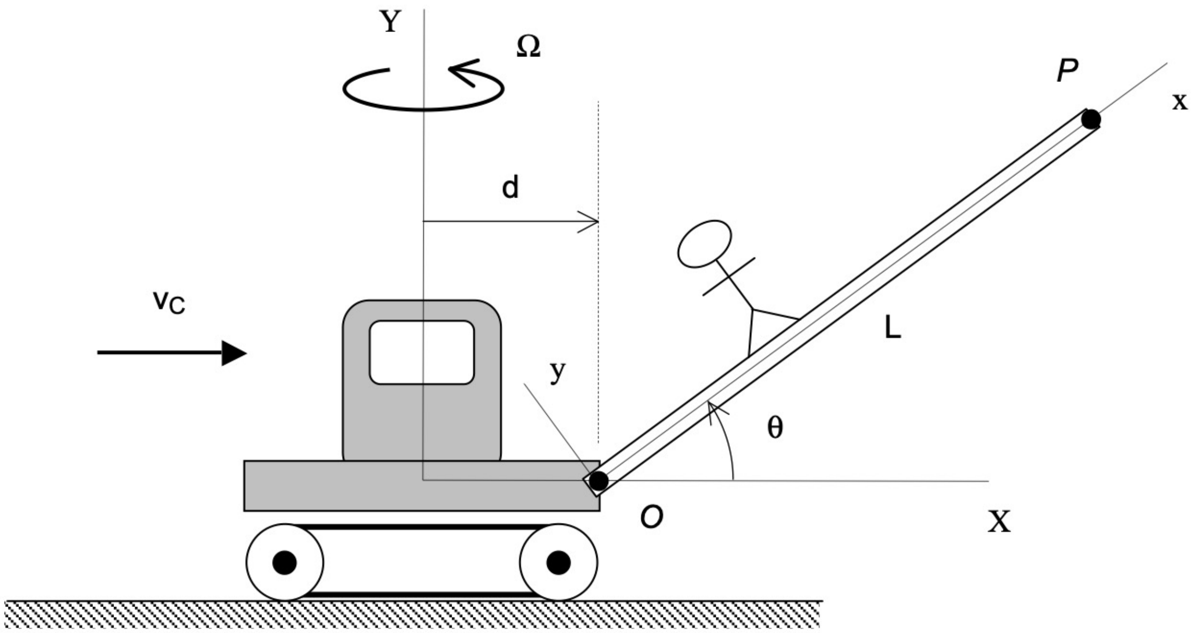


Use the following parameters in your analysis: $\omega_0 = 4$ rad/s, $\omega_{disk} = 3$ rad/s, $d = 18$ cm, $h = 10$ cm and $R = 6$ cm.

Homework H3.G

Given: A crane is moving to the right with a constant speed of v_C and is rotating about a vertical axis with a constant rate of Ω. The boom is being raised at a rate of $\dot{\theta}$ with $\dot{\theta}$ changing at a rate of $\ddot{\theta}$.

Find: Determine the acceleration of end P of the boom.



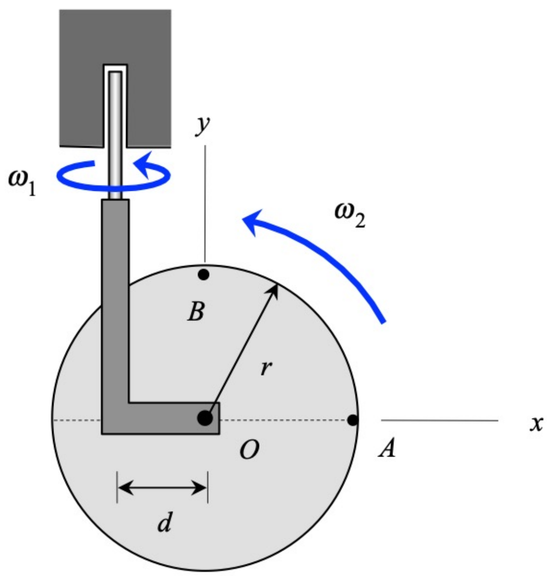
Use the following parameters in your analysis: $v_C = 0.5$ ft/s, $\Omega = 1.5$ rad/s, $\theta = 36.87°$, $\dot{\theta} = 0.6$ rad/s, $\ddot{\theta} = 1.2$ rad/s^2, $L = 20$ ft and $d = 4$ ft.

Homework H3.H

Given: A caster wheel is supported by an L-shaped bracket. The bracket is rotating about a fixed vertical axis with a constant rate of ω_1. The wheel rotates with respect to the bracket with a constant rate of ω_2.

Find: For this problem, determine:

1. The angular velocity and angular acceleration of the wheel. Write your answers as vectors.
2. The acceleration of point A on the wheel at the instant shown when A is immediately to the right of the center O of the wheel.
3. The acceleration of point B on the wheel at the instant shown when B is immediately above the center O of the wheel.

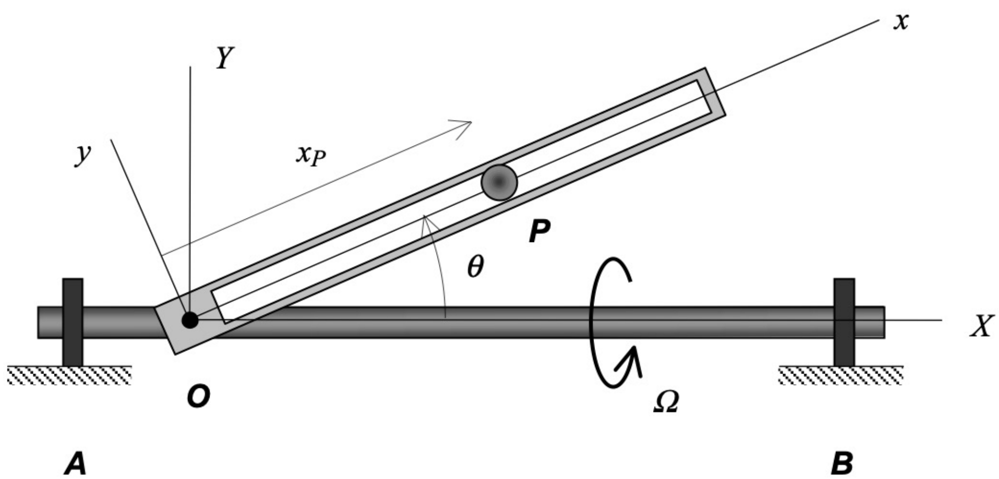


Use the following parameters in your analysis: $\omega_1 = 2$ rad/s, $\omega_2 = 5$ rad/s, $r = 200$ mm and $d = 100$ mm.

Homework H3.I

Given: Shaft AB rotates about a fixed axis with a constant rotational speed of Ω. A tube is hinged on shaft AB with the angle θ between the tube and shaft increasing at a constant rate of $\dot{\theta}$. Particle P moves within the tube at a constant rate of $\dot{x}_P$ relative to the tube. The XYZ coordinate system is fixed with the X-axis aligned with the fixed rotation axis of the shaft AB. The xyz coordinate system is attached to the tube with the x-axis aligned with the tube for all time. For the position shown below, the z- and Z-axes are aligned.

Find: For the position shown, determine the acceleration of particle P. Write your answer as a vector in terms of its xyz components.

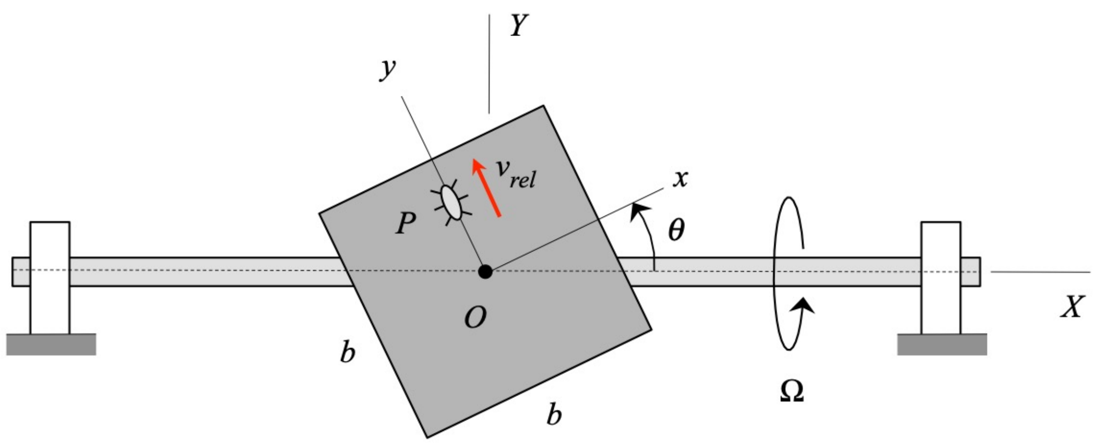


Use the following parameters in your analysis: Ω = 3 rad/s, $\theta = 60°$, $\dot{\theta}$ = -4 rad/s, x_P = 2 m and $\dot{x}_P$ = -2 m/s.

Homework H3.J

Given: A shaft is rotating about the fixed X-axis at a constant rate of Ω. A square plate is pinned at its center O to the centerline of the shaft and is rotating relative to the shaft about O at a constant rate of $\dot{\theta}$. A set of xyz axes are attached to the plate with its origin at O. An insect on the plate is walking along the y-axis with a constant speed of v_{rel} relative to the plate.

Find: Determine the velocity and acceleration of the insect when the insect has reached the edge of the plate. The insect reaches the edge of the plate when $\theta = 0°$.



Use the following parameters in your analysis: $b = 6$ in, $v_{rel} = 12$ in/s, $\Omega = 3$ rad/s and $\dot{\theta} = 5$ rad/s.

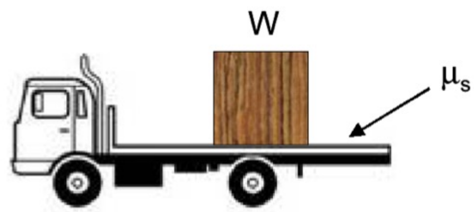
Chapter 4

Particle Kinetics Homework

Homework H4.A

Given: The truck shown below is travelling along I-65 when a deer runs out onto the highway. The truck is initially traveling at a speed of v_0 and decelerates at a constant rate.

Find: Find the minimum distance s over which the truck can stop to ensure that its load does not shift (and the deer is safely avoided).

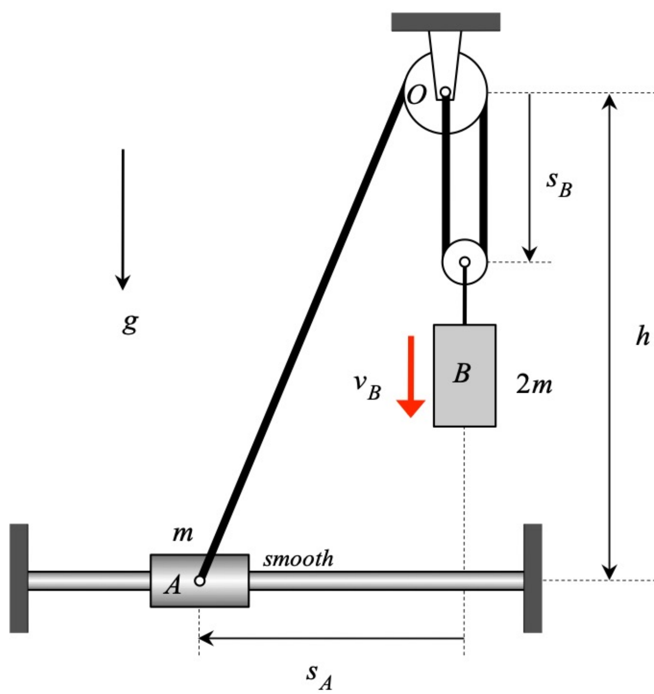


Use the following parameters in your analysis: $v_0 = 40$ m/s, $W = 1000$ N and $\mu_s = 0.2$.

Homework H4.B

Given: Block A (having a mass of m) is connected to block B (with a mass of $2m$) through the cable-pulley system shown. The system is released with block B moving downward with a speed of v_B and with block A displaced to the left of the path of B by an amount of s_A and moving to the right along the its horizontal guide. Consider all surfaces to be smooth. Assume that the cable remains taut during this motion and that the radii of the pulleys are small.

Find: Determine the acceleration of blocks A and B for this position.

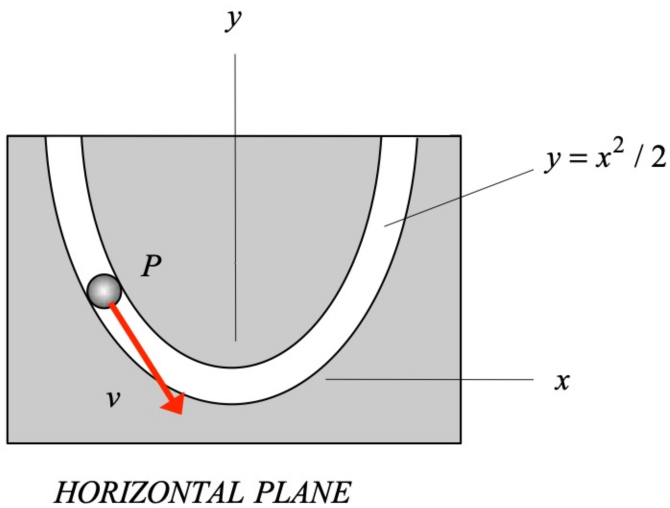


Use the following parameters in your analysis: $m = 10$ kg, $s_A = 0.5$ m, $h = 2$ m and $v_B = 10$ m/s.

Homework H4.C

Given: Particle P (of mass m) moves through a smooth, fixed slot in a horizontal plane with a constant speed of v. The shape of the slot is given in Cartesian coordinates as $y = x^2/2$, where x and y are in meters.

Find: Determine the reaction force on P due to the smooth slot at $x = 0$ m.



Use the following parameters in your analysis: $m = 4$ kg and $v = 30$ m/s.

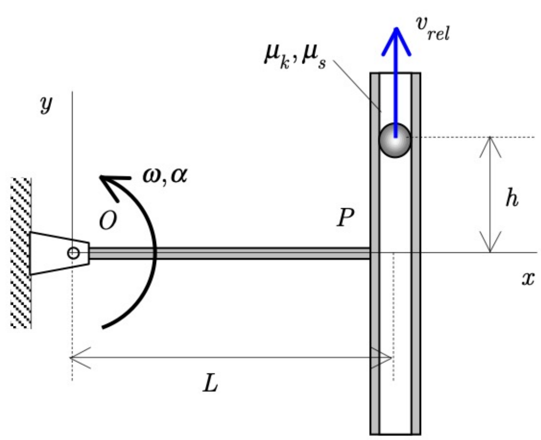
Homework H4.D

Given: Particle P of mass m slides in the direction shown within a tube with a speed of v_{rel} relative to the tube as the tube rotates in the CCW sense with an angular speed of ω and angular acceleration α.

Find: For this problem:

(a) Determine the acceleration of P;

(b) Determine the friction force acting on P.



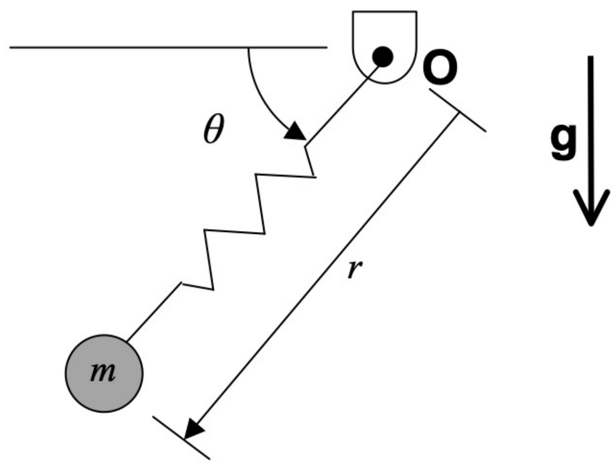
HORIZONTAL PLANE

Use the following parameters in your analysis: $m = 10$ kg, $\omega = 5$ rad/s, $\alpha = 2$ rad/s^2, $v_{rel} = 4$ m/s, $\mu_s = 0.6$, $\mu_k = 0.3$, $L = 0.4$ m and $h = 0.2$ m.

Homework H4.E

Given: A mass is attached to a spring rotating about the pin at O.

Find: Determine the unstretched length of the spring.

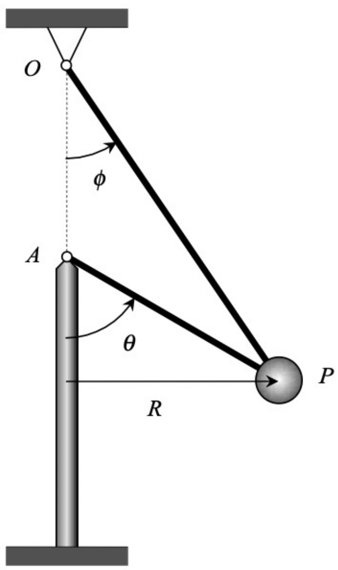


Use the following parameters in your analysis: $\theta = 30°$, $\dot{\theta} = 8$ rad/s, $m = 3$ kg, $k = 100$ N/m, $r = 0.75$ m, $\dot{r} = 3$ m/s and $\ddot{r} = 3$ m/s^2.

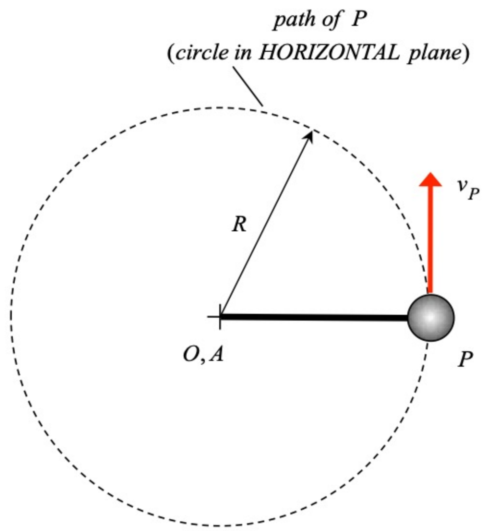
Homework H4.F

Given: Two wires, OP and AP, connect particle P (having a mass of m) to fixed points O and A, respectively, where OA is a vertical line. The particle rotates about axis OA such that P has a constant speed of v_P and with the two wires remaining taut as particle moves on a circular path of radius R. Let ϕ and θ be the angles that wires OP and OA, respectively, make with the vertical.

Find: Determine the range of values for v_P for which wires OP and OA remain taut.



SIDE view



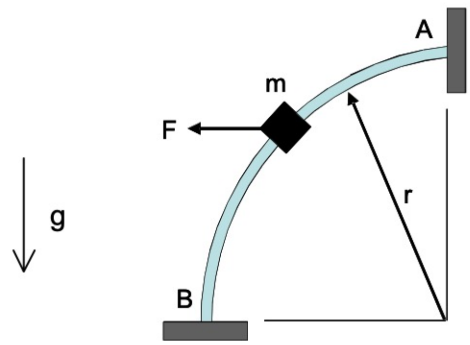
TOP view

Use the following parameters in your analysis: $R = 2$ m, $\theta = 60°$, $\phi = 30°$ and $m = 4$ kg.

Homework H4.G

Given: The collar, shown below, of mass m, starts from rest at point A. A constant force F is applied to the collar in the direction shown. Note that the mechanism lies in the vertical plane. Assume all surfaces to be smooth.

Find: Determine the speed of the collar when it reaches point B.

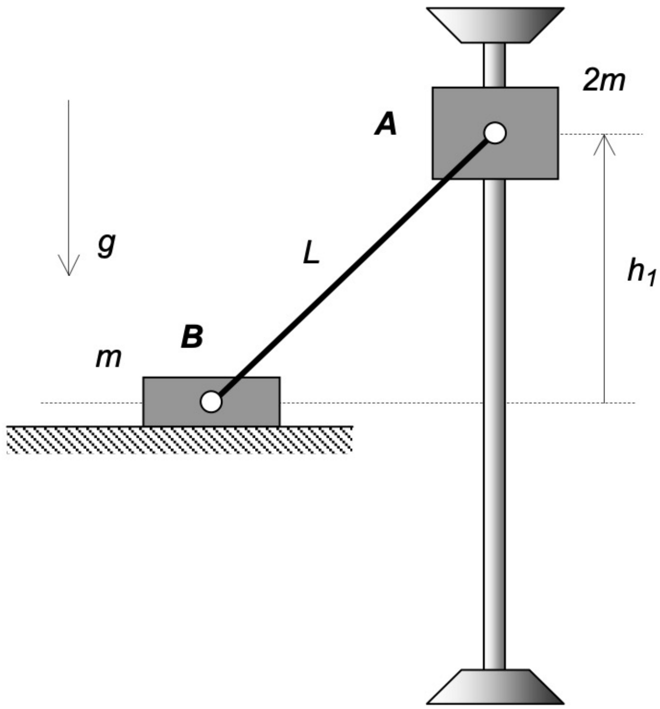


Use the following parameters in your analysis: $mg = 10$ lb, $F = 6$ lb, and $r = 2$ ft.

Homework H4.H

Given: Blocks A and B (having masses of $2m$ and m, respectively) are connected by rigid, massless rod AB of length L. Block A is constrained to move along a smooth vertical guide, and B moves along a smooth, horizontal surface. The system is released from rest when A is at a height of h_1 ABOVE the path of B.

Find: Determine the speed of block A when A has dropped to a position that is at a distance of h_2 BELOW the path of B.

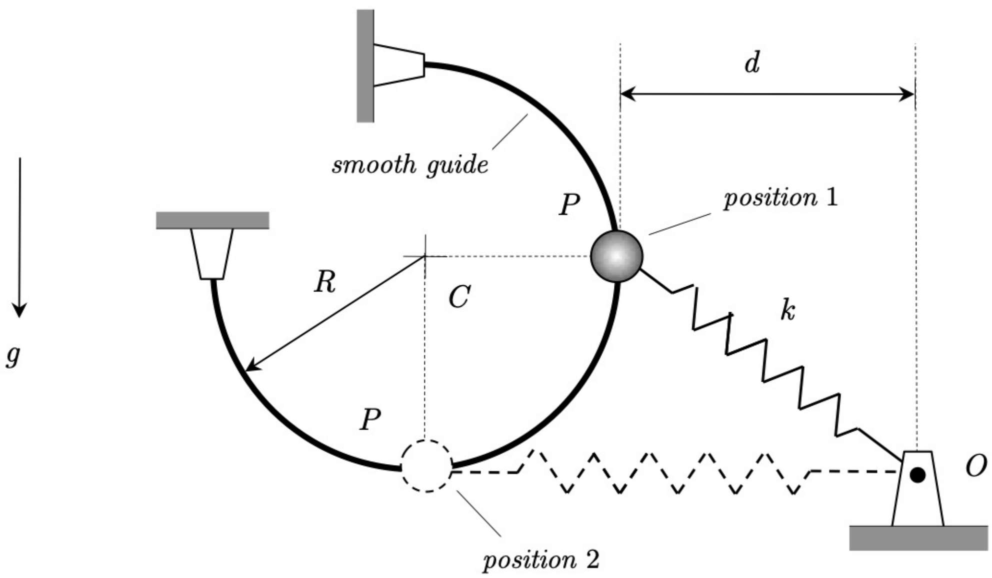


Use the following parameters in your analysis: $m = 10$ kg, $L = 0.5$ m, $h_1 = 0.4$ m and $h_2 = 0.3$ m.

Homework H4.I

Given: Particle P (of mass m) is constrained to move along a smooth circular guide (of radius R). A spring of stiffness k is attached between P and the fixed point O, where O is on the same horizontal line as the bottom of the circular guide, as shown in the figure. At position 1, P is at rest and is on the same horizontal line as the center of the guide C. At position 2, P is on the same horizontal line as O and the spring is unstretched.

Find: Determine the speed of particle P at position 2.

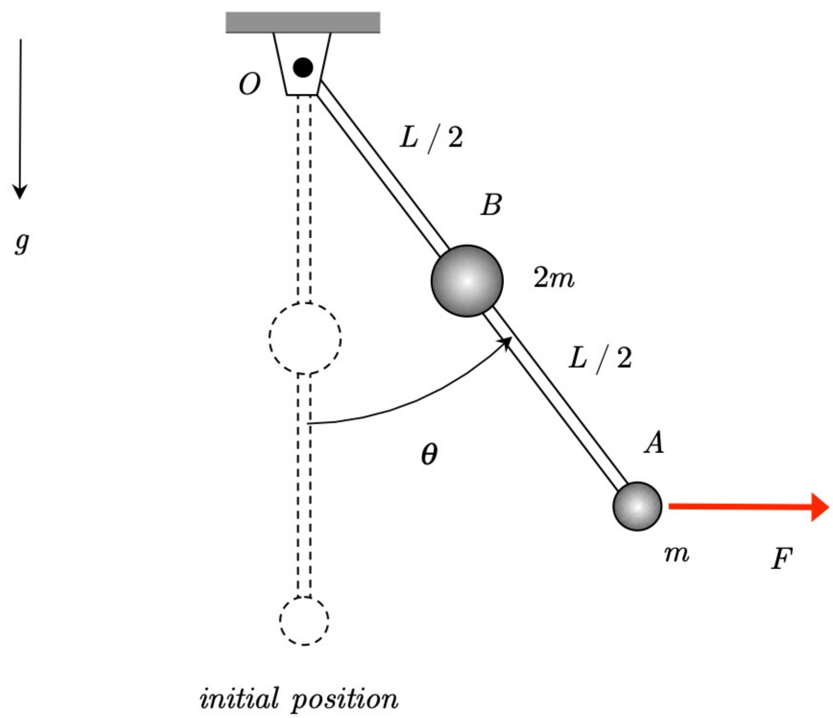


Use the following parameters in your analysis: m = 50 kg, R = 0.6 m, d = 0.8 m and k = 500 N/m.

Homework H4.J

Given: Particles A and B (having masses of m and $2m$, respectively) are attached to a lightweight rigid bar as shown in the figure. A constant horizontal force F acts on particle A. At the initial state of $\theta = 0°$, the system is at rest.

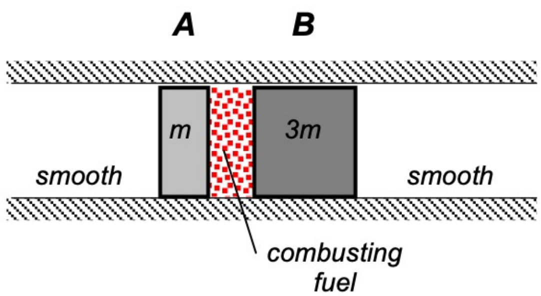
Find: Determine the angular speed of the bar as a function of θ and in terms of the parameters of the problem.

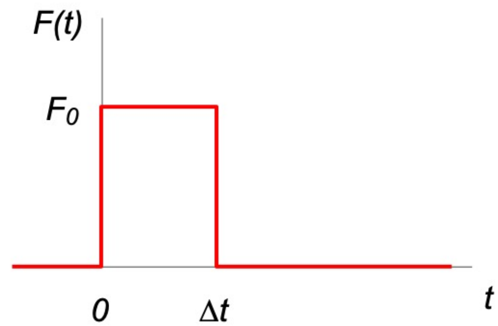


Homework H4.K

Given: Pellets A and B (having masses m and $3m$, respectively) are placed within a smooth tube trapping a small compartment of fuel. At a time when the pellets are initially at rest, the fuel is ignited. The combustion occurs over a short time Δt, and over this time the combustion applies equal and opposite forces on the pellets with this force idealized by the force time history $F(t)$ shown below.

Find: Determine the speed of each pellet at time t_2, where $t_2 > \Delta t$.



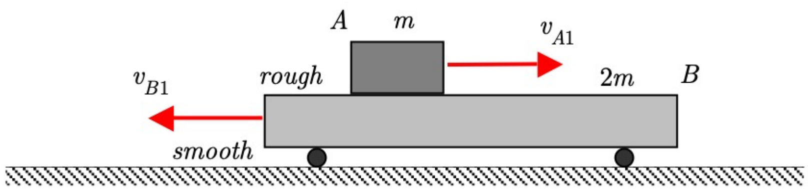


Use the following parameters in your analysis: $F_0 = 3000$ N, $\Delta t = 0.005$ s and $m = 0.75$ kg.

Homework H4.L

Given: Block B (of mass $2m$) is able to slide along a smooth horizontal surface. Block A (of mass m) is able to slide along the rough top surface of block B, as shown in the figure. Initially, A is traveling to the right with a speed of v_{A1}, and block B is traveling to the left with a speed of v_{B1}.

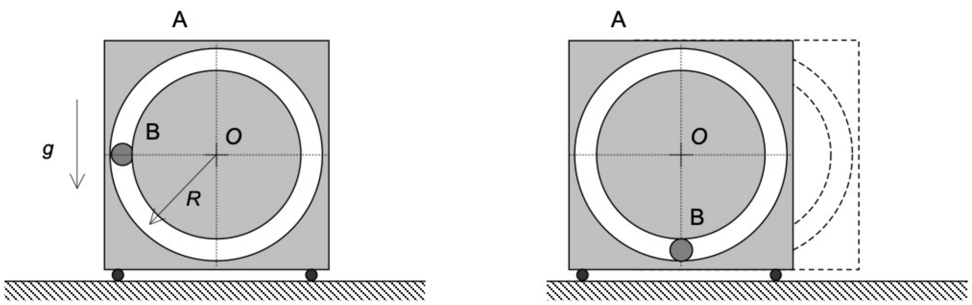
Find: Determine the velocity of block B when block A has to come rest relative to block B.



Homework H4.M

Given: Particle B (having a mass of m) is constrained to move within a circular slot (of radius R) that is cut into block A (having a mass of M). The system is released from rest with particle B on a horizontal line passing through the circle's center O. Consider all surfaces to be smooth.

Find: For this problem:
(a) Determine the velocities of A and B when B has moved position 2 where B is directly below O (write your answers as vectors);
(b) Determine the work done on block A in moving from position 1 to position 2.



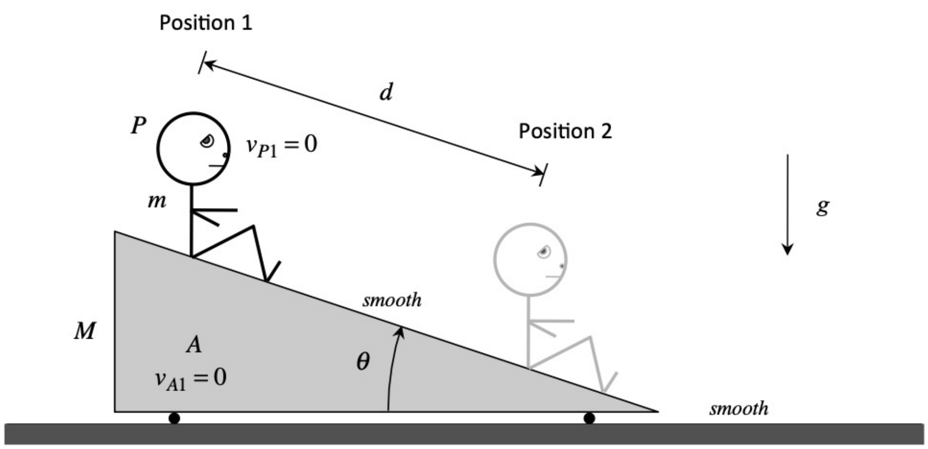
Position 1 ***Position 2***

Use the following parameters in your analysis: $m = 20$ kg, $M = 40$ kg and $R = 0.5$ m.

Homework H4.N

Given: A wedge-shaped block of mass M is able to slide freely on a horizontal surface. A person of mass m climbs onto the inclined surface of the wedge. With both the person and cart initially at rest, the person slides down the incline of the wedge. Consider the person to move as a particle as she moves down the inclined surface. Also, consider all surfaces to be smooth.

Find: Determine the velocity of the person and of the cart after the person has slid down a distance d along the surface of the inclined wedge. Write your answers as vectors.

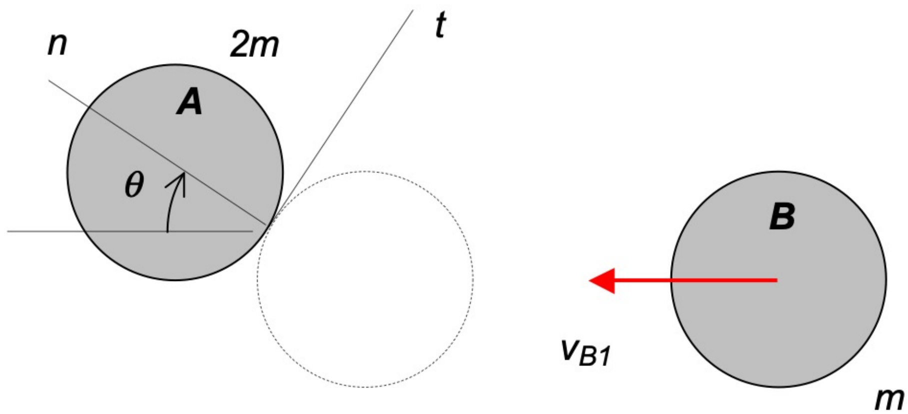


Use the following parameters in your analysis: $mg = 120$ lb, $Mg = 180$ lb, $\theta = 36.87°$ and $d = 8$ ft.

Homework H4.O

Given: Disks A and B have masses of $2m$ and m, respectively. Disk B is traveling in the direction shown with a speed of v_{B1} when it strikes the stationary disk A ($v_{A1} = 0$ ft/s). Let e represent the coefficient of restitution of impact between A and B.

Find: Determine the velocity of disk A after impact. Write your answer as a vector in terms of its n and t components.

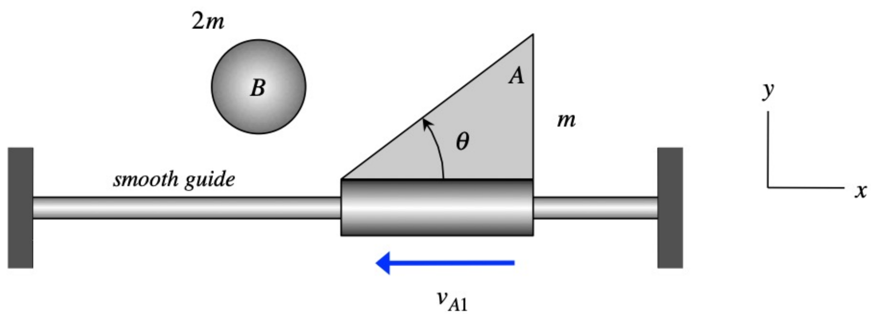


Use the following parameters in your analysis: $e = 0.6$, $\theta = 60°$ and $v_{B1} = 80$ ft/s.

Homework H4.P

Given: Slider A (having a mass of m) is sliding to the left with a speed of v_{A1} along a straight smooth guide. The slider then impacts a stationary particle B (having a mass of $2m$), with this impact having a coefficient of restitution of e.

Find: Determine the velocity of A and B after impact. Write your answers as vectors.



TOP view of a HORIZONTAL plane

Use the following parameters in your analysis: $v_{A1} = 30$ ft/s, $\theta = 36.87°$ and $e = 0.9$.

Homework H4.Q

Given: Particle P (of mass m) travels in the x-y plane with a velocity of $\vec{v}_P$.

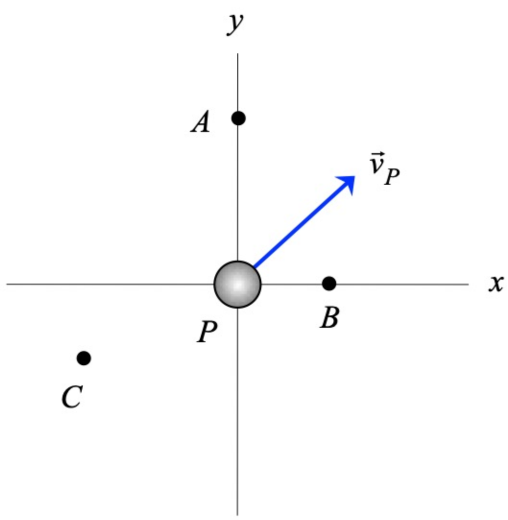
Find: For this problem:

(a) The angular momentum of P about point A, $\vec{H}_A$.

(b) The angular momentum of P about point B, $\vec{H}_B$.

(c) The angular momentum of P about point C, $\vec{H}_C$.

Write your answers as vectors.

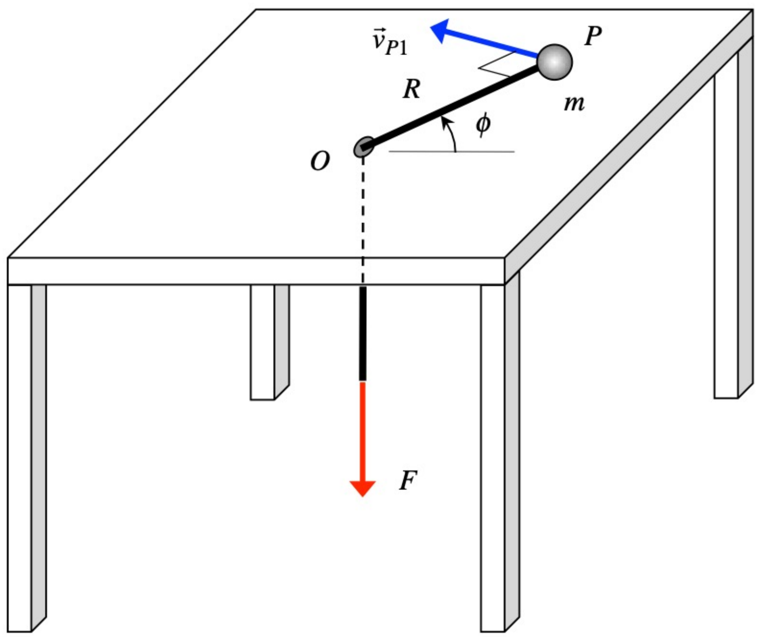


Use the following parameters in your analysis: $m = 5$ kg, $\vec{v}_P = \left(20\hat{i} + 15\hat{j}\right)$ m/s and the following (x, y) coordinates for points A, B, C and P: $A : (0, 1.2)$ m, $B : (0.9, 0)$ m, $C : (-0.4, -0.3)$ m, and $P : (0, 0)$ m.

Homework H4.R

Given: Particle P, having a mass of m, is able to slide on the smooth, horizontal top of a table. A flexible cable is attached to P, with the cable being fed through a hole in the table at O. A constant force F acts on the other end of the cable. The system is released with P being at a radial distance $R = R_1$ from O, and with P having a velocity perpendicular to OP with a speed of v_{P1}.

Find: Determine the numerical values for $\dot{R}$ and $\dot{\phi}$ when P has moved to a position for which $R = R_2$.

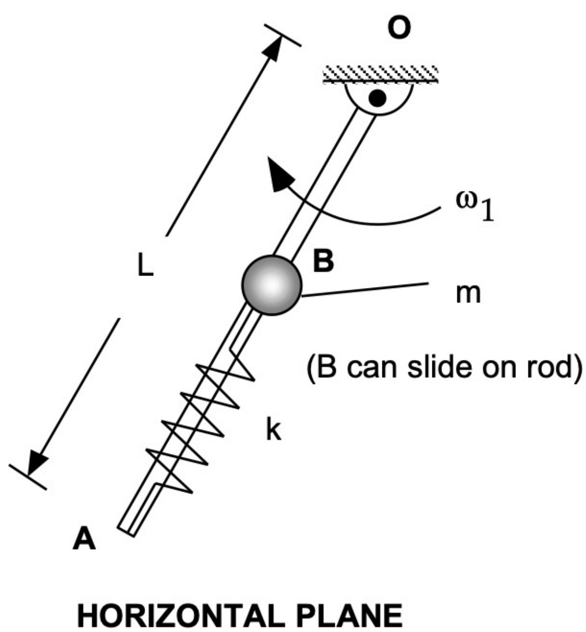


Use the following parameters in your analysis: $m = 2$ kg, $R_1 = 1.5$ m, $R_2 = 0.5$ m, $v_{P1} = 5$ m/s and $F = 236$ N.

Homework H4.S

Given: Bar OA (having negligible mass and length L) is pinned to ground at point O in such a way that the bar moves in a HORIZONTAL plane. Particle B (with a mass of m) is able to slide without friction on OA, and is attached to A with a spring having a stiffness of k and an unstretched length of $L/2$. At the instant shown, bar OA is rotating CW with a speed of ω_1, particle B is not moving relative to bar OA and the spring is unstretched.

Find: Determine the velocity of particle B after it has moved an additional distance of b outward on the bar.

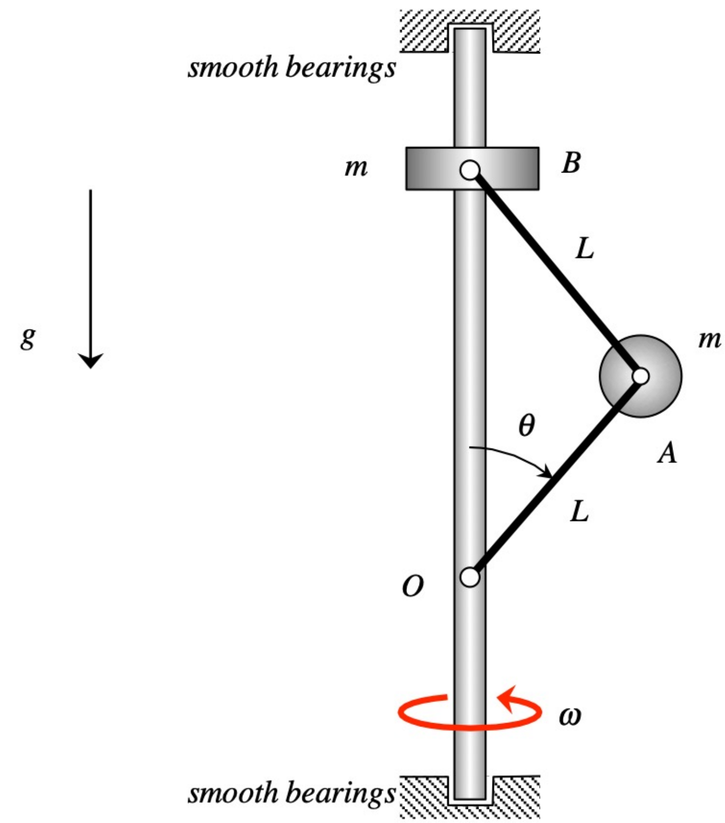


Use the following parameters in your analysis: $\omega_1 = 9$ rad/s, $m = 10$ kg, $M = 10$ kg, $L = 3$ m, $k = 500$ N/m, $d = 1.5$ m and $b = 0.5$ m.

Homework H4.T

Given: Particle A (having a mass of m) is attached to rod OA, with end O being pinned to a vertical shaft. Particle A is also attached to particle B (having a mass of m) through rod AB, with particle B being constrained to slide along the rotation axis of the shaft. At a time when the shaft is rotating with a rate of ω_1 and with $\theta = \theta_1$, and while particles A and B are stationary with respect to the shaft, the particles are released. Assume all surfaces to be smooth and the mass of rods OA and AB to be negligible.

Find: Determine the speed of particle B when $\theta = \theta_2$.



Use the following parameters in your analysis: $m = 20$ kg, $L = 0.5$ m, $\omega_1 = 6$ rad/s, $\theta_1 = 30°$ and $\theta_2 = 90°$.

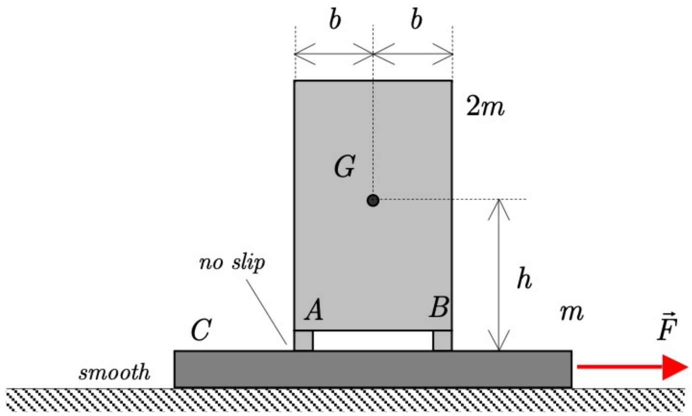
Chapter 5

Rigid Body Kinetics Homework

Homework H5.A

Given: A crate of mass $2m$ rests on block C of mass m that, in turn, rests on a smooth horizontal surface. A horizontal force $\vec{F}$ acts to the right on block C.

Find: Determine the maximum force F for which the crate does not tip on C. Assume that sufficient friction exists between the crate and C such that the crate does not slip on block C.

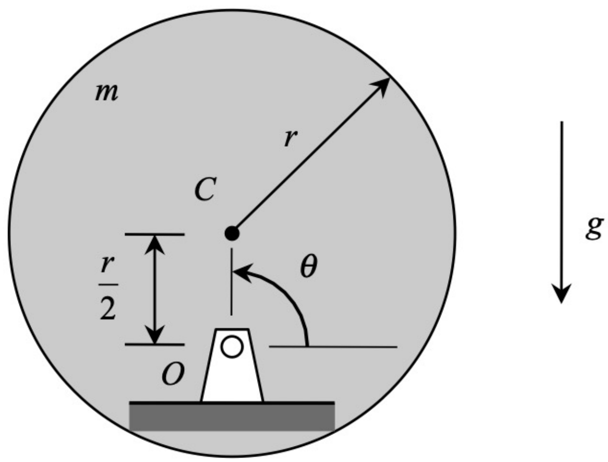


Use the following parameters in your analysis: $m = 100$ kg, $h = 3$ m and $b = 1.5$ m.

Homework H5.B

Given: A homogeneous disk of mass m and outer radius r is attached to a horizontal shaft at point O, where O is at a radial distance of $r/2$ from the center C of the disk. The shaft to which the disk is attached is rotating at a constant rate of $\dot{\theta} = \Omega$.

Find:
(a) Determine the total reaction force acting on the disk at O when $\theta = 90°$. Write this answer as a vector.
(b) Determine the total reaction force acting on the disk at O when $\theta = 180°$. Write this answer as a vector.

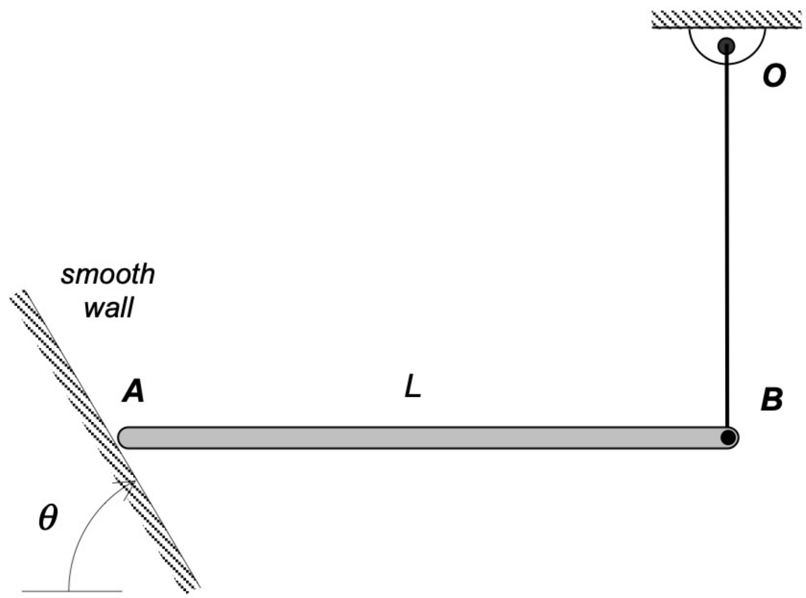


Use the following parameters in your analysis: $m = 20$ kg, $r = 0.8$ m and $\Omega = 10$ rad/s.

Homework H5.C

Given: The homogeneous thin bar AB shown below has a mass of M and a length of L. The bar is supported by lightweight, inextensible cable BO at end B and by a smooth, inclined wall at end A. The bar is released from rest with the bar in a horizontal orientation and with cable BO being vertical.

Find: Determine the tension in cable BO immediately after the bar is released.

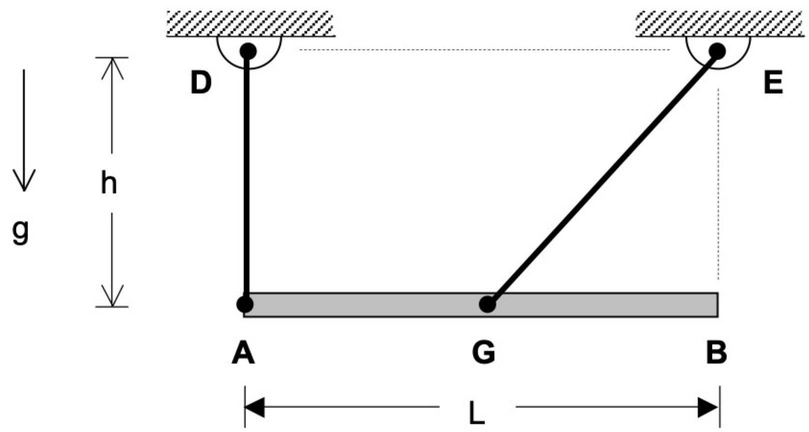


Use the following parameters in your analysis: $M = 100$ kg, $L = 3$ m and $\theta = 53.13°$.

Homework H5.D

Given: A thin, homogeneous bar AB with a mass of M and length L is supported by cables AD and EG, where G is the center of mass of the bar. At the position shown below, the bar is horizontal, point A is directly below D, and end B is directly below E. The bar is released from rest in the position shown.

Find: Determine the angular acceleration of the bar immediately after release.

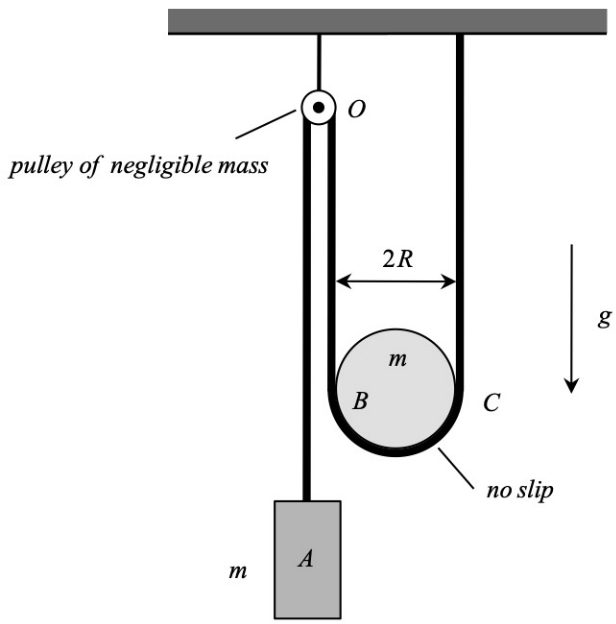


Use the following parameters in your analysis: $M = 20$ kg, $L = 4$ m and $h = 1.5$ m.

Homework H5.E

Given: A homogeneous disk having a mass m and outer radius R is supported by the cable-pulley system shown, where block A (with a mass of m) is attached to the free end of the cable. The disk does not slip on the cable as the system moves.

Find: Determine the acceleration of block A on release.

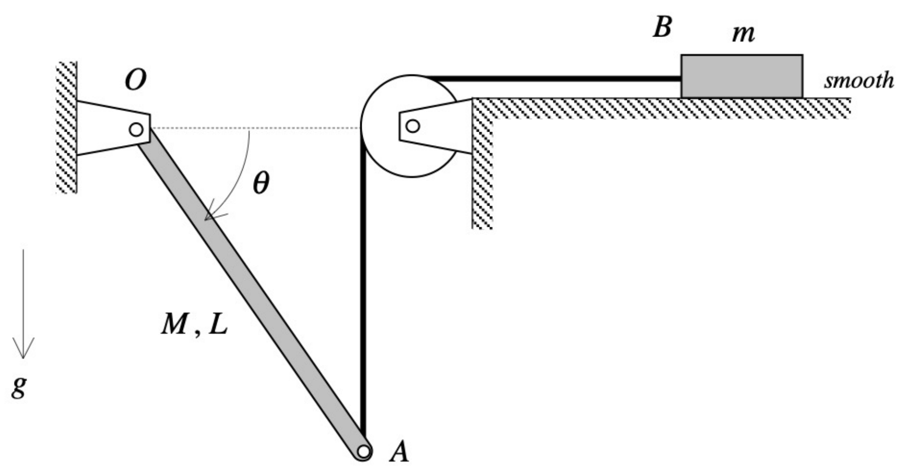


Use the following parameters in your analysis: $m = 20$ kg and $R = 0.2$ m.

Homework H5.F

Given: A homogeneous bar of mass M and length L is pinned to ground at O. The other end of the bar, point O, is supported by a cable that is pulled over a smooth pulley and attached to a block B of mass m. Block B is able to slide on a smooth, horizontal surface. At the instant of release with bar OB oriented at an angle of θ with the horizontal, the bar is stationary and the cable is taut with the two free sections of the cable oriented horizontally and vertically, as shown in the figure. Assume the mass of the pulley to be negligible compared to the mass of the bar and block.

Find: Determine the acceleration of block B on release.

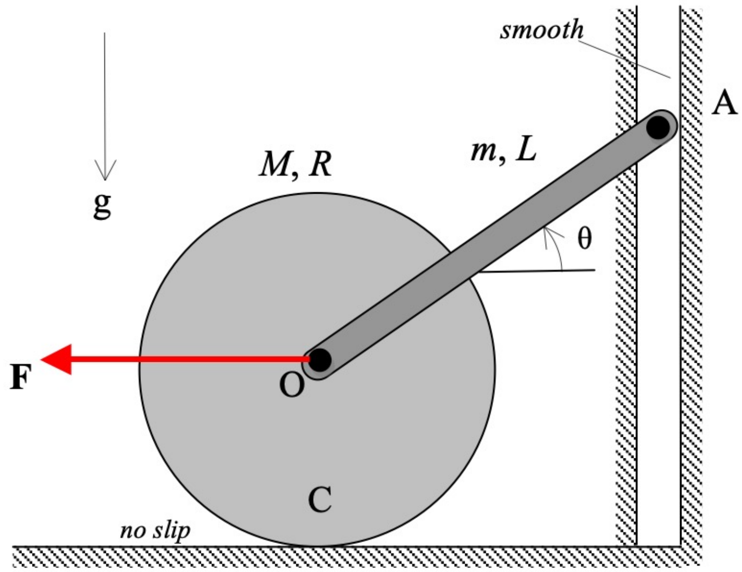


Use the following parameters in your analysis: $M = 50$ kg, $m = 20$ kg $L = 2$ m and $\theta = 60°$.

Homework H5.G

Given: A homogeneous disk (mass M and radius R) is attached to a homogeneous, thin rod OA (mass m) at its center O. A constant force F is applied at O and point A is confined so that it moves along a smooth, vertical slot. The disk rolls without slipping. If the system is released from rest with $\theta = \theta_1$.

Find: Determine the velocity of point A when $\theta = \theta_2$.

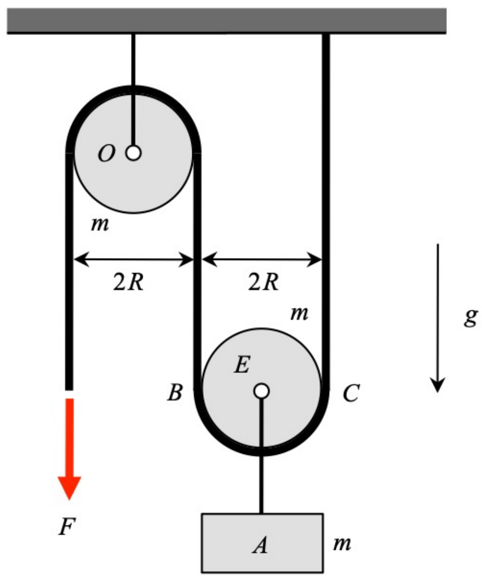


Use the following parameters in your analysis: $M = 12$ kg, $m = 6$ kg, $R = 1$ m, $L = 2.5$ m, $F = 80$ N, $\theta_1 = 53.13°$ and $\theta_2 = 0°$.

Homework H5.H

Given: A homogeneous disk of mass m and outer radius R is supported by the cable-pulley system shown. The pulley (having a mass of m and with an outer radius of R) is supported by a smooth shaft at its center O. Block A (with a mass of m) is supported at the center E of the disk. A constant force F is applied to the free end of the cable. The system is released from rest. Assume the pulley and disk do not slip on the cable.

Find: Determine the speed of block A after A has moved through a distance of s_A.

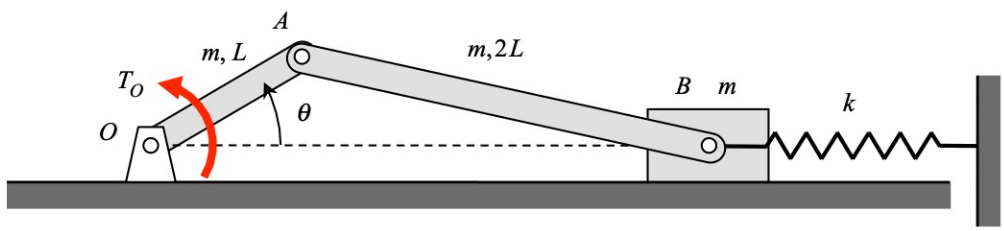


Use the following parameters in your analysis: m = 15 kg, R = 0.25 m, s_A = 0.5 m and F = 300 N.

Homework H5.I

Given: A mechanism is made up of links OA and AB, along with the slider B. A spring of stiffness k is attached between the slider B and ground, with the spring being unstretched when $\theta = 90°$. A constant torque T_O is applied to link OA at the shaft passing through OA at point O. The mechanism is released from rest when $\theta = 0°$. The mechanism moves in a horizontal plane.

Find: Determine the speed of slider B when $\theta = 90°$.

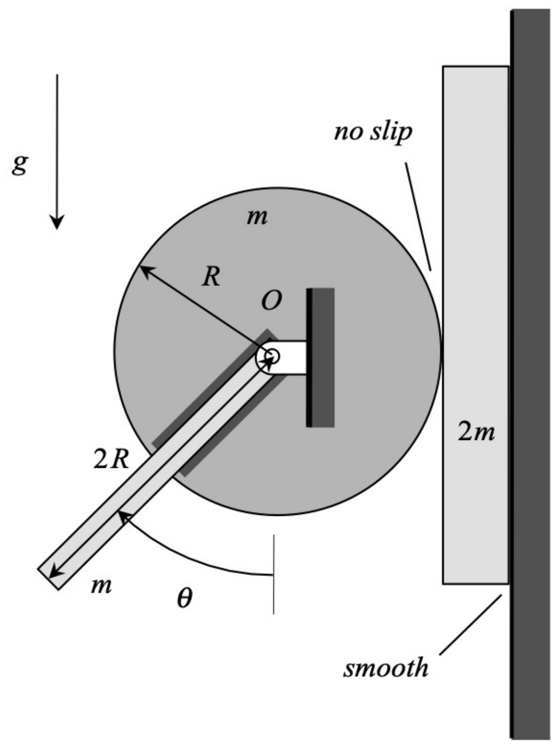


Use the following parameters in your analysis: $m = 10$ kg, $L = 0.2$ m and $k = 300$ N.

Homework H5.J

Given: A homogeneous disk of mass m and outer radius R is able to rotate about a frictionless bearing at its center O. A thin, homogeneous bar of mass m and length $2R$ is welded to the disk with the bar aligned with a radial direction on the disk and one end at O. A block of mass $2m$ is able to slide along a smooth, vertical wall, with the block being in no-slip contact with the outer surface of the disk, as shown in the figure. The system is released from rest with $\theta = 90°$.

Find: Determine the angular velocity of the disk after the disk has rotated through an additional angle of 90° after release.

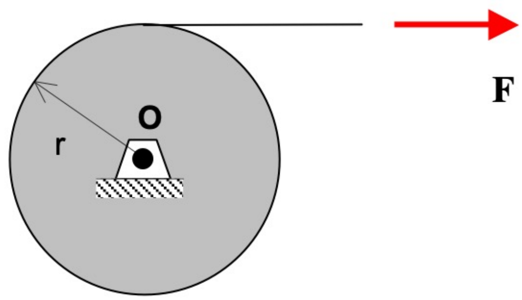


Use the following parameters in your analysis: $m = 5$ kg and $R = 0.1$ m.

Homework H5.K

Given: A uniform disk (of mass m and radius r) is pinned at its center of mass O. Force F is applied to a rope wrapped around the disk. The disk is released from rest. Assume that the rope does not slip on the disk.

Find: At time t, determine:
(a) the angular velocity of the disk; and
(b) the reaction force components on the disk at pin O.

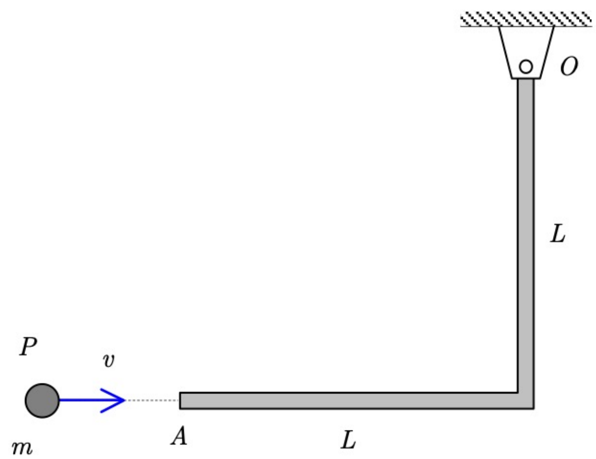


Use the following parameters in your analysis: $m = 15$ kg, $r = 0.4$ m, $F = 200$ N and $t = 2$ s.

Homework H5.L

Given: Particle P (of mass m) strikes end A of a stationary homogeneous L-shaped bar (of mass M) with a speed of v. The coefficient of restitution for the impact of P with end A of the bar is known to be e.

Find: Determine the angular speed of bar OA immediately after the impact.



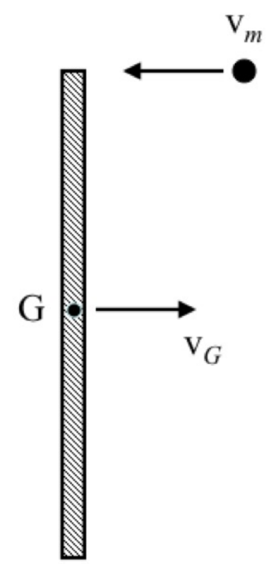
HORIZONTAL PLANE

Use the following parameters in your analysis: $M = 30$ kg, $m = 10$ kg, $L = 2$ m, $v = 40$ m/s and $e = 0.5$.

Homework H5.M

Given: A homogeneous, slender bar of mass M and length L is sliding across a frictionless horizontal surface with a speed v_G when it is suddenly struck at its end by a particle of mass m moving with velocity v_m in the direction shown. Assume that the particle sticks to the bar upon impact.

Find: Determine the velocity of the bar's center of mass G and the angular velocity of the bar post-impact.

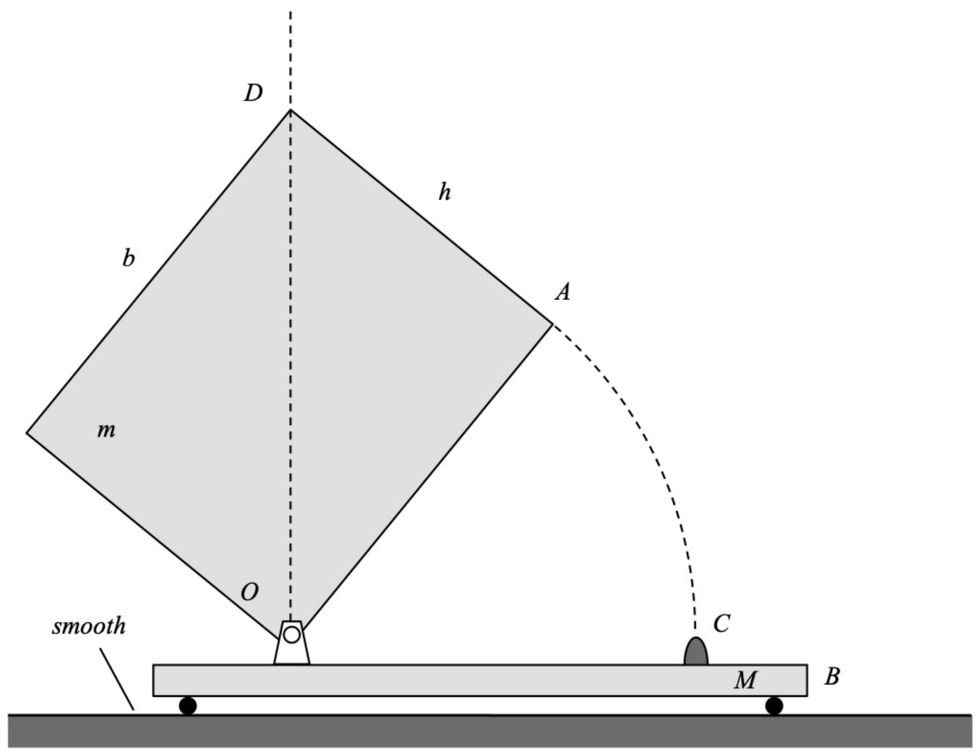


Use the following parameters in your analysis: $M = 4$ kg, $L = 0.75$ m, $v_G = 5$ m/s, $m = 0.4$ kg and $v_m = 25$ m/s.

Homework H5.N

Given: A homogeneous rectangular plate of mass m is pinned to cart B at corner O, where cart B is constrained to move along a smooth horizontal surface. The system is released from rest with corner D displaced slightly to the right of a vertical line passing through the pin at O. As a result, the plate eventually impacts bumper C on the cart, with the coefficient of restitution between the plate and the bumper being e.

Find: For this problem:
(a) Determine the velocity of the center of mass of the plate immediately before the plate contacts the bumper C. Write your answer as a vector.
(b) Determine the velocity of the center of mass of the plate immediately after the plate contacts the bumper C. Write your answer as a vector.



Use the following parameters in your analysis: $m = 10$ kg, $M = 25$ kg, $b = 2$ m, $h = 1$ m and $e = 0$.

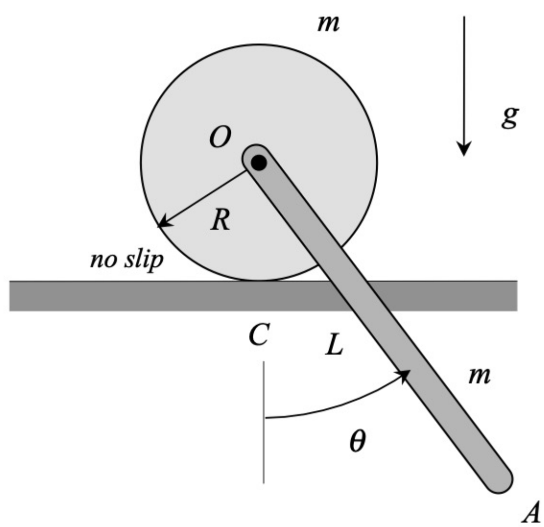
Homework H5.O

Given: A thin homogeneous bar OA (of mass m and length L) is pinned to a homogeneous disk (of mass m and radius R) at the disk's center O. The disk is able to roll without slipping on a rough horizontal surface. The system is released from rest with the bar at an angle of θ measured counterclockwise from vertical.

Find: For this problem:

(a) Determine the angular acceleration of the disk on release.

(b) Determine the angular acceleration of bar OA on release.

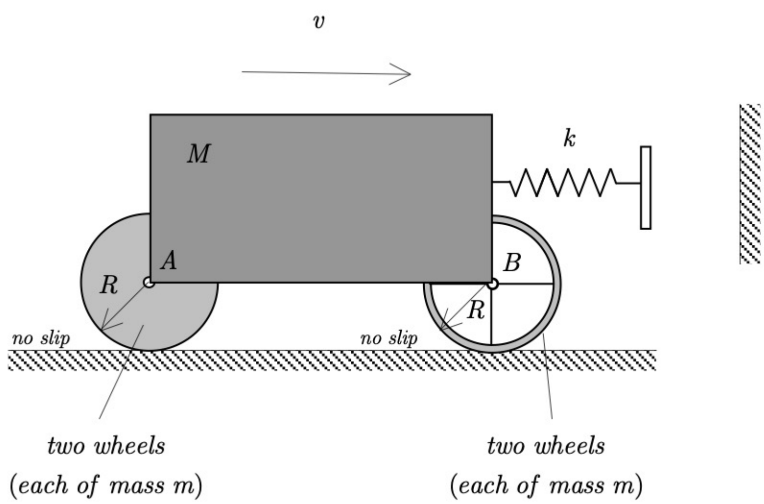


Use the following parameters in your analysis: $m = 30$ kg, $R = 0.5$ m, $L = 1.5$ m and $\theta = 60°$.

Homework H5.P

Given: A cart is made up of a body (of mass M) and FOUR wheels (each wheel of mass m and outer radius R). The rear wheels are solid and homogeneous, whereas the front wheels are spoked rims (with the spokes being massless). The cart is initially moving to the right with a speed of v. At some point, a spring of stiffness k attached to the front of the cart contacts a fixed wall.

Find: Determine the maximum compression of the spring after contacting the wall.



Use the following parameters in your analysis: $M = 14$ kg, $m = 3$ kg, $R = 0.15$ m, $k = 800$ N/m and $v = 0.5$ m/s.

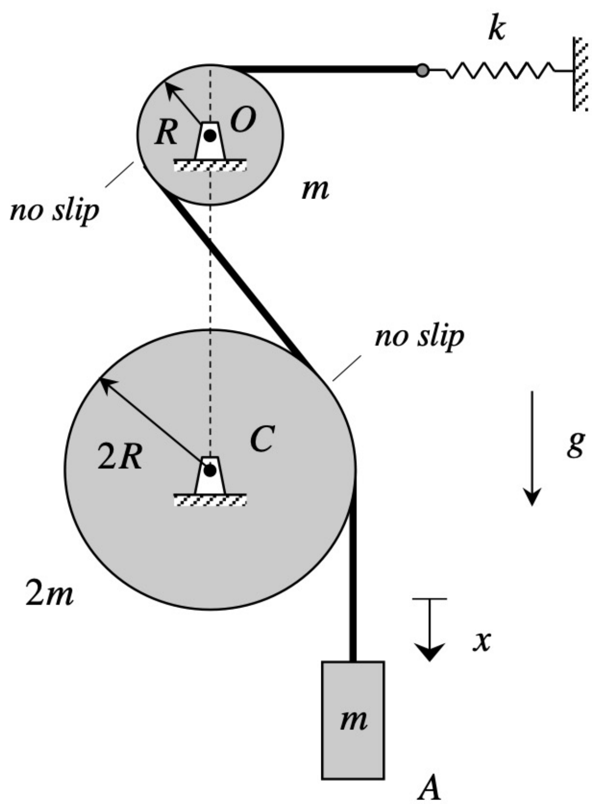
Chapter 6

Vibrations Homework

Homework H6.A

Given: Homogeneous disks (of outer radii R and $2R$ and masses m and $2m$, respectively) are pinned to ground at their centers O and C. A cable is wrapped around both is disks as shown, with particle A (of mass m) attached to one end and a grounded spring of stiffness k attached to the other. The cable is known to not slip on either disk nor does the cable go slack. Let x represent the motion of A, where $x = 0$ when the spring is unstretched.

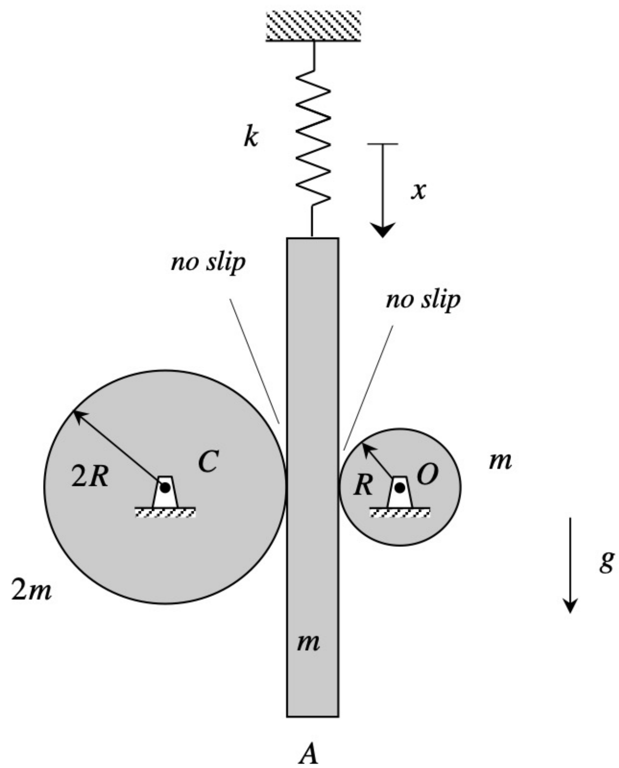
Find: For this problem, derive the dynamical equation of motion for the system in terms of the coordinate x.



Homework H6.B

Given: Block A (of mass m) is attached to a ground spring with a stiffness of k. Block A is also in rolling contact with a pair of homogeneous disks (of masses and radii of m and R, and $2m$ and $2R$) pinned to ground at their centers O and C. As A moves in the vertical direction, it does not slip with respect to the disks. Let x represent the vertical motion of A where $x = 0$ when the spring is unstretched.

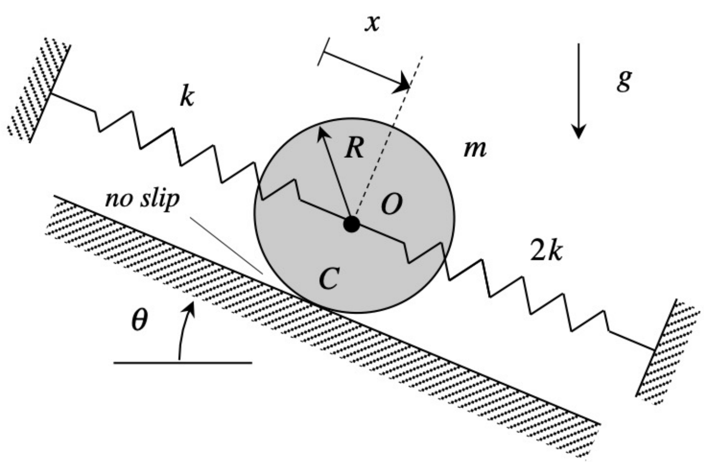
Find: For this problem, derive the dynamical equation of motion for the system in terms of the coordinate x.



Homework H6.C

Given: A homogeneous disk of mass m and outer radius R is able to roll without slipping on a rough, inclined surface. The center of the disk O is attached to ground with two springs of stiffnesses k and $2k$, as shown in the figure. Let x represent the motion of O along the incline as the disk rolls, where $x = 0$ when the springs are unstretched.

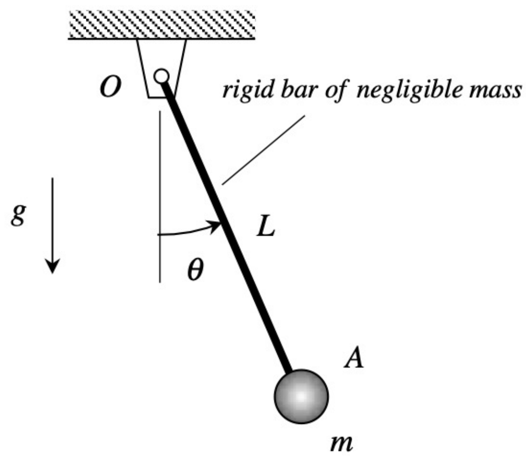
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate x;
(b) From the EOM, determine the static displacement of O, x_{st};
(c) Rewrite the EOM of the system in terms of the variable $z = x - x_{st}$, where z represents the position of O relative to its static equilibrium position; and,
(d) Determine the natural frequency of the system in terms of, at most, the given parameters of the problem: m, k and R .



Homework H6.D

Given: A simple pendulum is made up of a particle A (of mass m) being attached to the lower end of rod OA (of length L), with the other end of the rod pinned to ground at O. Let θ represent the angle of OA measured positively counterclockwise from the vertical, as shown in the figure.

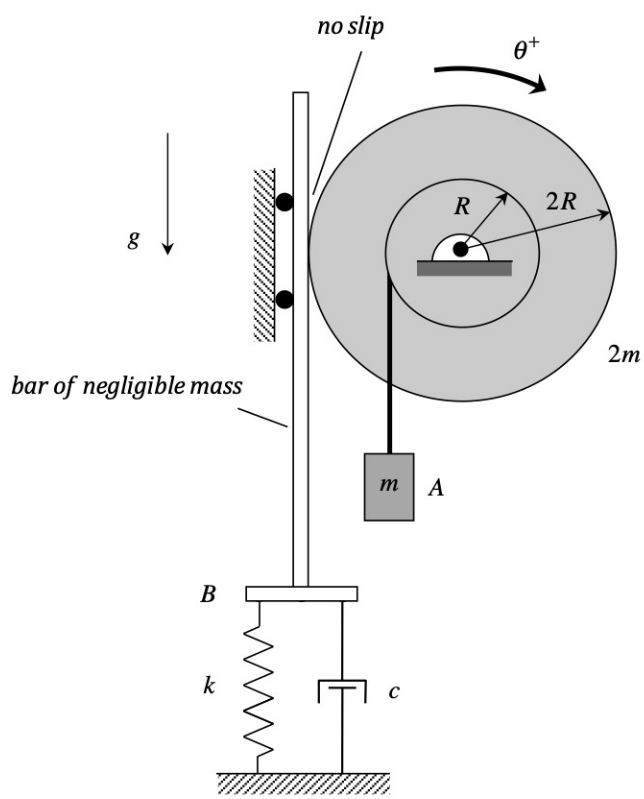
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate θ;
(b) Assume that the angle of rotation of OA is small for all motion, such that $sin\theta \approx \theta$. Determine the EOM of the pendulum using this small angle approximation;
(c) Determine the natural frequency of the system in terms of, at most, the given parameters of the problem: m, L and g; and,
(d) Determine the response of the system θ(t) if the pendulum is released from rest with $\theta(0) = \theta_0$.



Homework H6.E

Given: A stepped drum (of mass $2m$) is pinned to ground at its center, with the inner and outer radii of the drum given by R and $2R$, respectively. The radius of gyration of the drum about its center of mass is given by k_O. A cable is wrapped around the inner radius of the drum with the other end of the cable connected to particle A that has a mass of m. A second cable is wrapped around the outer radius of the drum with the other end of the cable being attached to connector B, with B, in turn, connected to a grounded spring of stiffness k and a grounded dashpot having a damping coefficient c, as shown in the figure. As the system moves, the cables are known to not slip on the drum nor do the cables do slack. Let θ represent the rotation of the drum, with θ being defined positive in the clockwise direction. The mass of B is to be considered to be negligible.

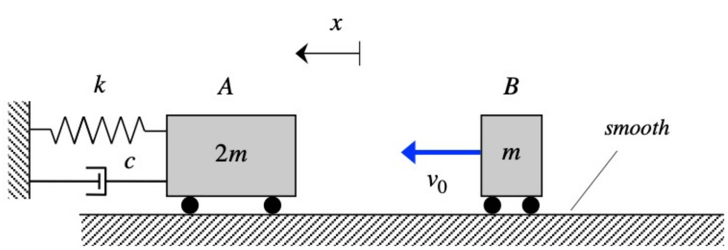
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate θ;
(b) From the EOM, determine the static rotation of the drum, θ_{st};
(c) Rewrite the EOM of the system in terms of the variable $z = \theta - \theta_{st}$, where z represents the rotation of the drum relative to its static equilibrium rotation;
(d) Determine the natural frequency of the system in terms of, at most, the given parameters of the problem: m, R, k and k_O; and,
(e) Determine the ratio of the parameters $c/\sqrt{km}$ that is required for critical damping to exist in the response of the system. Use $R/k_O = 1$.



Homework H6.F

Given: Particle A (having a mass of m) is attached to a grounded spring of stiffness k and a dashpot with a damping coefficient c. Let x represent the motion of A, with $x = 0$ when the spring is unstretched. At an instant when A is at rest and with the spring being unstretched/uncompressed (that is, when $x = 0$), particle B (of mass m), traveling with a speed of v_0, strikes A. On the impact with A, block B immediately sticks to block A.

Find: For this problem:
(a) Determine the speed of A immediately after B sticks to it.
(b) Derive the dynamical equation of motion (EOM) of A+B in terms of the coordinate x describing motion after the two blocks stick together;
(c) Determine undamped natural frequency ω_n, the damping ratio ζ and the damped natural frequency ω_d for the system;
(d) Determine the response $x(t)$ of the system after A and B stick together.

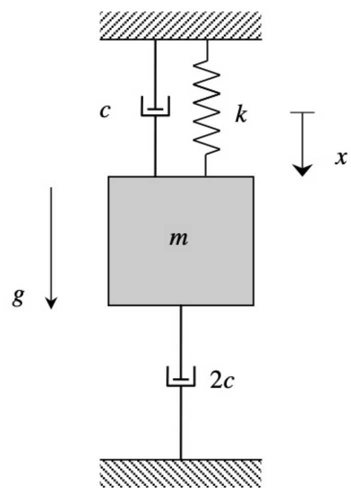


Use the following parameters in your analysis: $m = 10$ kg, $k = 3000$ N/m and $c = 360$ kg/s.

Homework H6.G

Given: A block of mass m is attached to a grounded spring (of stiffness k) and two dashpots (having damping coefficients c and $2c$), as shown in the figure. Let x represent the motion of the block, with $x = 0$ when the spring is unstretched.

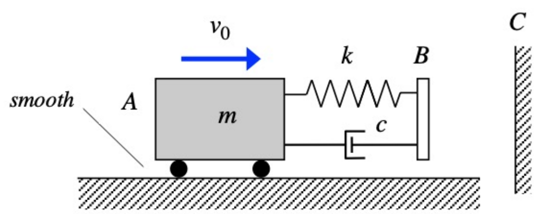
Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate x;
(b) Determine the static equilibrium position of the block, x_{st};
(c) Rewrite the EOM of the system in terms of the variable $z = x - x_{st}$, where z represents the position of the block relative to its static equilibrium position; and,
(d) Determine undamped natural frequency ω_n, the damping ratio ζ and the damped natural frequency ω_d for the system in terms of, at most, the parameters of the problem: m, c and k.



Homework H6.H

Given: A block of mass m is attached to a spring of stiffness k and a dashpot with a damping coefficient c, with the opposite ends of the spring and dashpot joined to connector B, where the mass of B can be considered to be negligible. A is initially traveling to the right with a speed of v_0 when it strikes a stationary wall at C. B immediately sticks to C after impact. Let x represent the motion of A after B has stuck to the wall, with x being measured positively to the right.

Find: For this problem:
(a) Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate x for motion occuring after B sticks to C;
(b) Determine undamped natural frequency ω_n, the damping ratio ζ and the damped natural frequency ω_d for the system; and,
(c) Determine the maximum compression of the spring after B impacts and sticks to C.

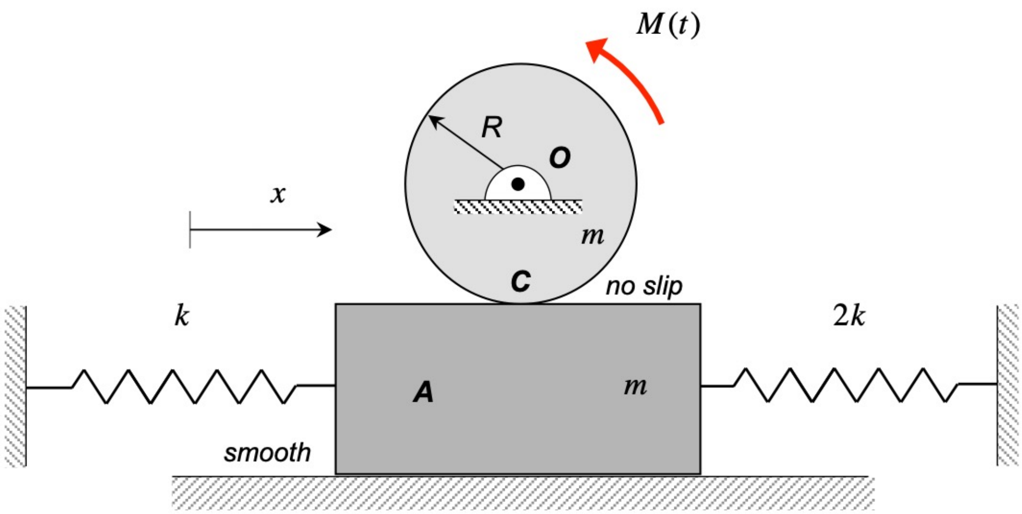


Use the following parameters in your analysis: $v_0 = 5$ m/s, $m = 10$ kg, $k = 4000$ N/m and $c = 240$ kg/s.

Homework H6.I

Given: A homogeneous disk of mass m and with an outer radius of R is pinned to ground at its center O. Block A (of mass m) is connected to grounded springs of stiffnesses k and $2k$, as shown, as it slides on a smooth horizontal surface. The disk and block A are in contact as the system moves, with the disk not slipping on the surface of block A. A torque $M(\mathrm{t})$ acts on the disk. Let x measure the position of block A, with x measured positively to the right and with $x = 0$ corresponding to the springs being unstretched.

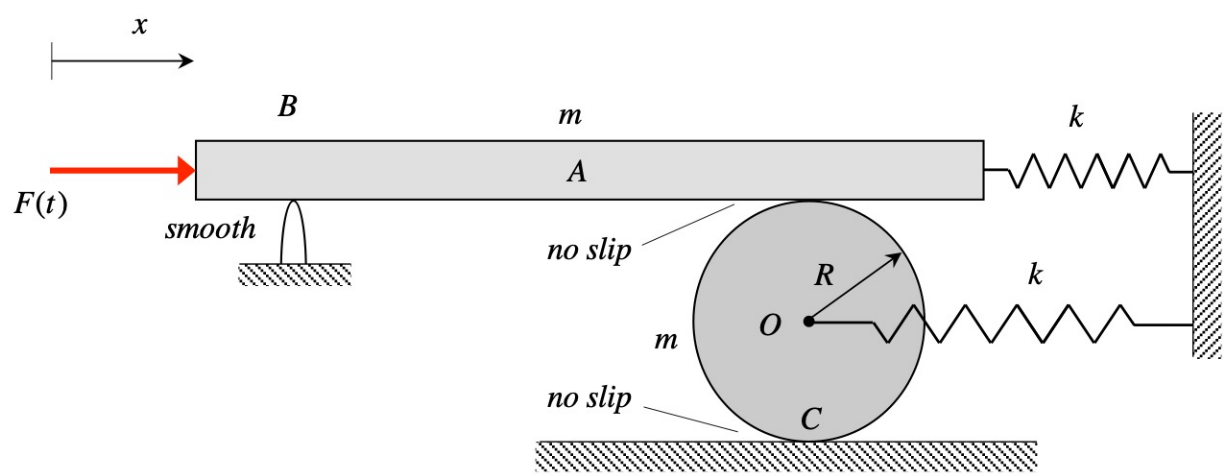
Find: For this problem:
(a) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x;
(b) Determine the natural frequency of the system; and,
(c) Determine the particular solution of the EOM corresponding to an input torque of $M(\mathrm{t}) = M_0 \sin\Omega t$.



Homework H6.J

Given: A homogeneous disk of mass m and with an outer radius of R rolls without slipping on a rough horizontal surface. A grounded spring of stiffness k is attached to the center O of the disk. Bar A (of mass m) is attached to a grounded spring of stiffness k. Bar A is supported by the top of the disk, with A not slipping on the disk as the system moves. A force $F(\text{t})$ acts on block A. Let x measure the position of A from its position when the springs are unstretched; i.e., $x = 0$ when the springs are unstretched.

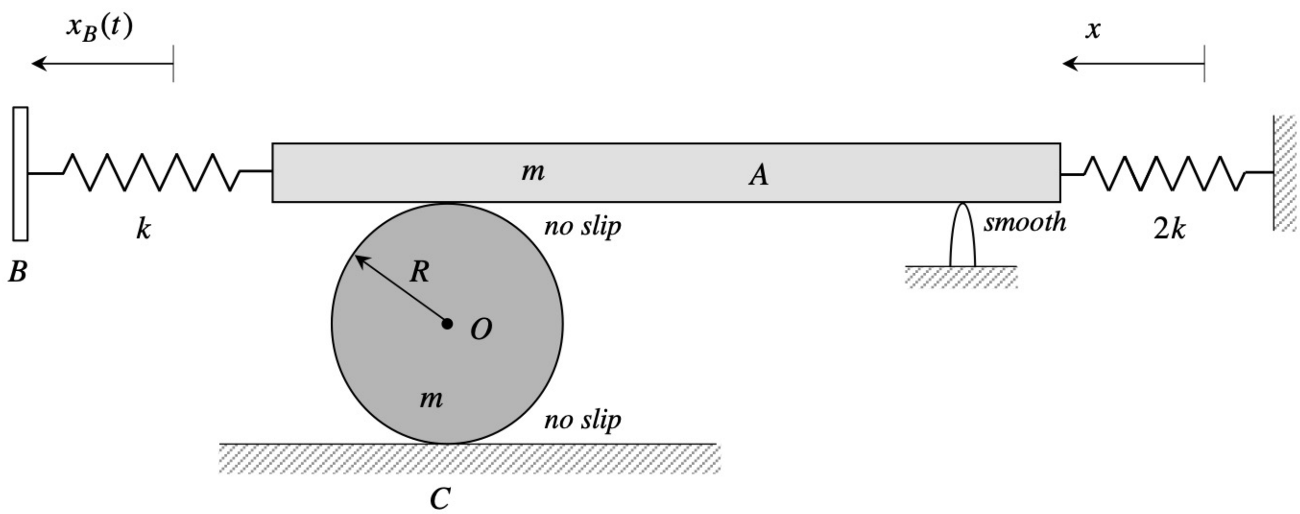
Find: For this problem:
(a) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x;
(b) Determine the natural frequency of the system; and,
(c) Determine the particular solution of the EOM corresponding to an input force of $F(\text{t}) = F_0 \sin\Omega t$.



Homework H6.K

Given: A homogeneous disk of mass m and with an outer radius of R rolls without slipping on a rough horizontal surface. Bar A (of mass m) is attached to a grounded spring of stiffness $2k$ on its right end. The left end of A is attached to a spring (of stiffness k) connected to a moveable base B. Bar A is supported by the top of the disk, with A not slipping on the disk as the system moves. The base B is given a prescribed motion of x_B(t) = b $sin\Omega t$. Let x measure the position of A from its position when the springs are unstretched; i.e., $x = 0$ when the springs are unstretched.

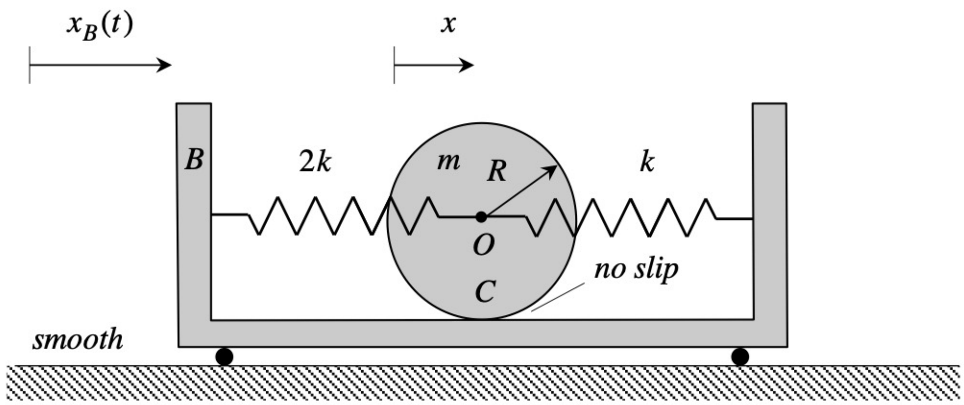
Find: For this problem:
(a) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x;
(b) Determine the natural frequency ω_n of the system;
(c) Determine the particular solution of the EOM; and,
(d) If $\Omega = 0.5\ \omega_n$, does A move in phase or out of phase with B?



Homework H6.L

Given: A homogeneous disk of mass m and with an outer radius of R rolls without slipping on a rough horizontal surface on cart B. The disk is connected at its center O with two springs (of stiffnesses $2k$ and k) to B, as shown in the figure. The base B is given a prescribed motion of x_B(t) = b $sin\Omega t$. Let x measure the position of O from its position when the springs are unstretched.

Find: For this problem:
(a) Derive the differential equation of motion (EOM) for the system in terms of the coordinate x;
(b) Determine the natural frequency ω_n of the system;
(c) Determine the amplitude of the motion described by particular solution of the EOM; and,
(d) For the motion found in (c) above, does O move in phase or out of phase with B?



Use the following parameters in your analysis: m = 200 kg, k = 10,000 N/m, b = 0.1 m, and Ω = 15 rad/s.

Homework H6.M

Given: Ground motion during an earthquake event is often represented by a prescribed motion x_B(t) for the ground. Structures attached to the ground experience an excitation due to this ground motion. Consider here a structure of mass m and support stiffness k attached to ground during an earthquake event. Let x represent the motion of this structure shown in the figure.

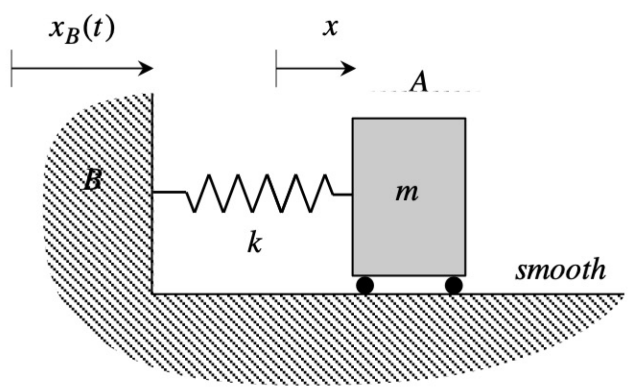
Find: For this problem:

(a) Derive the differential equation of motion (EOM) for the structure in terms of the coordinate x;

(b) Recognizing that the strain in the structure support is an important measure of the response and that strain is related to the relative motion between A and B, write the EOM in (a) above in terms of the relative coordinate z(t) = x(t) - x_B(t). Compare this EOM with that found in (a);

(c) Consider a ground motion of x_B(t) = b sinΩt. Determine the particular solution for the EOMs in (a) and (b); and,

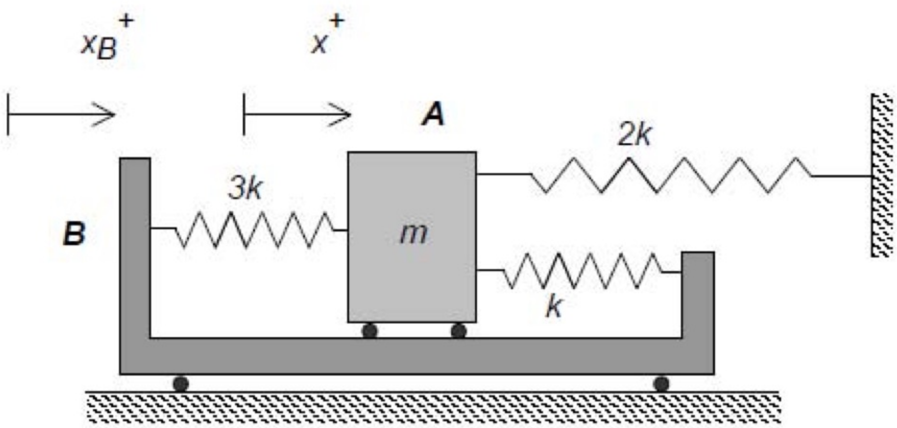
(d) Make sketches of the amplitude of response for these two solutions. Compare the results. Are they consistent?



Homework H6.N

Given: Block A (having a mass of m) is attached to cart B with two springs of stiffnesses $3k$ and k, as shown below. A third spring of stiffness $2k$ is attached between A and ground. Cart B is given a prescribed displacement of $x_B(t) = b \cos \omega t$. The absolute motion of block A is described by the coordinate x. All springs are unstretched when $x = x_B = 0$ m. Consider all surfaces to be smooth.

Find: For this problem:
(a) Derive the differential equation of motion for block A in terms of the coordinate x;
(b) Determine the numerical value for the natural frequency of this system; and
(c) Derive the particular solution of the system $x_p(t)$.



Use the following parameters in your analysis: $m = 12$ kg, $k = 800$ N/m, $b = 0.1$ m, and $\omega = 25$ rad/s.