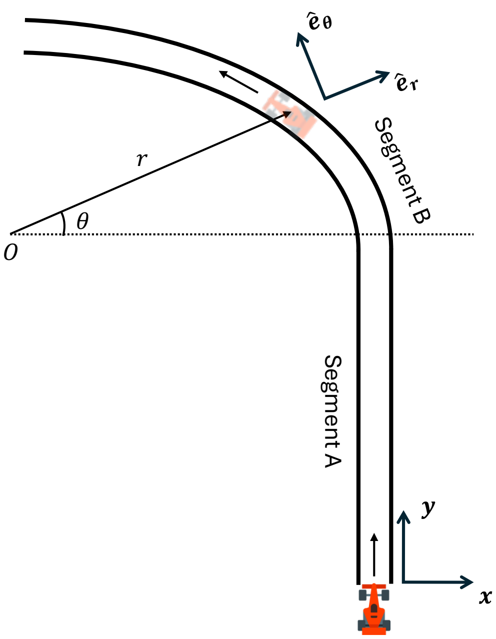
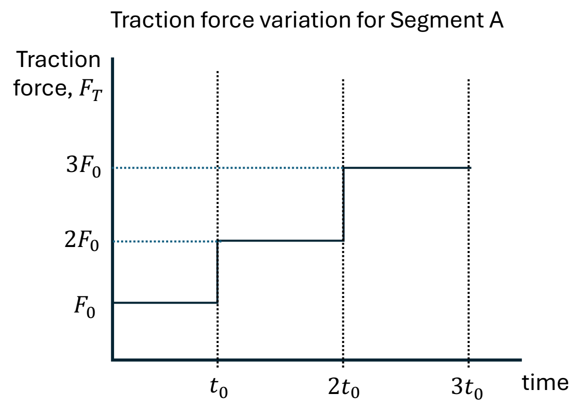
*ME 274: Basic Mechanics II* **Name** *(print)* **SOLUTION**

***Exam 2 - Spring 2025***

***PROBLEM NO. 1*** ***(****20 points) – PLEASE SCAN ALL PAGES*

After taking ME274, you decide to join a new Indy 500 race car team, TEAM DYNAMIC BOILERS, for this May 2025 edition. The team lead asks you to analyze the dynamics of your team's car by treating it as a particle and calculate its performance over the first two segments of the race. Segment A is a straight-line path, and Segment B is a curved path with a variable radius of curvature. Assume both Segments A and B are *flat* (i.e., no banking), mass of car is $m$, and that the car travels *without slipping* throughout Segments A and B.




**Segment A:** You are given a graph of how the traction force $F_T$ (resulting from friction that propels the car) varies versus time during segment A. For segment A, $F_T$ acts along the straight-line path of the car.

**A.** Draw an FBD of the car for Segment A in the plane of motion.

**B.** Assuming the car starts from rest, calculate the speed of the car at the end of segment A in terms of $F_0$, $t_0$, and $m$.

**Segment B:** For Segment B (beginning at the dashed line), the car's trajectory is defined by $\theta = a(t - 3t_0)$ and $r = b\theta^2 + c$, where time $t \geq 3t_0$, and $a$, $b$, and $c$ are constants. The total friction force ensures that the car stays on the given curved trajectory and travels with the kinematics provided.

**C.** Draw an FBD of the car for Segment B in the plane of motion.

**D.** Calculate the magnitude of total friction force acting on the car during Segment B as a function of time $t$ in terms of constants $m$, $a$, $b$, $c$, and $t_0$.

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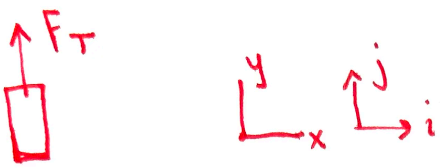
**PROBLEM NO. 1** (*continued) – PLEASE SCAN ALL PAGES*

A.




B: LIM:

$$m\vec{v}_2 = m\vec{v}_1 + \int_1^2 \Sigma \vec{F}\, dt$$

j direction:

$$mv_2 = mv_1 + \int_0^{3t_0} F_T\, dt \quad ; \quad 1 :\Rightarrow t = 0, \quad 2 :\Rightarrow t = 3t_0$$

$v_1$ (= 0) ; starts from rest

$$\Rightarrow v_2 = \frac{\int_0^{3t_0} F_T\, dt}{m} = \frac{6F_0 t_0}{m}$$

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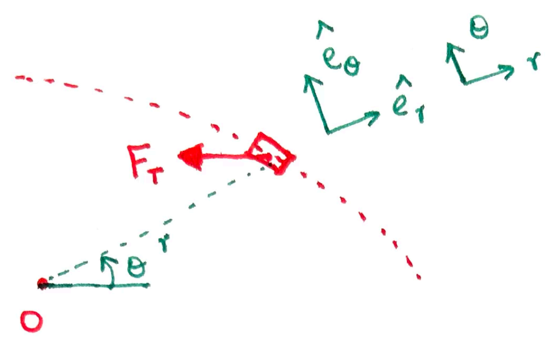
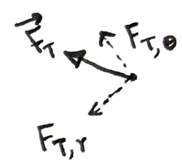
**PROBLEM NO. 1** *(continued) – PLEASE SCAN ALL PAGES*

C :




$\Rightarrow \vec{F}_T = F_{T,r}\hat{e}_r + F_{T,\theta}\hat{e}_\theta$

(total friction force)




D: Given kinematics:

$\theta = a(t - 3t_0)$

$r = b\theta^2 + c$ ; a, b, c, $t_0$ are constants.

$\Rightarrow \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$ [ ① ]

and $\vec{F}_T = m\vec{a} \Rightarrow |\vec{F}_T| = m|\vec{a}|$

We have :

$\dot{\theta} = a$ ; $\ddot{\theta} = 0$

$\dot{r} = 2b\theta\dot{\theta} = 2ab\theta = 2a^2b(t-3t_0)$ ; $\ddot{r} = 2b\theta\ddot{\theta} + 2b\dot{\theta}^2 = 0 + 2ba^2$, $\ddot{r} = 2a^2b$ ②

② → ①

$\therefore \vec{a} = [2a^2b - a^2(b\,a^2(t-3t_0)^2 + c)]\hat{e}_r + [2\cdot 2a^2b(t-3t_0)a]\hat{e}_\theta$

$\vec{a} = [2a^2b - a^2c - ba^4(t-3t_0)^2]\hat{e}_r + [4ba^3(t-3t_0)]\hat{e}_\theta$

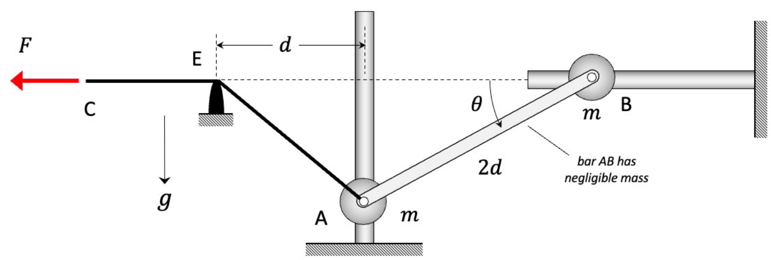
$\Rightarrow |\vec{F}_T| = m|\vec{a}|$

$= m\sqrt{[2a^2b - a^2c - ba^4(t-3t_0)^2]^2 + [4ba^3(t-3t_0)]^2}$ .

*ME 274: Basic Mechanics II* **SOLUTION**
***Exam 2 - Spring 2025***
***PROBLEM NO. 2** (20 points) – PLEASE SCAN ALL PAGES*

***Given:*** Particles A and B (each having a mass of *m*) are pinned to rigid bar AB that has negligible mass and a length of 2*d*. A and B are constrained to move along *smooth* vertical and horizontal guides, as shown in the figure. An inextensible cable is attached to particle A, and is pulled over a *smooth* guide E by a *constant* force *F* acting in a horizontal direction to the left. At the initial state, $\theta = \theta_0 = 30°$, and the system is at rest. The physical dimensions of A and B are small compared with the other lengths in the problem.

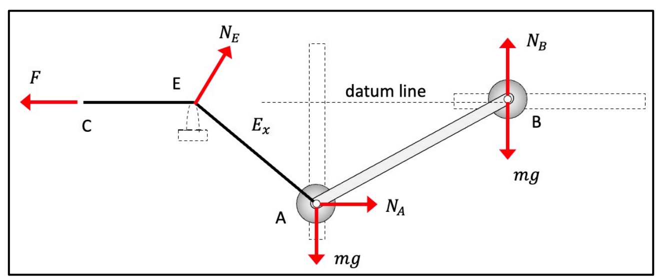
***Find:*** It is desired to know the speed of A at its position when $\theta = 0$. To this end, please complete the following solution steps. *For full credit, you must provide the correct responses within the correct steps provided below.*




***Solution***:

*STEP 1*: Free body diagram (FBD)

Complete the FBD provided of the system made up of A, B, bar AB and the cable. Define the location of any gravitational datum lines that will be needed for the system.




*NOTE*: Only F does work on the system, since the other forces are perpendicular to the paths of points on which they act, or their work will be included in the potential energy.

*STEP 2*: Kinetics

Write down the appropriate kinetics equation(s) and expressions for the terms in these equation(s). More space is provided on the next page. Do not do kinematics at this point…that is in the next step.

$$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$$

where:

$$T_1 = \tfrac{1}{2}mv_{A1}^2 + \tfrac{1}{2}mv_{B1}^2 = 0 \quad ; \quad \text{I.A.R.}$$

$$T_2 = \tfrac{1}{2}mv_{A2}^2 + \tfrac{1}{2}mv_{B2}^2$$

$$V_1 = -mgh_{A1}$$

$$V_2 = 0$$

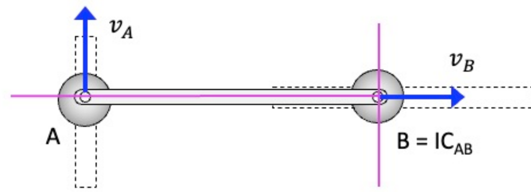
$$U_{1\to2}^{(nc)} = \int_1^2 (\vec{F}\cdot\hat{e}_t)ds_C = F\Delta_C$$

Therefore:

$$-mgh_{A1} + F\Delta_C = \tfrac{1}{2}mv_{A2}^2 + \tfrac{1}{2}mv_{B2}^2$$

*STEP 3*: Kinematics
Write down the kinematics equations that will be needed to solve this problem.

At Position 2, the IC for AB is at point B. Therefore: $v_{B2} = 0$




Also:

$$h_{A1} = 2d\sin\theta_0 = d$$

$$\Delta_C = \sqrt{d^2 + (2d\sin\theta_0)^2} - d = d(\sqrt{2} - 1)$$

*STEP 4*: Solve
Using the equations from STEPS 2 and 3, determine the speed of A for the position corresponding to $\theta = 0$. Write your final answer in terms of, *at most*: $F$, $m$, $d$ and $g$.

Substituting the kinematics into the W/E equation gives:

$$-mgd + Fd\left(\sqrt{2} - 1\right) = \frac{1}{2}mv_{A2}^2 \quad \Rightarrow$$

$$v_{A2} = \sqrt{-2gd + 2Fd\left(\sqrt{2} - 1\right) / m}$$

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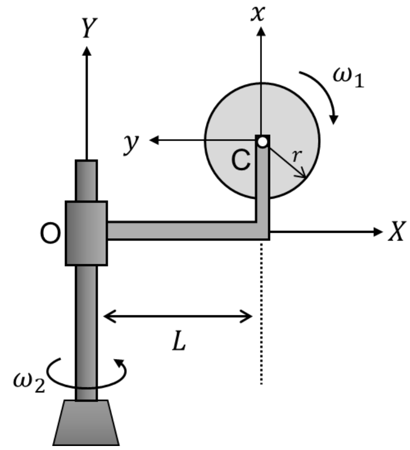
***PROBLEM NO. 3*** **(***20 points) – PLEASE SCAN ALL PAGES*

***PART 3.A*** *(4 points)*

A disc rotates with a constant rate of $\omega_1$ (rad/s) with respect to the arm OC. The arm OC rotates about a FIXED vertical axis with a constant rate of $\omega_2$ (rad/s). The *XYZ* axes are fixed and the *xyz* are rotating. At this instant, the rotating (*xyz*) and fixed (*XYZ*) axes are 90° from each other as shown.




***3.A.1*** If the *xyz* axes are **attached to the arm OC** at point C, circle the correct expression for the angular velocity of the *xyz* rotating coordinate system (1 point).

a) $\vec{\omega} = \omega_2 \hat{J}$

b) $\vec{\omega} = -\omega_1 \hat{k} + \omega_2 \hat{J}$

c) $\vec{\omega} = -\omega_1 \hat{K}$

d) $\vec{\omega} = -\omega_1 \hat{K} + \omega_2 \hat{J}$

***3.A.2*** If the *xyz* axes are **attached to the disc** at point C, circle the correct expression for the angular velocity of the *xyz* rotating coordinate system **for all time** (1 point).

a) $\vec{\omega} = \omega_2 \hat{J}$

b) $\vec{\omega} = -\omega_1 \hat{k} + \omega_2 \hat{J}$

c) $\vec{\omega} = -\omega_1 \hat{K}$

d) $\vec{\omega} = -\omega_1 \hat{K} + \omega_2 \hat{J}$

***3.A.3*** If the *xyz* axes are **attached to the disc** at point C, circle the correct expression for the angular acceleration of the *xyz* rotating coordinate system **at this instant** (2 points).

a) $\vec{\alpha} = -\omega_1 \omega_2 \hat{I}$

b) $\vec{\alpha} = 0$

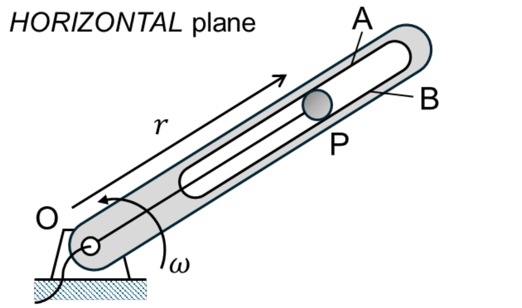
c) $\vec{\alpha} = -\omega_1 \omega_2 \hat{K}$

d) $\vec{\alpha} = \omega_1^2 \hat{J}$

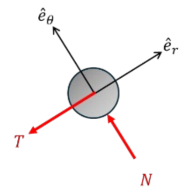
$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = -\dot{\omega}_1 \hat{k} - \omega_1 \dot{\hat{k}} + \dot{\omega}_2 \hat{J} + \omega_2 \dot{\hat{J}} = -\omega_1 \dot{\hat{k}}$$
$$= -\omega_1 (\vec{\omega} \times \hat{k}) = -\omega_1 (-\omega_1 \hat{k} + \omega_2 \hat{J}) \times \hat{k}$$
$$= -\omega_1 (-\omega_1 \hat{K} + \omega_2 \hat{J}) \times \hat{K} = -\omega_1 \omega_2 \hat{I}$$

*PART 3.B* *(3 points)*

Particle P slides within a smooth straight slot in an arm and is pulled towards end O by a string at a constant rate $\dot{r} = -1$ (m/s). The arm rotates within a HORIZONTAL plane about O at a constant rate $\omega = 1$ (rad/s).




***3.B.1*** Draw a FBD of particle P using the schematic below (1 point).




***3.B.2*** Write Newton's 2nd Law for particle P along the $\hat{e}_\theta$ direction (1 point).

$$\sum F_\theta = N = m(a_\theta) = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -2m$$

***3.B.3*** Circle the correct statement (1 point).

a) P is in contact with side A of the slot
b) P is in contact with side B of the slot
c) Neither a) nor b)

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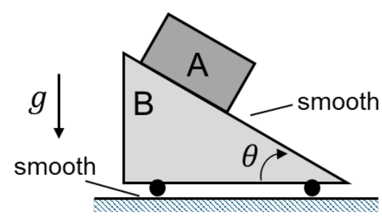
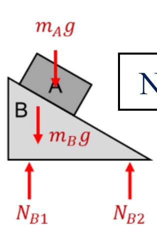
*PART 3.C* *(4 points)*

Wedge-shaped block B can move along a smooth horizontal surface. Block A can slide on the smooth inclined surface of B. The system is released from rest.



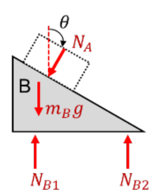

***PART 3.C.1*** Draw FBDs for block B and for the system of blocks A and B using the schematics below. (2 points)

FBD of system A+B




No NC forces do work

FBD of B



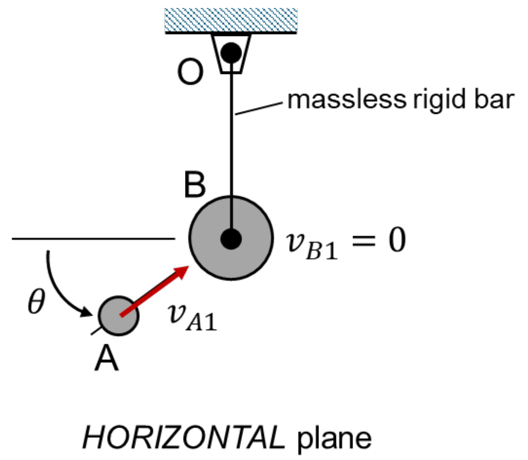

$N_A sin\theta$ does NC work

***PART 3.C.2*** Circle the correct TRUE/FALSE response for the statements regarding the motion of the system. (2 points).

a) Mechanical energy is conserved for block B alone: TRUE or FALSE
b) Mechanical energy is conserved for the system of blocks A and B: TRUE or FALSE

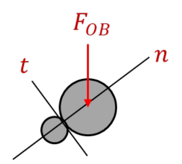
***PART 3.D*** (5 points)

Particle B (mass $3m$) is attached to massless rigid bar BO (length $L$) pinned to the ground at O. Particle A (mass $m$) strikes the **stationary** particle B with a speed of $v_{A1}$ in the direction shown. After impact, the particle A **sticks** to particle B. All surfaces are *smooth* and all motion occurs in the HORIZONTAL plane. For this impact event:




***3.D.1*** Draw the FBD for the system of particles A and B using the schematic below, indicating the correct line of impact ($n$) and plane of contact ($t$) axes. (1 point)

*FBD of system A+B*




***3.D.2*** Circle the correct TRUE/FALSE response for the statements regarding the motion of the system. (2 points).

a) Mechanical energy for system A + B is conserved: TRUE or FALSE
b) Linear momentum in the $n$-direction for system A+B is conserved: TRUE or FALSE
c) Linear momentum in the $t$-direction for system A+B is conserved: TRUE or FALSE
d) Angular momentum about point O for system A+B is conserved: TRUE or FALSE

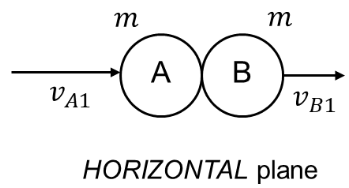
***3.D.3*** Circle the correct angular speed of the system immediately after impact (2 points).

a) $\omega_2 = v_{A1} sin\theta/4L$ b) $\omega_2 = v_{A1} cos\theta/2L$ c) $\omega_2 = v_{A1} sin\theta/2L$ d) $\omega_2 = v_{A1} cos\theta/4L$

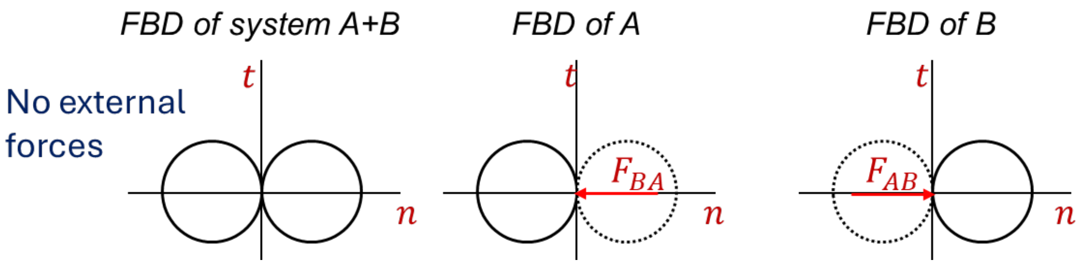
$$\vec{H}_{O1} = \vec{H}_{O2}$$
$$mLv_{A1}cos\theta = mL^2\omega_2 + 3mL^2\omega_2$$
$$\omega_2 = v_{A1}cos\theta/4L$$

***PART 3.E*** *(4 points)*

Particles A and B have the same mass $m$ and are traveling to the right at speeds $v_{A1} > v_{B1}$.




***3.E.1*** Draw the FBDs for the system of particles A and B using the schematics below, indicating the correct line of impact ($n$) and plane of contact ($t$) axes. (1.5 points)




***3.E.2*** If, after impact, particle B moves to the right at speed $v_{B2} = v_{A1}$ and particle A moves to the right at speed $v_{A2} = v_{B1}$, what statement about the coefficient of restitution is correct? (1.25 points)

a) $e = 1$    b) $e < 1$    c) $e > 1$    d) $e = 0$

$e = (v_{B2} - v_{A2})/(v_{A1} - v_{B1}) =$
$(v_{A1} - v_{B1}/(v_{A1} - v_{B1}) =1$

***3.E.3*** If instead, after impact, particle B and particle A stick together, what is the speed $v_2$ of the two particles after impact? (1.25 points)

b) $v_2 = v_{A1} - v_{B1}$    b) $v_2 = v_{A1} + v_{B1}$    c) $v_2 = 2(v_{A1} + v_{B1})$    d) $v_2 = \frac{1}{2}(v_{A1} + v_{B1})$

LM conserved in x for system A-B:

$$mv_{A1} + mv_{B1} = 2mv_2$$

$$v_2 = \frac{1}{2}(v_{A1} + v_{B1})$$