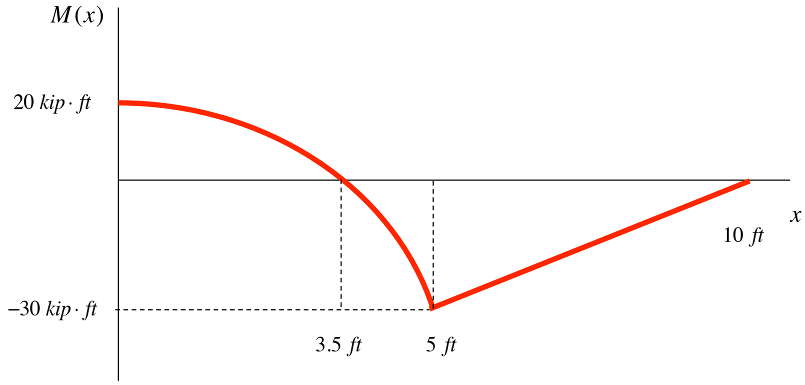
ME 323 - Final Exam **Name** ______________________________

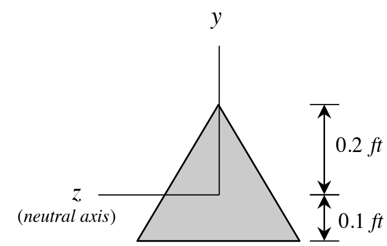
December 15, 2015 **Instructor** (circle)

Krousgrill *11:30AM-12:20PM* *Gonzalez* *12:30-1:20PM*

Ghosh *2:30-3:20PM* *Zhao* *4:30-5:20PM*

PROBLEM NO. 4 – Part A (2 points max.)





beam cross-section

The longitudinal axis of a beam is aligned with the x axis. The beam has a triangular cross-section, as shown above. The loading on the beam produces the bending moment diagram for M(x) shown above.

a) Provide the x-y coordinates of the point on the beam that experiences the largest magnitude *compressive* normal stress.

b) Provide the x-y coordinates of the point on the beam that experiences the largest magnitude *tensile* normal stress.

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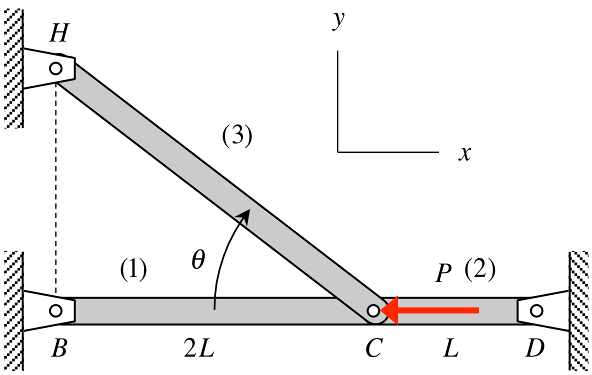
Ghosh 2:30-3:20PM	*Zhao* 4:30-5:20PM

PROBLEM NO. 4 – Part D (6 points max.)



The truss shown above is made up of truss elements (1), (2) and (3). A horizontal force P is applied to joint C.

(i) Draw a free body diagram (FBD) for joint C.

(ii) Let σ_3 be the axial stress in element (3). Circle the correct description of σ_3 below (consider your FBD from above):

a) $\sigma_3 > 0$ (*tension*)

b) $\sigma_3 = 0$

c) $\sigma_3 < 0$ (*compression*)

(iii) Let v_C be the *vertical* component of displacement of joint C. Circle the correct description of v_C below:

a) $v_C > 0$ (*UP*)

b) $v_C = 0$

c) $v_C < 0$ (*DOWN*)

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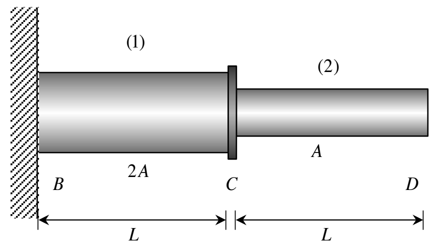
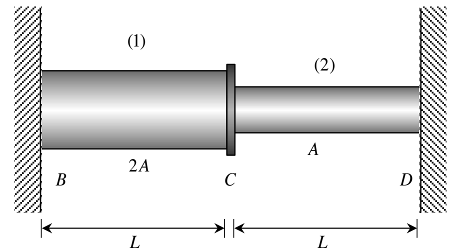
December 15, 2015 | **Instructor** (circle) *Krousgrill 11:30AM-12:20PM* | *Gonzalez 12:30-1:20PM* | *Ghosh 2:30-3:20PM* | *Zhao 4:30-5:20PM*

PROBLEM NO. 4 – Part E (6 points max.)

Rod I



Rod II



i) Rod I shown above is made up of a material with a Young's modulus of E and thermal expansion coefficient α. The cross-sectional areas of elements (1) and (2) are given by 2A and A, respectively. Both elements are heated in such a way that each has a temperature increase of ΔT. Let σ_1 and σ_2 represent the stress in elements (1) and (2), respectively. Circle the correct description below of these two stresses:

a. $|\sigma_1| > |\sigma_2|$

b. $|\sigma_1| = |\sigma_2|$

c. $|\sigma_1| < |\sigma_2|$

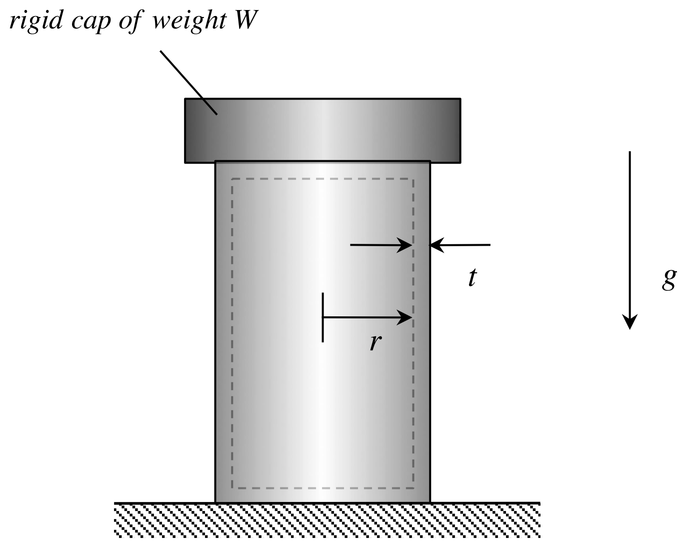
ii) Rod II is exactly the same as Rod I, except its right end is attached to a rigid wall. Again, both elements are heated to the same temperature increase ΔT. Circle the correct description below of the stresses in the two elements:

a. $|\sigma_1| > |\sigma_2|$

b. $|\sigma_1| = |\sigma_2|$

c. $|\sigma_1| < |\sigma_2|$

PROBLEM NO. 4 – Part F (3 points max.)



A thin-walled tank (having an inner radius of r and wall thickness t) constructed of a ductile material contains a gas with a pressure of p. A rigid cap of weight $W = 3\pi pr^2$ rests on top of the tank. Ignore the weight of the tank. Determine the principal components of stress in the wall of the tank.

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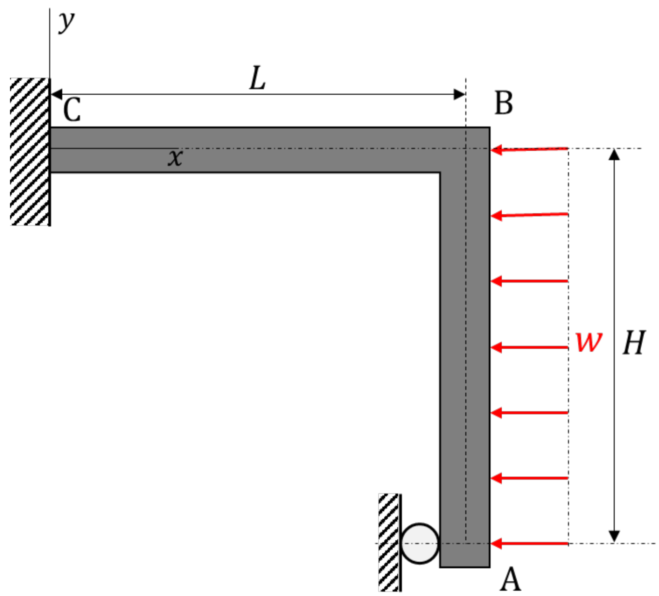
Problem 1 ***(28 points):*** The member shown below is fixed to ground at end C, is supported by a roller at end A, and is subject to a distributed load. It is known that the member has a uniform cross section area A, second area moment I, and a shear shape factor f_s. The member is composed of a material whose Young's modulus is E and shear modulus G.

Using Castigliano's theorem, determine the reaction on the beam at A. Please include the shear, normal and flexural strain energies in your solution. Leave your answer in terms of $w, L, H, E, G, A, I,$ and f_s.



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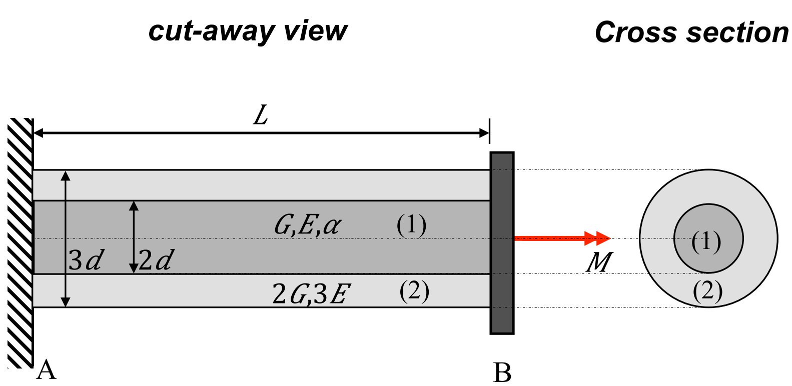
Problem 2 ***(28 points):*** A composite shaft of length L is composed of an inner shaft (1) and an outer shaft (2). The shaft is fixed to a wall at A, and to a rigid connector at B which ensures the same axial and torsional deformation for the two shafts. A torque M is applied to the rigid connector. Simultaneously, the temperature of shaft (1) is increased from the ambient by ΔT, while shaft (2) is maintained at ambient temperature.

The load and material properties are the following: $M = 1000\ Nm, L = 1\ m, G = 50\ GPa, E = 100\ GPa, d = 20\ mm, \alpha = 5\times 10^{-5}/°\text{C}$, while ΔT is unknown, and is desired to be determined.

a) Using an appropriate FBD, write down the equilibrium conditions that relate the reactions of the two shafts.
b) Write down the compatibility conditions that relate the elongations and the rotations of shafts (1) and (2).
c) Determine the location on the cross section of shafts (1) and (2) where both the axial and shear stresses are maximum. Also, find the magnitude of the stresses. Leave your answers in terms of ΔT.
d) Calculate the maximum rise in temperature ΔT (Celsius), such that the absolute maximum shear stress $\tau_{max,abs}$ in either shaft does not exceed $50\ MPa$.



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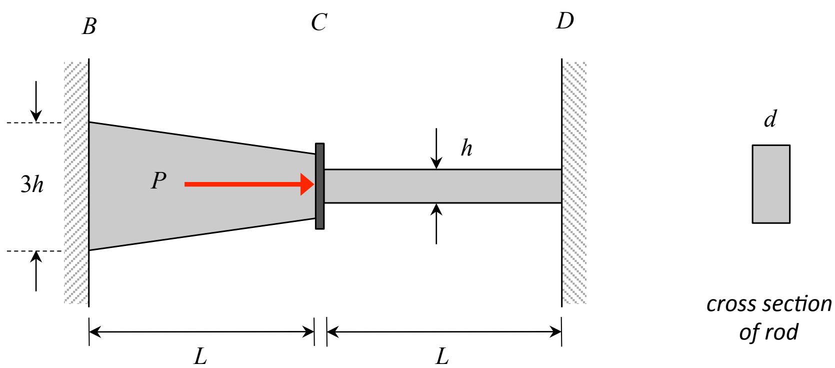
Problem 3 – *PART A (10 points)*:
A rod is made up of two rectangular cross-section segments BC and CD, with each segment having a length of L and a constant depth dimension of the cross section d. Segment CD has a thickness of h, whereas segment BC is tapered with its thickness going from 3h at B to 2h at C. The segments are joined by a rigid connector C, and the rod is connected to fixed walls at ends B and D. A load P acts at connector C. A finite element model of the rod using a single element per segment produces the following equilibrium equations:

$$\left[K\right]\vec{u} = \vec{F}$$

For this problem, you are asked to:

a) determine the *stiffness matrix* $\left[K\right]$ and the *loading vector* $\vec{F}$ after the enforcement of the boundary conditions for the problem.

b) determine the *axial displacement of connector C* using the finite element equilibrium equations in a) above.



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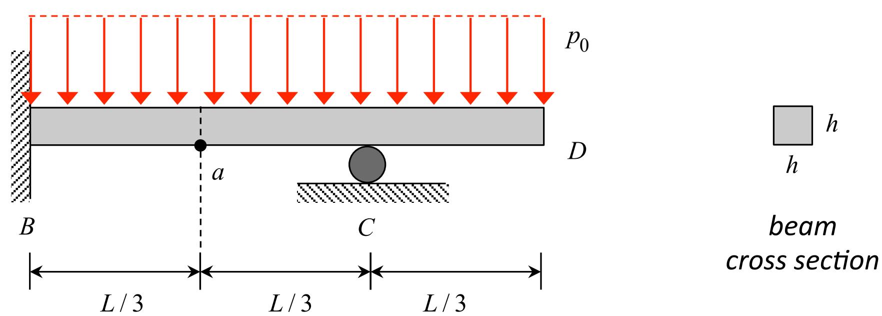
Problem 3 – *PART B (10 points)*:
A homogeneous beam (of length L, having a square cross section and made up of a material with an elastic modulus of E) is fixed to a rigid wall at end B and is supported by a roller at location C. A line load of constant magnitude p_0 acts along the length of the beam. Determine the *shear and normal components of stress* on the bottom side of the beam at point "a".

<u>*HINT*</u>: You may use the *superposition method* in the deflection analysis for this problem.



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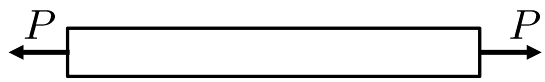
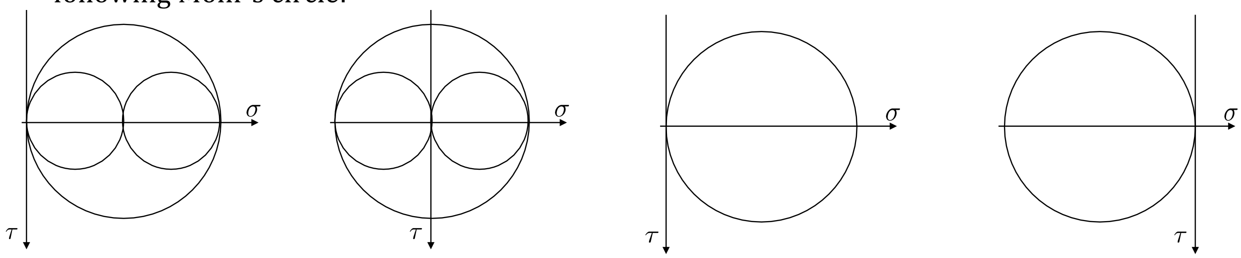
Problem 4 – ***PART A (6 points):***

A *steel* cylindrical specimen is subjected to tension until <u>*ductile failure*</u> is observed.



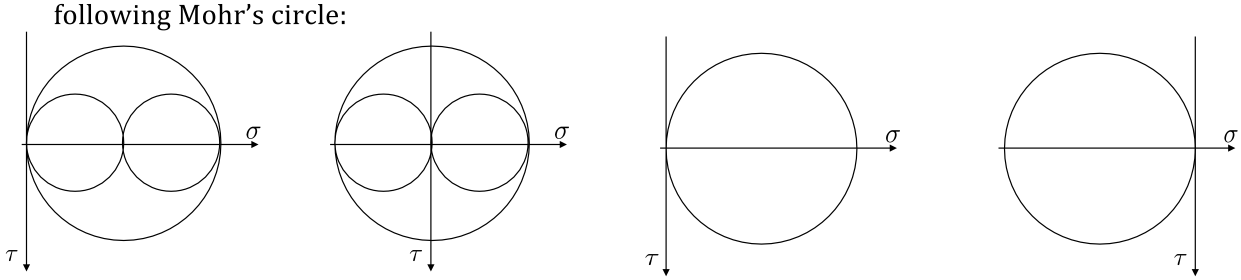
Circle the correct answer in the following statements:

(a) The state of stress of a point on the *top fiber* of the specimen is represented by the following Mohr's circle:



(i) (ii) (iii) (iv)

(b) The state of stress of a point on the *bottom fiber* of the specimen is represented by the following Mohr's circle:



(i) (ii) (iii) (iv)

(c) Before (i) / After (ii) necking occurs, a crack oriented at 0° (iii) / 45° (iv) / 90° (v) from the direction of loading will develop.

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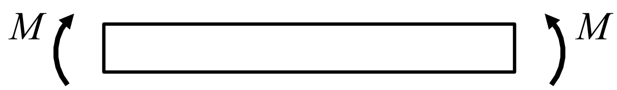
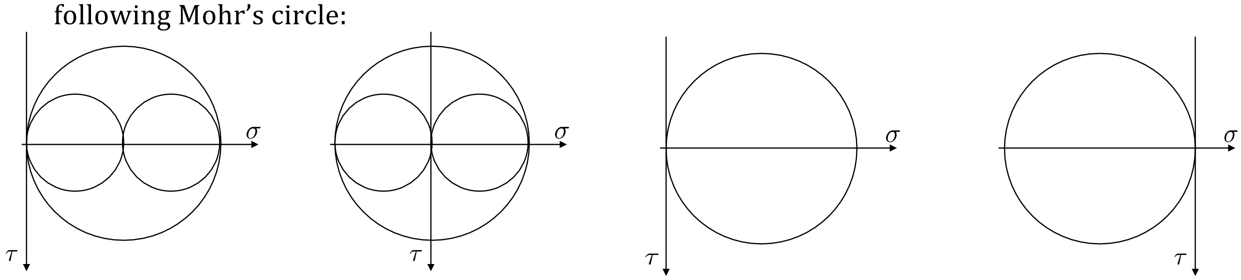
Problem 4 – *PART B (6 points):*

A *concrete* slab is subjected to bending until *brittle failure* is observed. As in most brittle materials, the ultimate compressive strength is larger than the ultimate tensile strength.



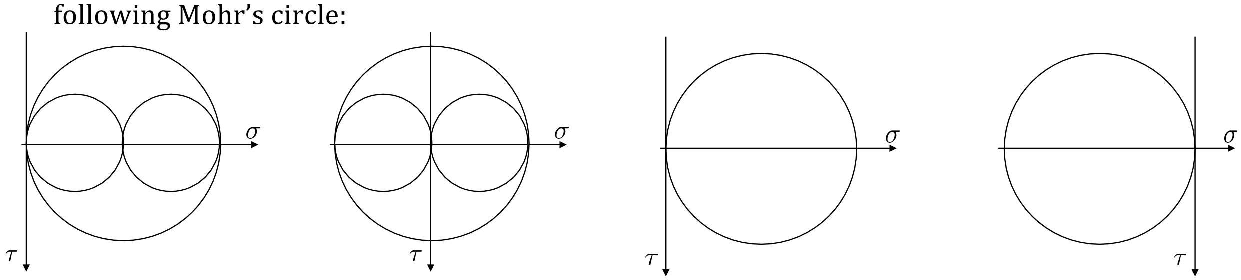
Circle the correct answer in the following statements:

(a) The state of stress of a point on the *top face* of the specimen is represented by the following Mohr's circle:



(i) (ii) (iii) (iv)

(b) The state of stress of a point on the *bottom face* of the specimen is represented by the following Mohr's circle:



(i) (ii) (iii) (iv)

(c) The first cracks develop on the top face / bottom face and are oriented vertically/ obliquely / horizontally.

top face (i) / bottom face (ii); vertically (iii) / obliquely (iv) / horizontally (v)

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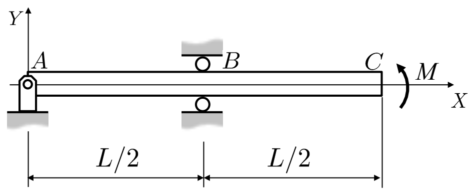
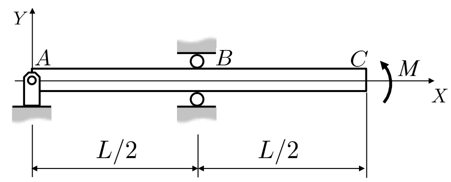
Problem 4 – ***PART C (8 points):***



Beam (i) – Steel



Beam (ii) – Aluminum

Beams (i) and (ii) shown above are identical, except that beam (i) is made up of steel and beam (ii) is made up of aluminum. Note that $E_{steel} \geq E_{aluminum}$.

Let $|\sigma|_{max,(i)}$ and $|\sigma|_{max,(ii)}$ represent the maximum magnitude of flexural stress in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $|\sigma|_{max,(i)} > |\sigma|_{max,(ii)}$

b) $|\sigma|_{max,(i)} = |\sigma|_{max,(ii)}$

c) $|\sigma|_{max,(i)} < |\sigma|_{max,(ii)}$

Let $|\tau|_{max,(i)}$ and $|\tau|_{max,(ii)}$ represent the maximum magnitude of the xy-component of shear stress in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $|\tau|_{max,(i)} > |\tau|_{max,(ii)}$

b) $|\tau|_{max,(i)} = |\tau|_{max,(ii)}$

c) $|\tau|_{max,(i)} < |\tau|_{max,(ii)}$

Let $|\delta|_{max,(i)}$ and $|\delta|_{max,(ii)}$ represent the maximum magnitude of deflection in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $|\delta|_{max,(i)} > |\delta|_{max,(ii)}$

b) $|\delta|_{max,(i)} = |\delta|_{max,(ii)}$

c) $|\delta|_{max,(i)} < |\delta|_{max,(ii)}$

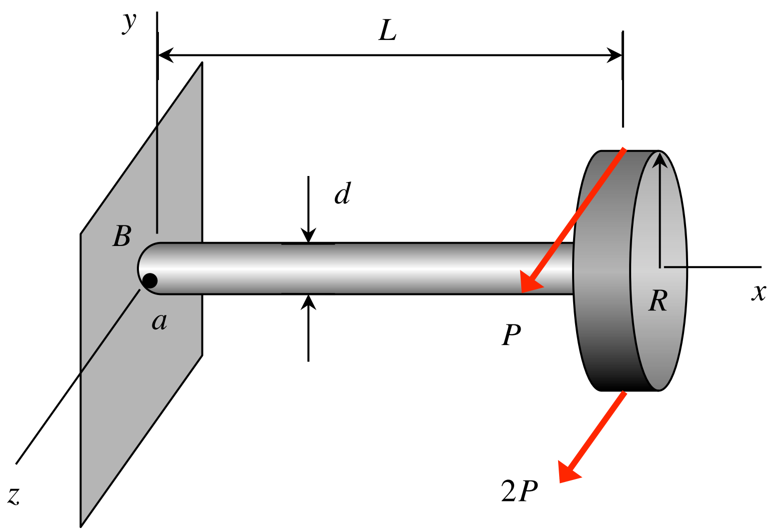
Let $|B_y|_{(i)}$ and $|B_y|_{(ii)}$ represent the vertical reaction at B in beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $|B_y|_{(i)} > |B_y|_{(ii)}$

b) $|B_y|_{(i)} = |B_y|_{(ii)}$

c) $|B_y|_{(i)} < |B_y|_{(ii)}$

PROBLEM NO. 1 – 20 points max.

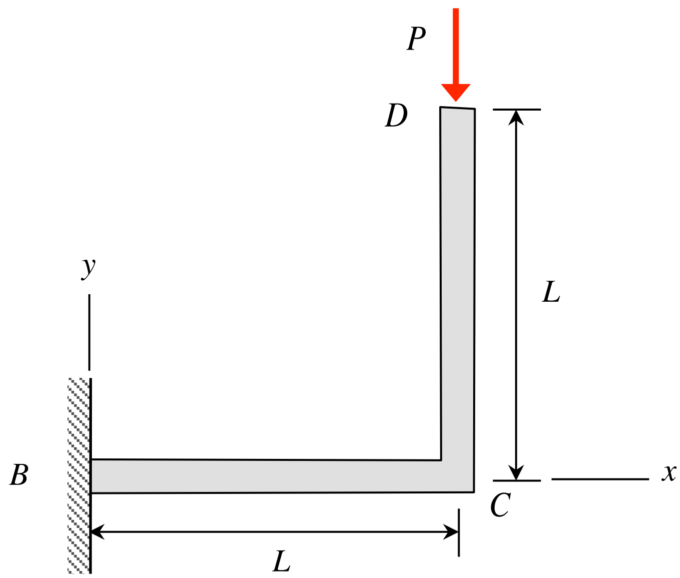


A circular cross-section shaft (of diameter d and length L) is built into a fixed wall at end B. A rigid pulley is attached to the other end of the shaft. A cable (not shown) wrapped around the pulley applies a pair of tension forces of P and 2P on the pulley, as shown in the figure (note that these two forces both act in the positive z-direction). Let E and G represent the Young's modulus and shear modulus, respectively, for the ductile material of the shaft. Also, σ_Y is the yield strength of the material.

a) Determine the state of stress at point "a" on the shaft (point "a" is at end B on the z-axis).
b) If the shaft is to fail at point "a", determine the value for P at which failure will occur. Use the maximum distortional energy theory for failure prediction.

Use $L/R = 3$ in your calculations.

PROBLEM NO. 2 – 20 points max.

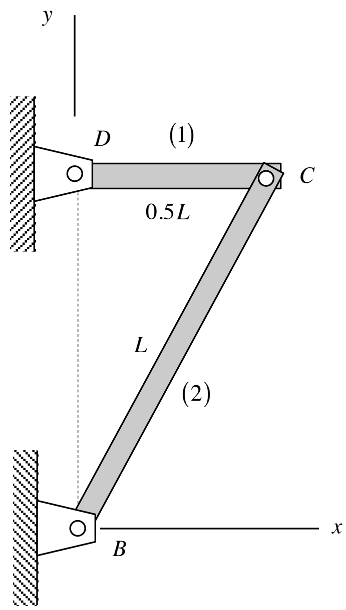


A thin, L-shaped bracket is rigidly attached to a fixed wall at B. A concentrated force P is applied to end D of the bracket. The material of the bracket has a Young's modulus of E, a shear modulus of G, and the circular cross-section of the bracket has an area of A and second area moment I everywhere. Ignore the weight of the bracket. Use Castigliano's theorem to:

a) Determine the vertical deflection of end D for this loading.

b) Determine the horizontal deflection of end D for this loading

PROBLEM NO. 3 – 8 points max.

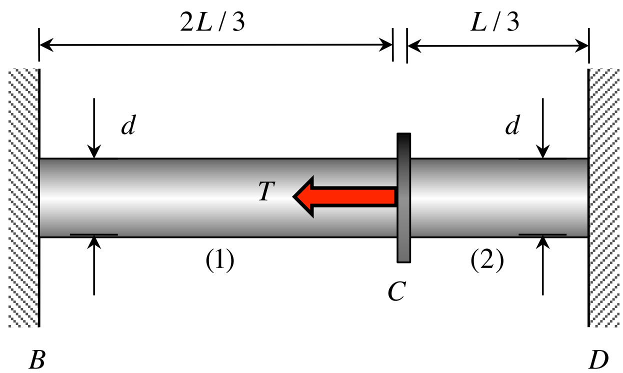


A truss is made up of members (1) and (2) as shown above, with each member being composed of a material with a Young's modulus of E and thermal expansion coefficient of α, and having a cross-sectional area of A. With the members being initially unstressed and unstrained, the temperature of (1) is *increased* by an amount of ΔT and the temperature of (2) is *decreased* by an amount of $2\Delta T$.

a) Using equations, show that members (1) and (2) remain unstressed after the temperature change described above.

b) What is the strain in each member after the temperature change?

PROBLEM NO. 4 – 12 points max.



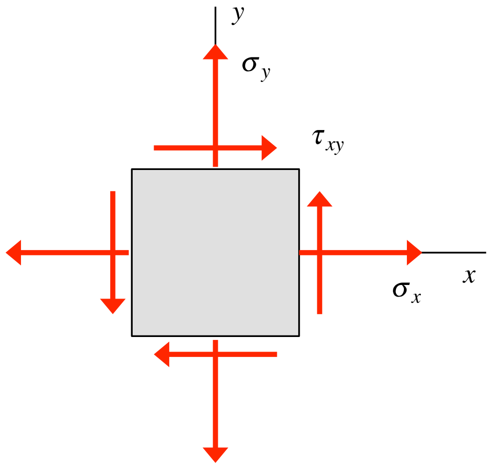
The shaft shown above is made up of solid components (1) and (2), with each component having the same diameter d. The components are joined by the rigid connector C as shown above, with a torque T being applied to connector C.

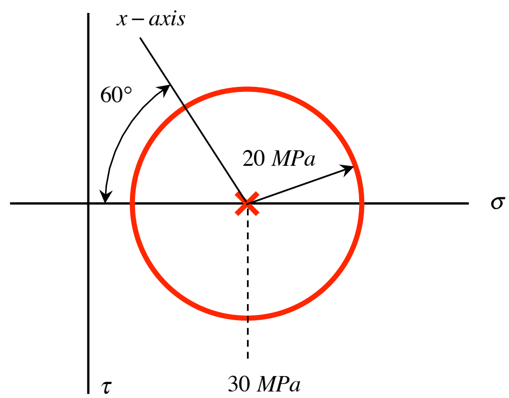
a) If the two components are made up of the same material (i.e., having the same shear modulus, $G_1 = G_2 = G$) what is the value of $|T_1 / T_2|$ where T_1 and T_2 are the torques carried by components (1) and (2), respectively?

b) If both components are replaced with a material having half of the original shear modulus (i.e., $G_1 = G_2 = 0.5G$), what is the new value of $|T_1 / T_2|$?

c) If, instead, the material for component (1) is kept the same as in (a) ($G_1 = G$) and the shear modulus for component (2) is changed to $G_2 = 0.5G$, what is the new value of $|T_1 / T_2|$?

PROBLEM NO. 7 – 8 points max.





The Mohr's circle for a stress state is presented above.

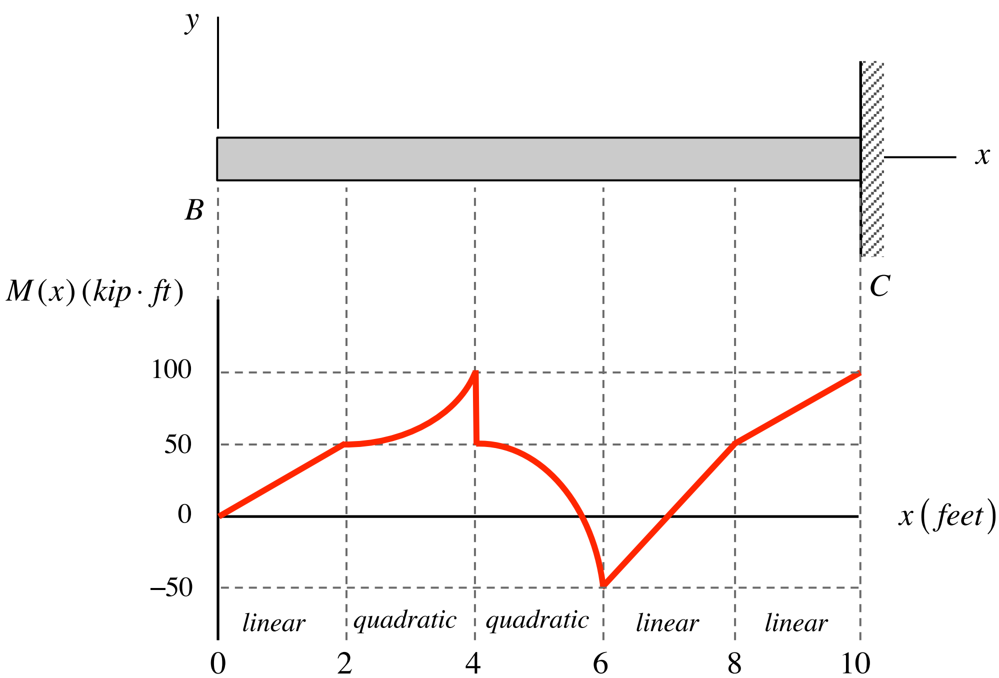
a) Show the location of the y-axis in the Mohr's circle above.

b) Determine the principal stresses and the absolute maximum shear stress for this state.

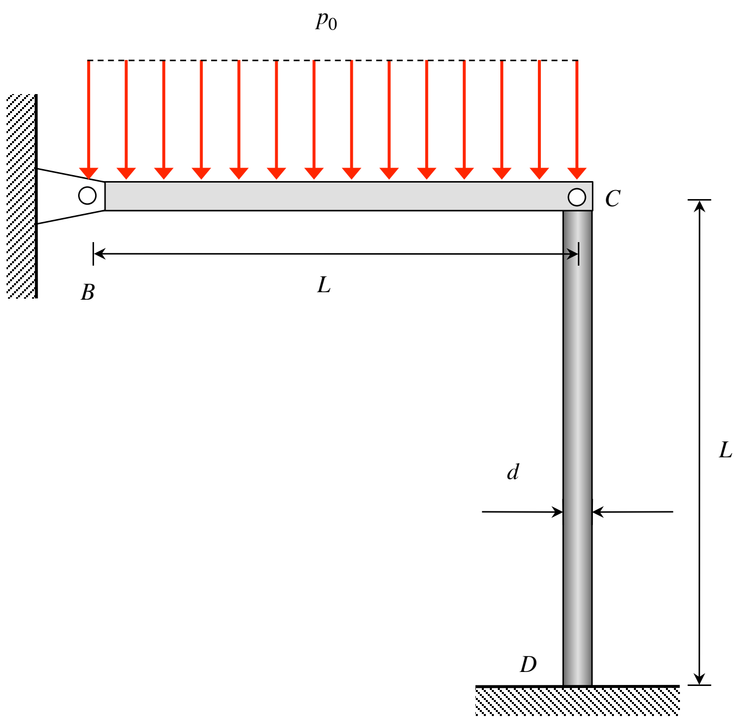
c) Determine the σ_x, σ_y and τ_{xy} for this stress state.

PROBLEM NO. 8 – 8 points max.

A cantilevered beam is loaded with concentrated couples and concentrated/distributed transverse forces. Although this loading is not shown below, a plot of the resulting internal bending moment M(x) is provided. Determine the reactions acting on the beam by the wall at its right end (at $x = 10\,ft$). Express your answers in terms of their xyz components.



PROBLEM NO. 9 – 8 points max.



A structure is made up of a horizontal rigid bar BC and a vertical elastic member CD, where CD has a circular cross section of diameter d and is made up of a material having a Young's modulus of E. A distributed loading having a transverse force/length of p_0 acts on bar BC. Ignore the weight of the members. Determine the critical load value p_0 that corresponds to the buckling of member CD.