## Example 6.2

Example shown below is made up of two members extending the full length of the composite element. The two members below experience identical strains $\varepsilon_{1}=\varepsilon_{2}=\varepsilon$ due to end connections to rigid plates. Determine the stresses in each member and determine the distance d locating the point of application of the load P needed for equal strains in the two members.


For equilibrium of rigid plate on right end:

$$
\begin{aligned}
& \sum F_{x}=P-\sigma_{1} A_{1}-\sigma_{2} A_{2}=0 \Rightarrow \\
& P=\left(E_{1} A_{1}+E_{2} A_{2}\right) \varepsilon \Rightarrow \\
& \varepsilon=\frac{P}{E_{1} A_{1}+E_{2} A_{2}}=\frac{P}{\left(E_{1} b_{1}+E_{2} b_{2}\right) h}
\end{aligned}
$$

and

$$
\begin{aligned}
& \sum M=-P d+\left(\sigma_{1} A_{1}\right)\left(\frac{b_{1}}{2}+b_{2}\right)+\left(\sigma_{2} A_{2}\right)\left(\frac{b_{2}}{2}\right)=0 \Rightarrow \\
& P d=\left[\left(E_{1} A_{1}\right)\left(\frac{b_{1}}{2}+b_{2}\right)+\left(E_{2} A_{2}\right)\left(\frac{b_{2}}{2}\right)\right] \varepsilon \Rightarrow \\
& d=\frac{E_{1} b_{1}\left(b_{1} / 2+b_{2}\right)+E_{2} b_{2}\left(b_{2} / 2\right)}{E_{1} b_{1}+E_{2} b_{2}}
\end{aligned}
$$

The stresses in each of the two members are given by, respectively:

$$
\begin{aligned}
& \sigma_{1}=E_{1} \varepsilon=\frac{P E_{1}}{\left(E_{1} b_{1}+E_{2} b_{2}\right) h} \\
& \sigma_{2}=E_{2} \varepsilon=\frac{P E_{2}}{\left(E_{1} b_{1}+E_{2} b_{2}\right) h}
\end{aligned}
$$

