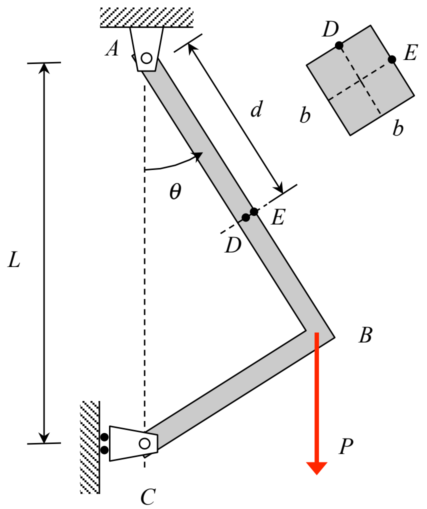
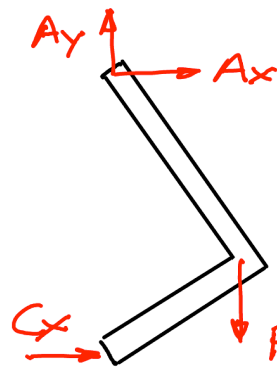
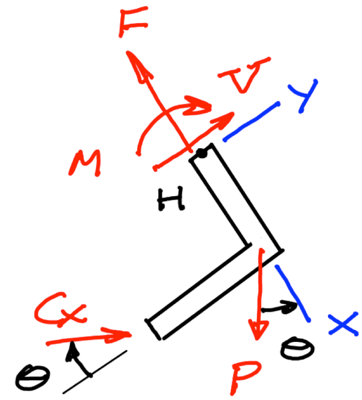
Example 14.7

The L-shaped frame ABC has a square cross section of side dimension b. This frame is supported by a fixed pin at A and by a roller at C. Determine the maximum principal stresses and the maximum in-plane shear stress at points D and E in the cross section at the location at a distance of d from A.







SOLUTION

From the FBD of the frame:

$$\sum M_A = C_x L - P(L\sin\theta) = 0 \quad \Rightarrow \quad C_x = P\sin\theta$$

From the FBD of the cut section:

$$\sum M_H = -M - (P\sin\theta)(L\cos\theta - d) + (C_x\cos\theta)(L\cos\theta - d) - (C_x\sin\theta)(L\sin\theta) = 0 \quad \Rightarrow$$

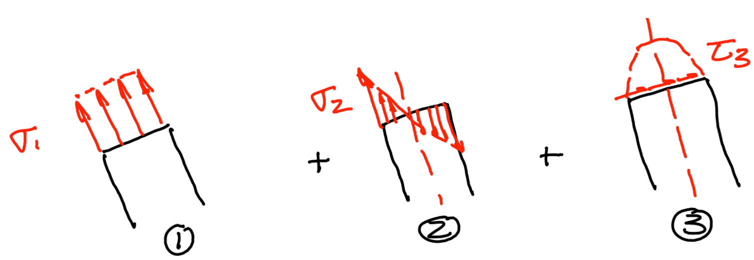
$$M = \left[\sin\theta(\cos\theta - 1)(L\cos\theta - d) - L\sin^3\theta\right]P$$

$$\sum F_x = C_x - F\sin\theta + V\cos\theta = 0 \quad \Rightarrow \quad -F\sin\theta + V\cos\theta = -P\sin\theta$$

$$\sum F_x = F\cos\theta + V\sin\theta - P = 0 \quad \Rightarrow \quad F\cos\theta + V\sin\theta = P$$

Solve the above two equations for the internal reactions F and V.

Stress distribution at cut:



<u>At D</u>:

$$\sigma = \sigma_1 = \frac{F}{A} \quad ; \quad A = b^2$$

$$\tau = \left(\tau_3\right)_{max} = \frac{3V}{2A}$$

Therefore:

$$\sigma_{ave} = \frac{\sigma}{2} = \frac{F}{2A} \quad ; \quad R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} > \sigma_{ave}$$

$$\sigma_{P1} = \sigma_{ave} + R > 0$$

$$\sigma_{P2} = \sigma_{ave} - R < 0$$

$$\left(\tau_{max}\right)_{in-plane} = R$$

<u>At E</u>:

$$\sigma = \sigma_1 - \left(\sigma_2\right)_{max} = \frac{F}{A} - \frac{M\left(b/2\right)}{I} \quad ; \quad A = b^2 \quad and \quad I = b^4/12$$

$$\tau = 0$$

Therefore:

$$\sigma_{ave} = \frac{\sigma}{2} = \frac{F}{2A} - \frac{Mb}{4I} \quad ; \quad R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + 0} = \sigma_{ave}$$

$$\sigma_{P1} = \sigma_{ave} + R = 2\sigma_{ave}$$

$$\sigma_{P2} = \sigma_{ave} - R = 0$$

$$\left(\tau_{max}\right)_{in-plane} = R = \sigma_{ave}$$