Lecture 6: Axial deformation—Statically determinate structures

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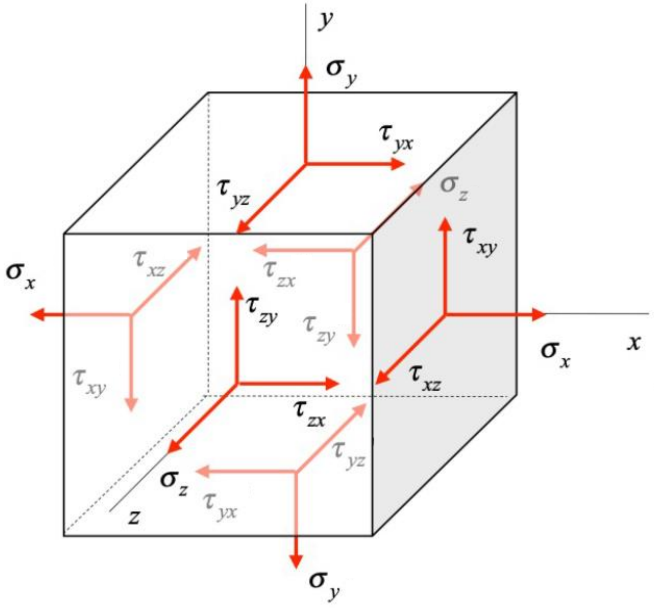
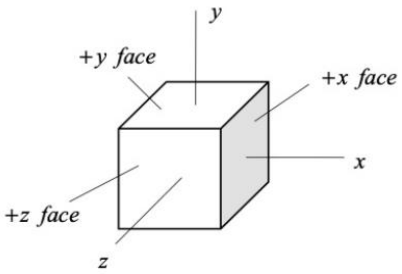
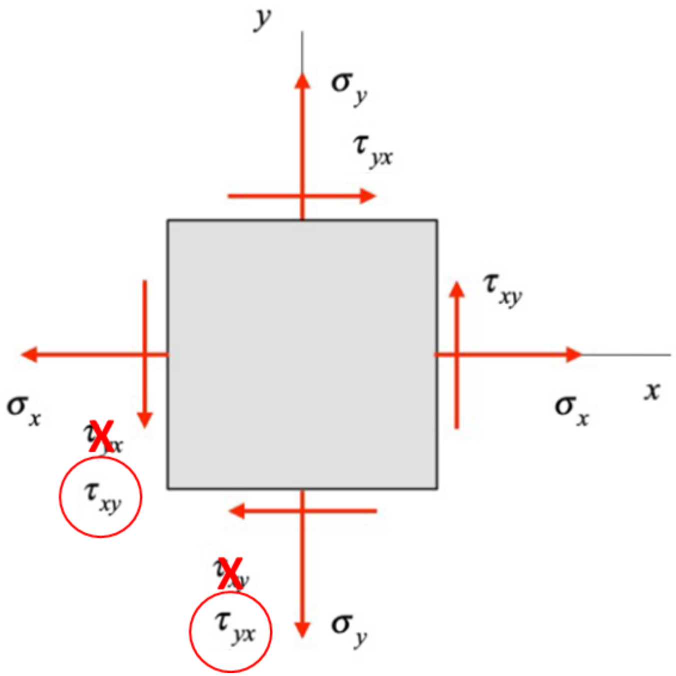
PURDUE ENGINEERING

Corrections from last class

Shear stresses







Example 5.8, Part (c)

$$\Delta t = t\frac{\sigma_0 \nu (1+\nu)}{E}$$

Review of last class

Hooke's law

Normal strains

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\sigma_z - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \alpha\Delta T = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

Shear strains

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

Shear modulus

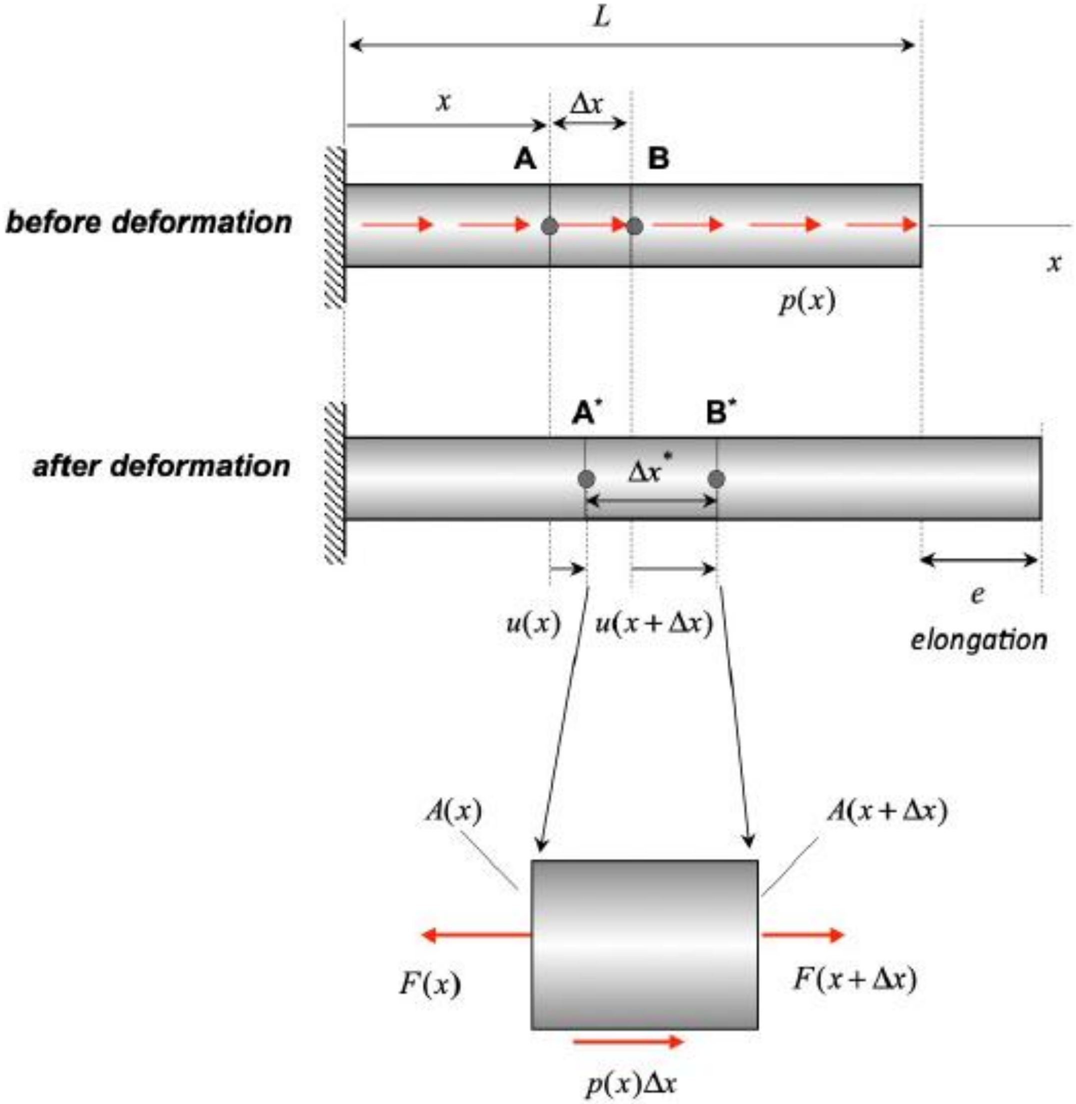
$$G = \frac{E}{2\left(1+\nu\right)}$$

Objectives

- Review assumptions for axial deformation
- Relate the elongation of an axial member with the axial force on the member
  - What further simplifying assumptions can we make?

General axial deformation

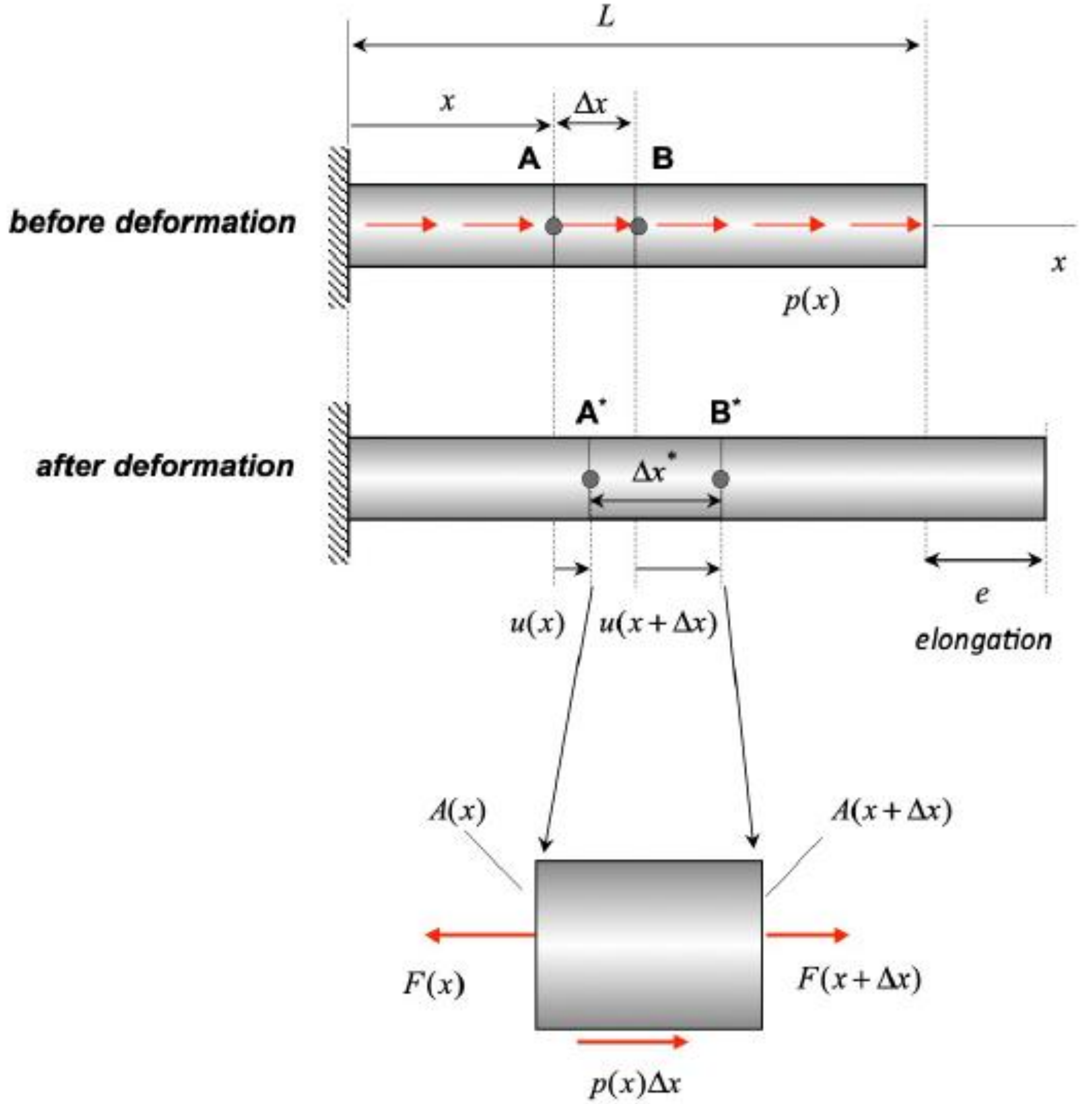
Deformation due to a distributed axial load $p(x)$



Key equations

General axial deformation (cont.)

Deformation due to a distributed axial load $p(x)$

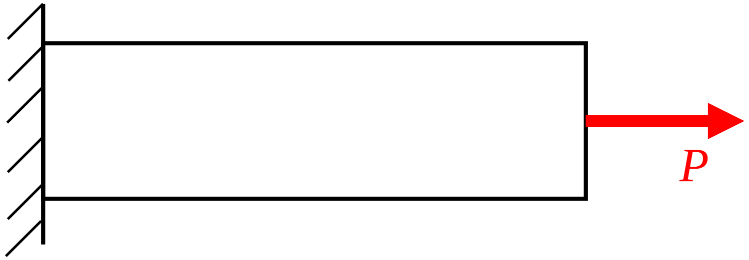


Key equations

Analogy with a spring

Spring

Axially-loaded bar



Procedure for axial loading problems

- Drawn an FBD of each element
  - One “element” has constant or smoothly varying internal axial force, properties, and cross-sectional area
- Enforce static equilibrium to find the internal axial force in each element
- Use the force-elongation equations to find the elongation of each element
  - Constant internal force, properties, and cross-sectional area: $e = u(L) - u(0) = \frac{FL}{AE}$
  - Otherwise: $e = u(L) - u(0) = \int_0^L \frac{F(x)}{A(x)E(x)} dx$
  - We can also use the elongation of each member to find displacement at points of interest

- Note: practice doing this in reverse!