Lectures 10-12: Torsion members

Lecture Book: Chapter 8

Joshua Pribe

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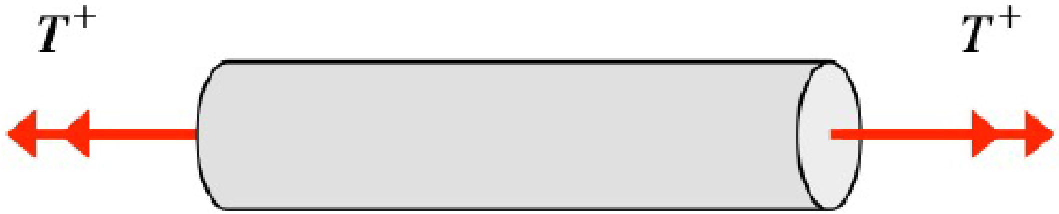
PURDUE ENGINEERING

Objectives

- Derive important relations for pure torsion of a circular bar
  - Shear strain and angle of twist
  - Shear stress and shear strain
  - Shear stress and torque
  - Torque and angle of twist
- Compare with our equations for uniaxial loading
- Complete several examples on calculating shear stress



Torsion members: Assumptions

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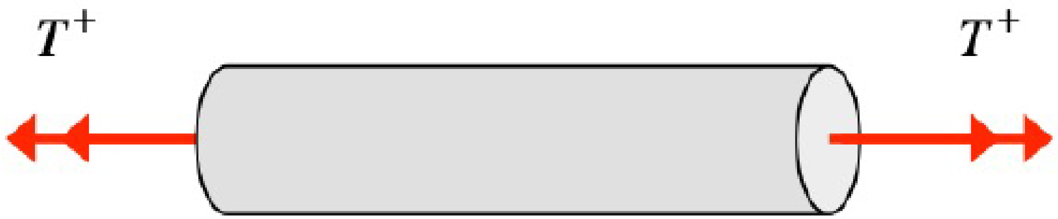
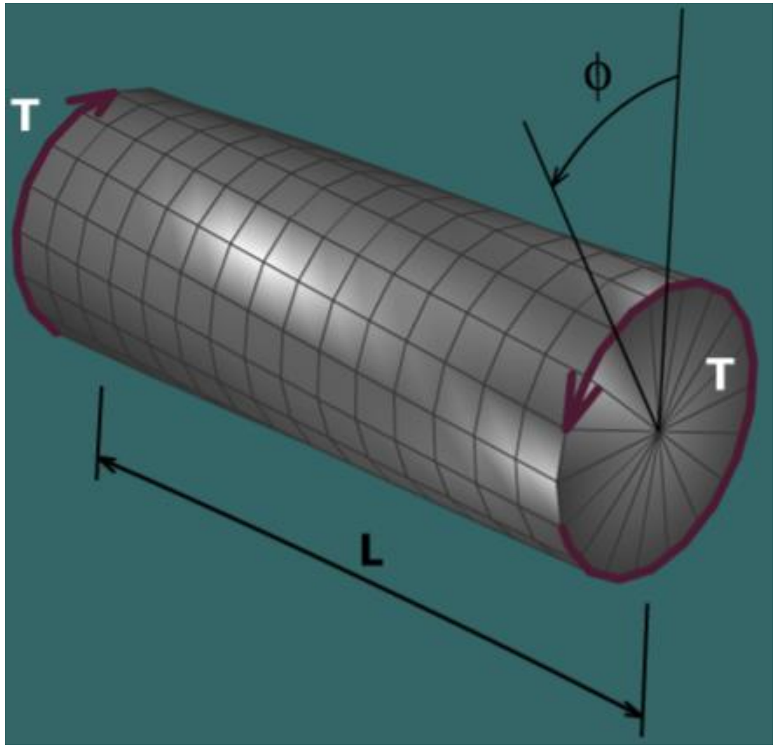
Assumptions:

- Only circular cross section shafts will be considered.
- Cross sections remain perpendicular to shaft axis after deformation (this is not a valid assumption, in general, for non-circular cross section bars).
- Radial lines remain radial (i.e. every point on a cross section rotates by the same amount).
- Shaft axis remains straight.





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Shear strain and angle of twist; shear strain and shear stress

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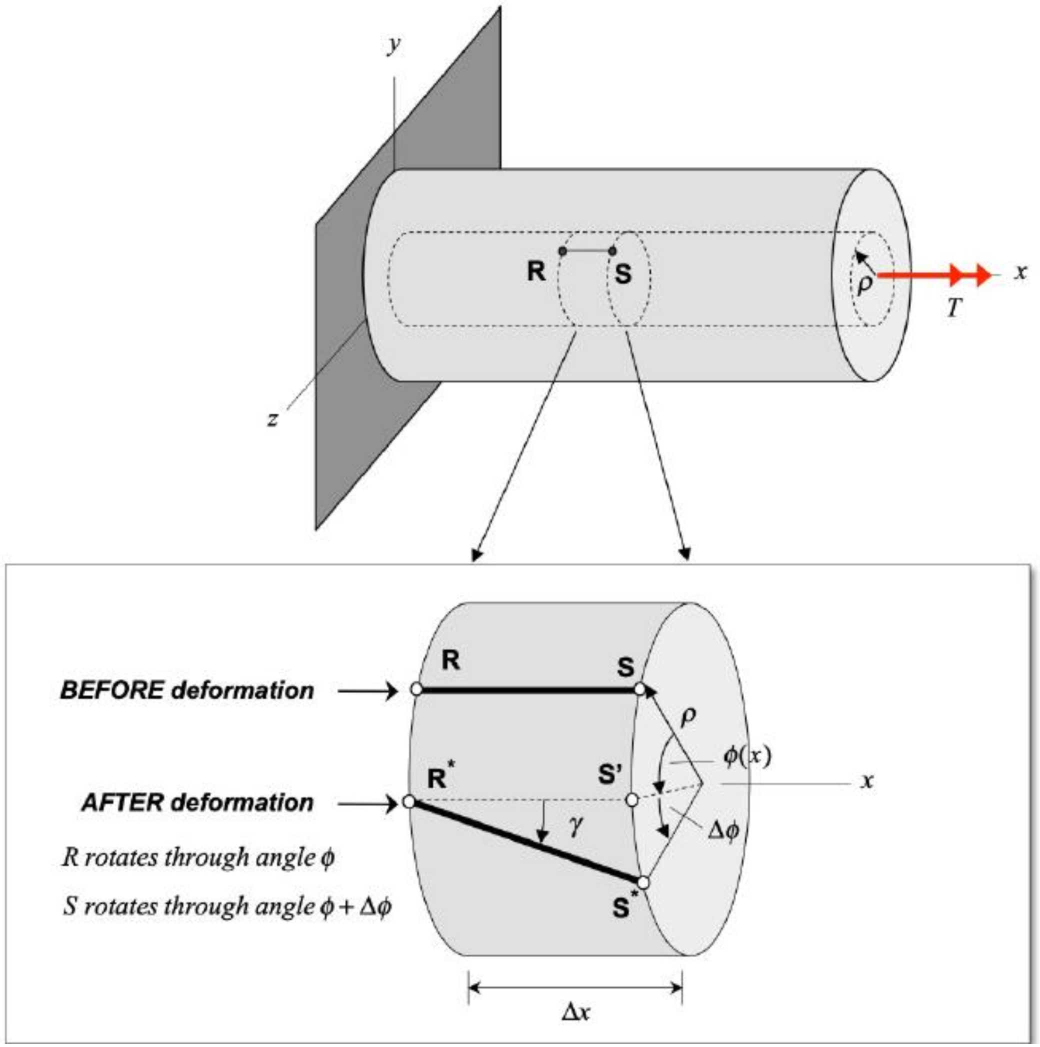

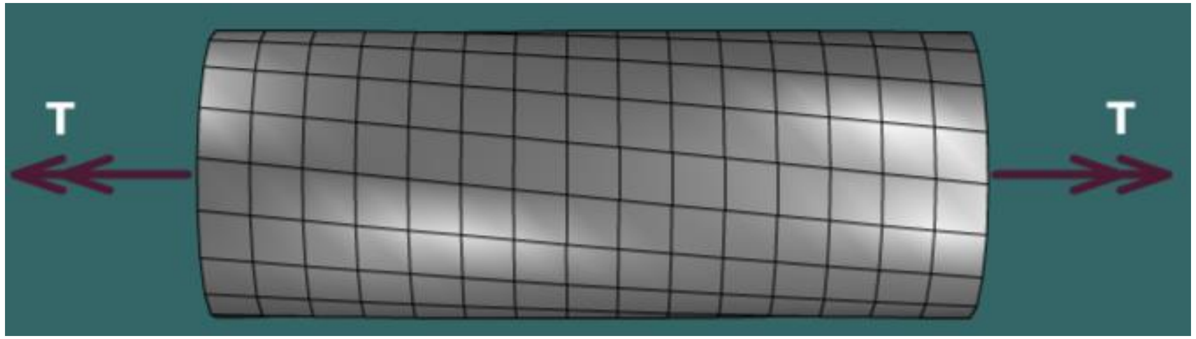

Shear stress and torque

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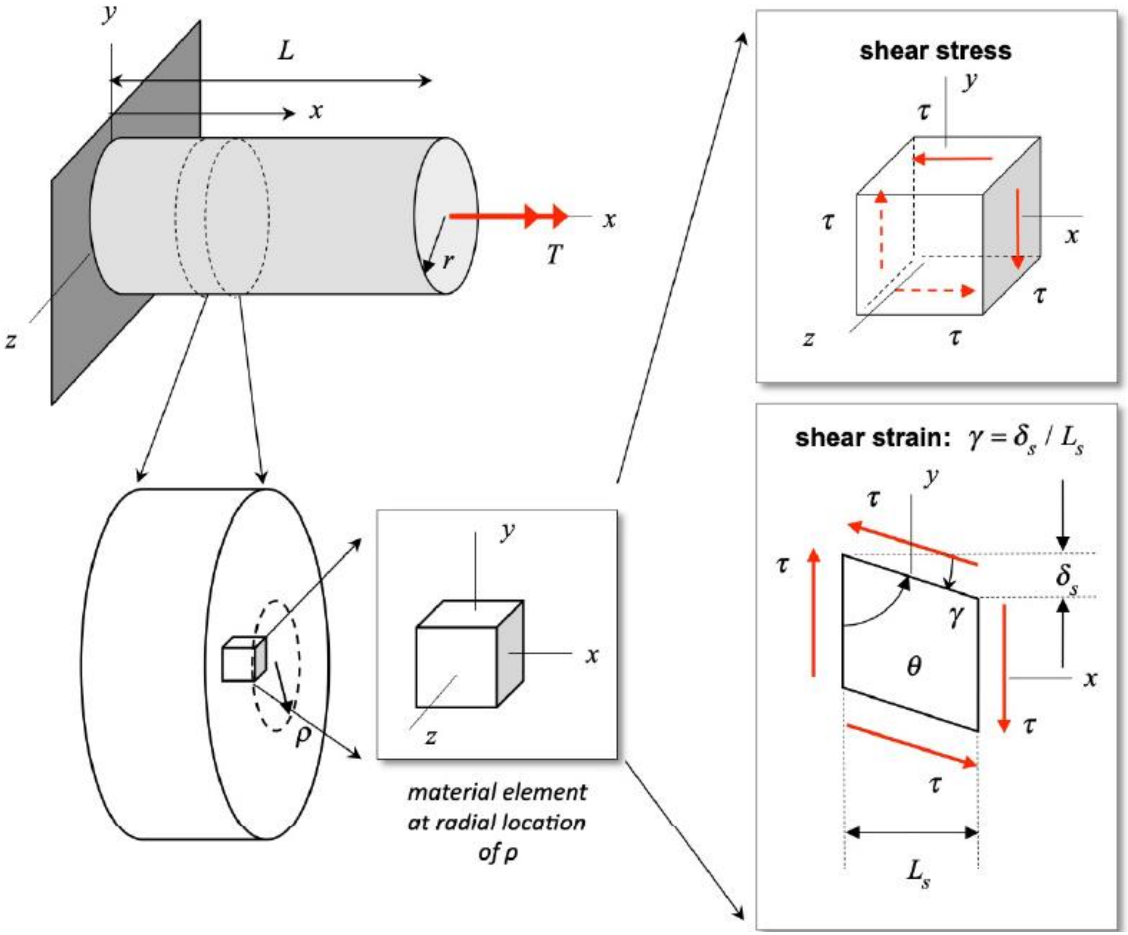

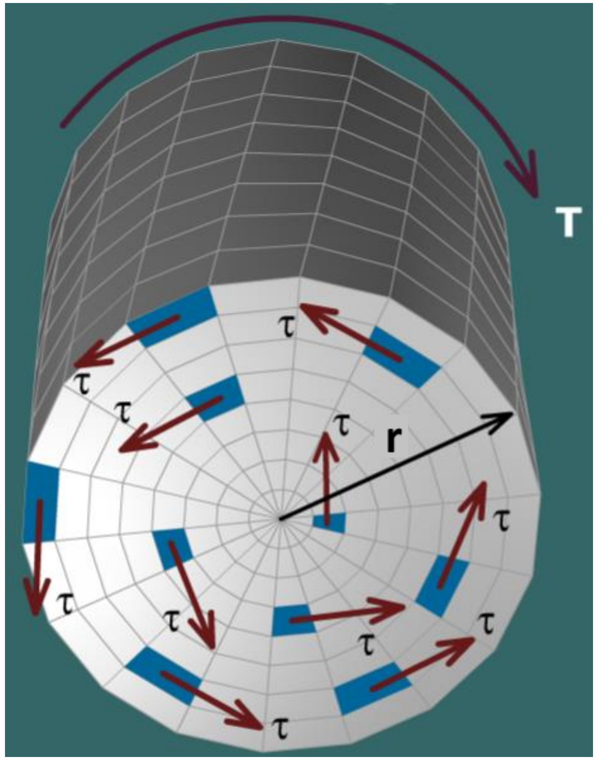

Torque-angle of twist relation

Using all of the previous equations, we can relate the torque T to the angle of twist ϕ

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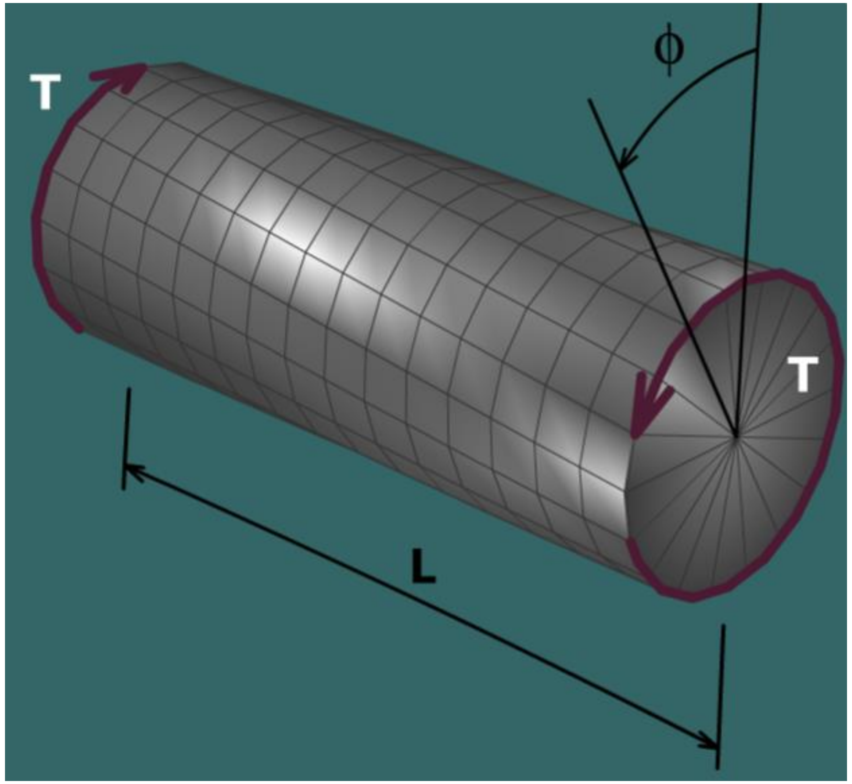
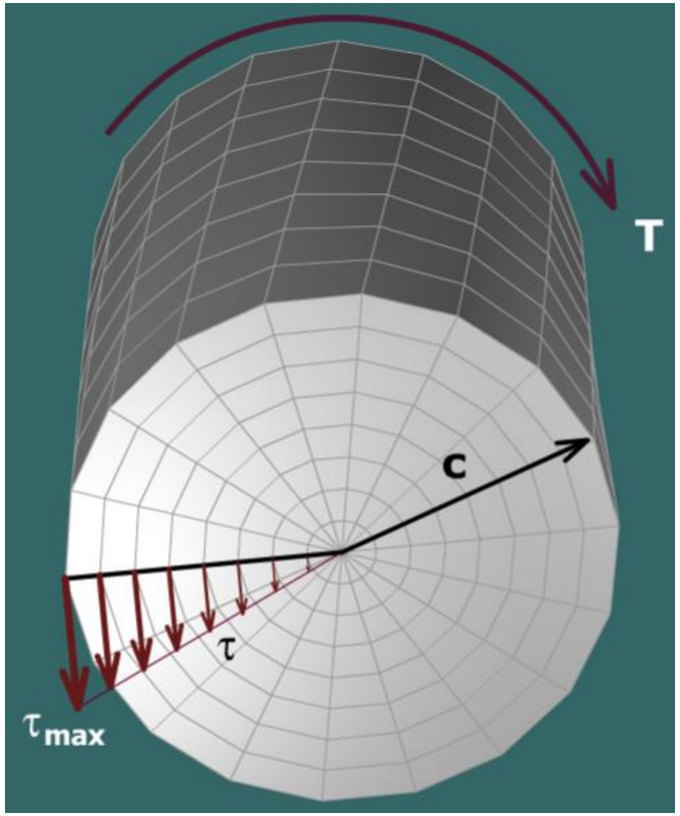


Notes on shear stress

For constant G throughout the cross section, the shear stress τ varies linearly with the radial distance ρ



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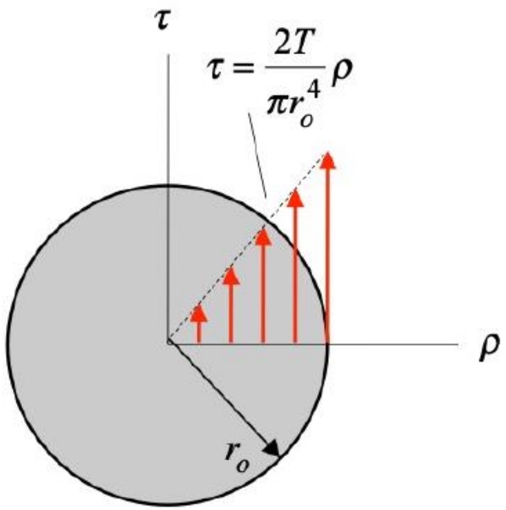
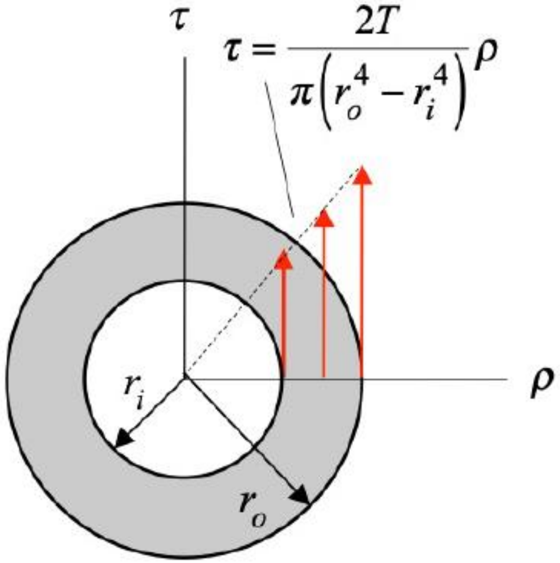
We will consider both solid and hollow (tubular) shafts





Notes on shear stress

States of stress at various points on a cross section—in all cases, what is the magnitude of τ?

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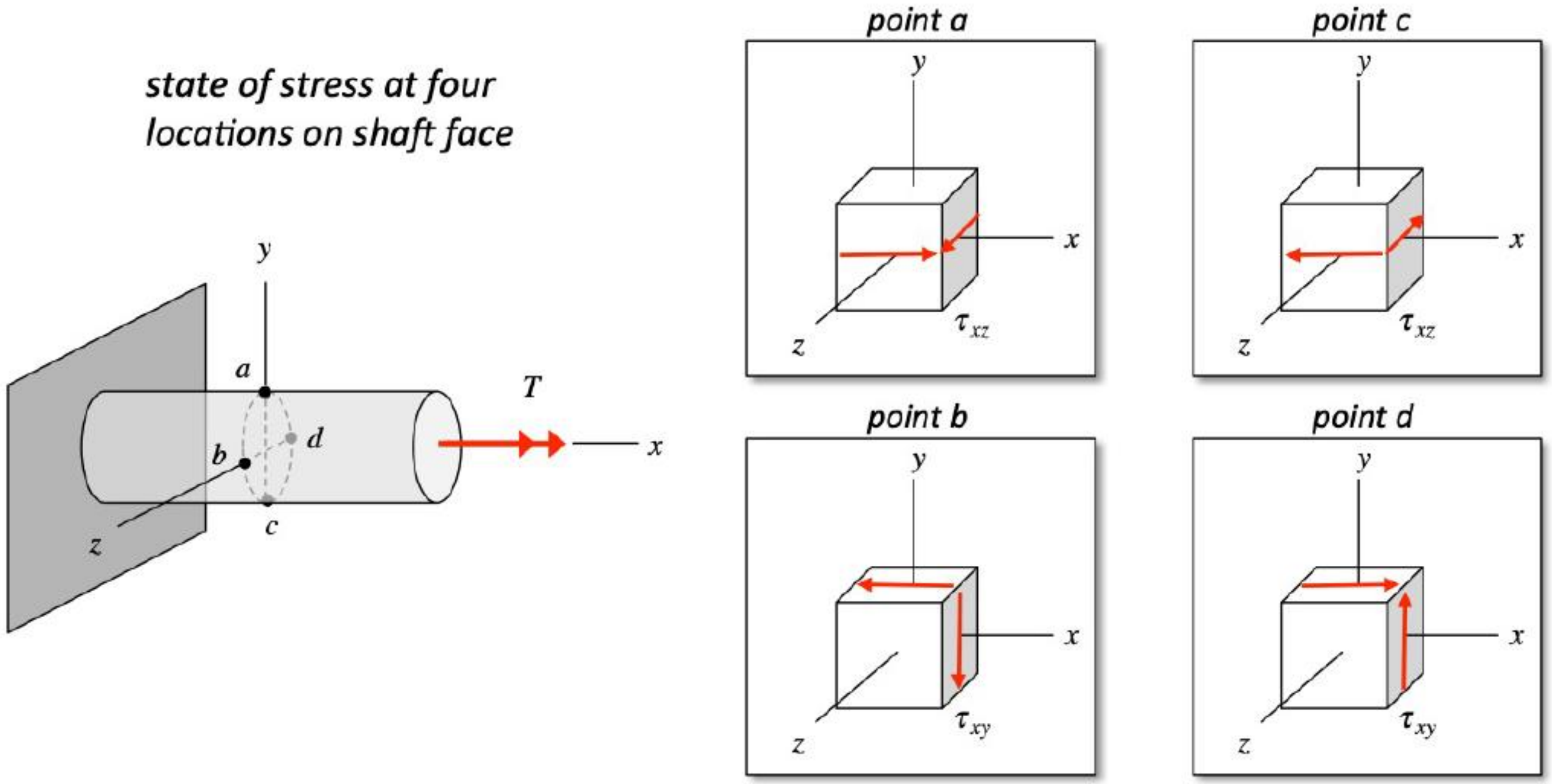

Summary: Comparison with uniaxial deformation

Normal strain-displacement

$$\varepsilon = \frac{du}{dx}$$

Stress-strain (uniaxial stress)

$$\sigma = E\varepsilon$$

Axial force-normal stress

$$\sigma = \frac{F}{A}$$

Force-deformation (elongation)

$$e = u(L) - u(0) = \int_0^L \frac{F}{AE} dx$$

Constant F, A, E: $e = \frac{FL}{AE}$

Shear strain-angle of twist

$$\gamma = \rho \frac{d\phi}{dx}$$

Stress-strain (shear)

$$\tau = G\gamma$$

Torque-shear stress

$$\tau = \frac{T\rho}{I_p} \qquad \tau_{\max} = \frac{Tr}{I_p}$$

Torque-twist

$$\Delta\phi = \phi(L) - \phi(0) = \int_0^L \frac{T}{GI_p} dx$$

Constant T, I_p, G: $\Delta\phi = \frac{TL}{GI_p}$

Summary: Comparison with uniaxial deformation

Uniaxial loading procedure:

1. FBDs and equilibrium
2. Force-deformation equations
3. Compatibility equation(s)
4. Solve for unknowns

Torsional loading procedure:

1. FBDs and equilibrium
2. Torque-twist equations
3. Compatibility equation(s)
4. Solve for unknowns