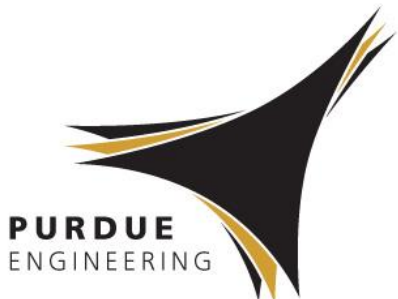


Lecture 31: Stress transformation – Principal stresses and maximum shear stress

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Fall 2019

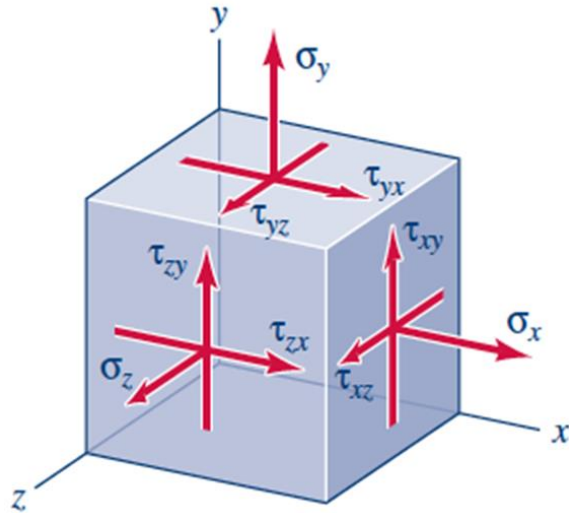
Lecture Book: Chapter 13



Motivation

- There is only *one* state of stress at a point
- BUT we can express that state of stress in different coordinate systems
 - A material could fail on a surface that does *not* correspond to our x , y , or z axes

Recall a general 3-D state of stress (Chapter 5)

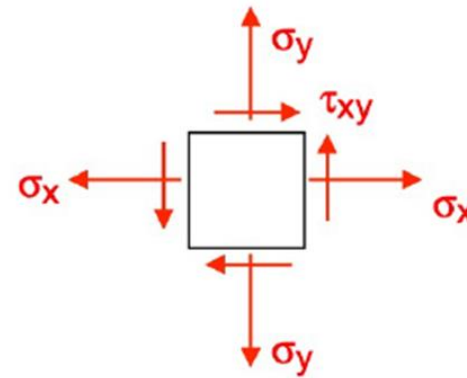
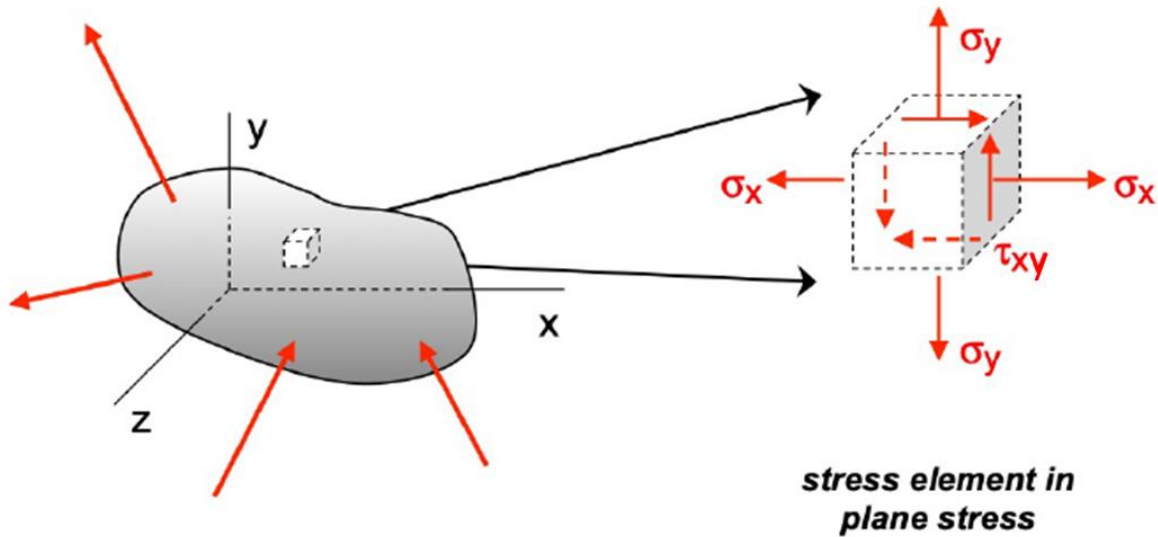


States of stress: Plane stress

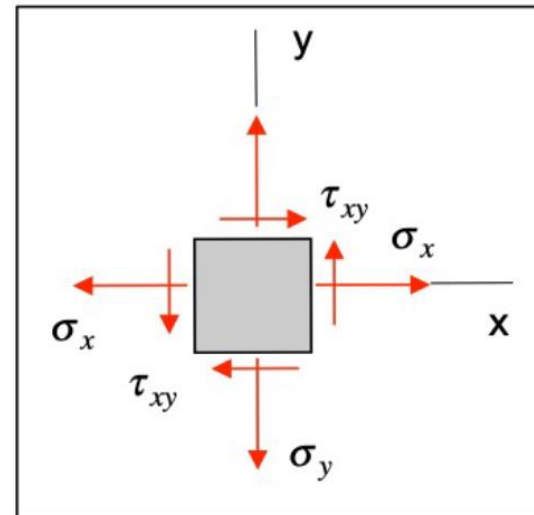
Many engineering components are in a state of “plane stress”

For plane stress in the x-y plane: $\sigma_x \neq 0, \sigma_y \neq 0, \tau_{xy} \neq 0$

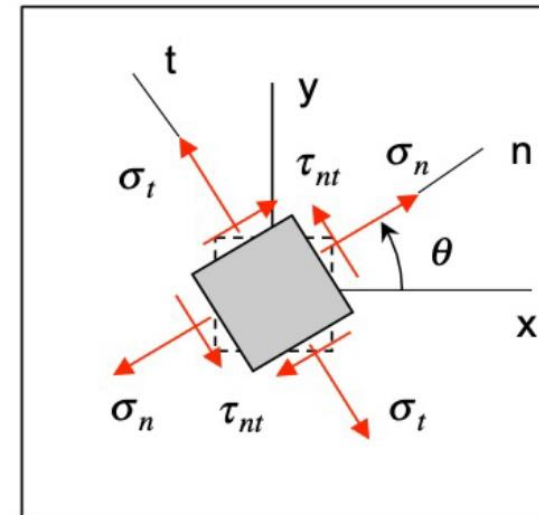
$\sigma_z = 0, \tau_{xz} = 0, \tau_{yz} = 0$



BEFORE rotation



AFTER rotation



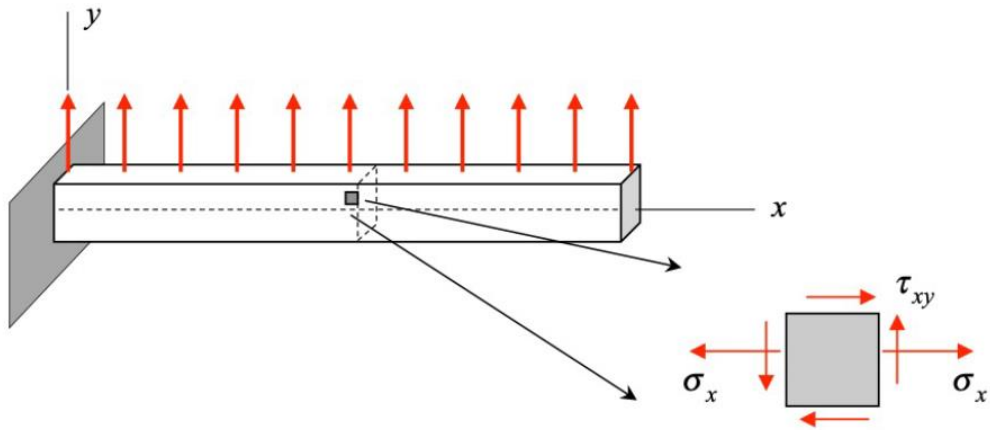
Lecture Book: Ch. 13, pg. 1

What if we want to express this state of stress with respect to different coordinates?

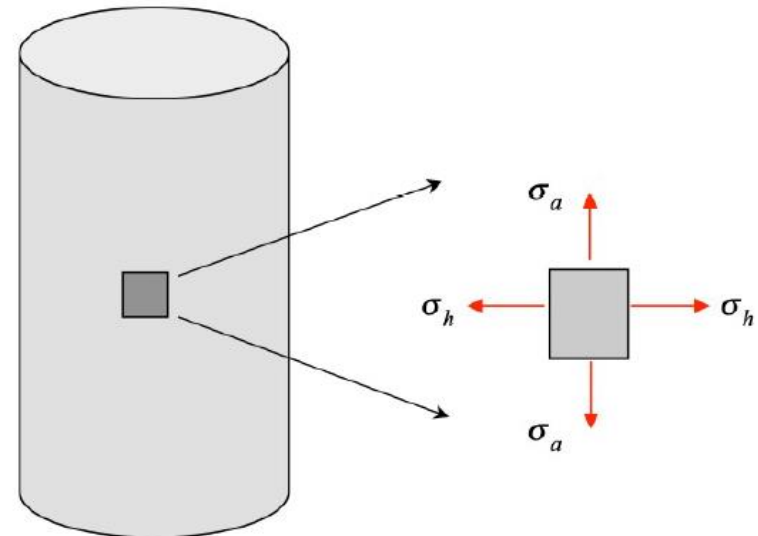
Plane stress examples

Each type of deformation we have looked at so far, along with combinations of each of them

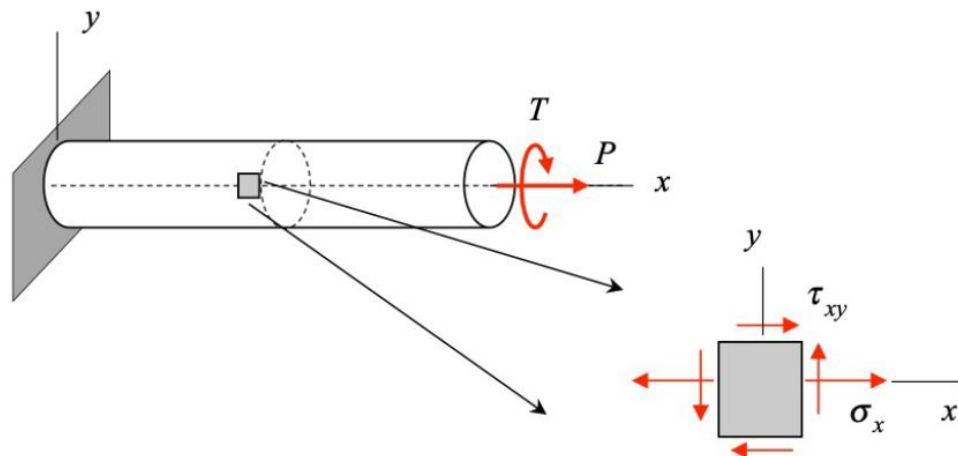
BENDING BEAM



THIN-WALLED PRESSURE VESSEL



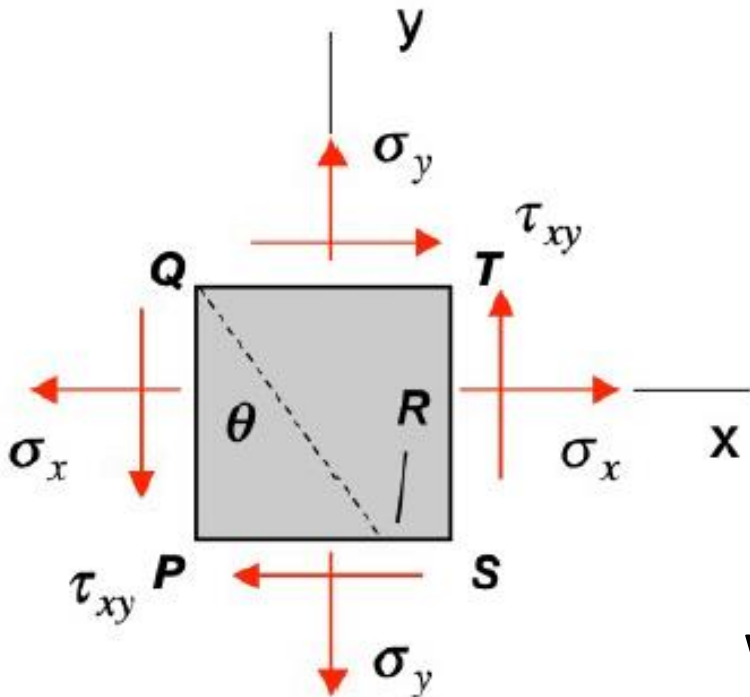
TORSIONALLY AND AXIALLY LOADED SHAFT



Stress transformation for plane stress

Goal: Find an expression for $\sigma_n, \sigma_t, \tau_{nt}$
from $\sigma_x, \sigma_y, \tau_{xy}$

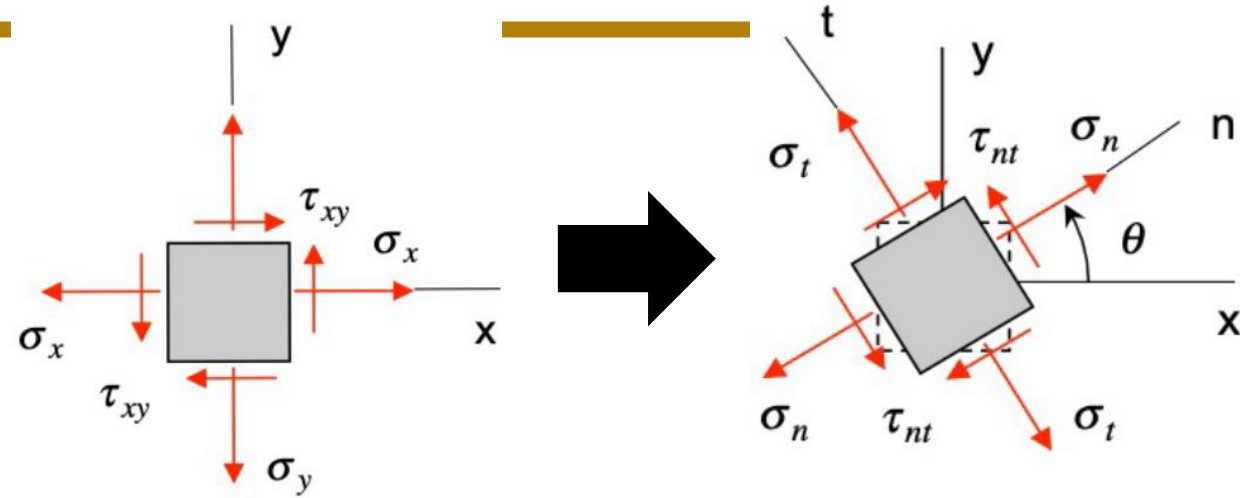
To do this, enforce equilibrium on the following cut:



We end up with:

$$\sigma_n = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau_{xy} (2 \sin \theta \cos \theta)$$

$$\tau_{nt} = -(\sigma_x - \sigma_y) \cos \theta \sin \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta)$$



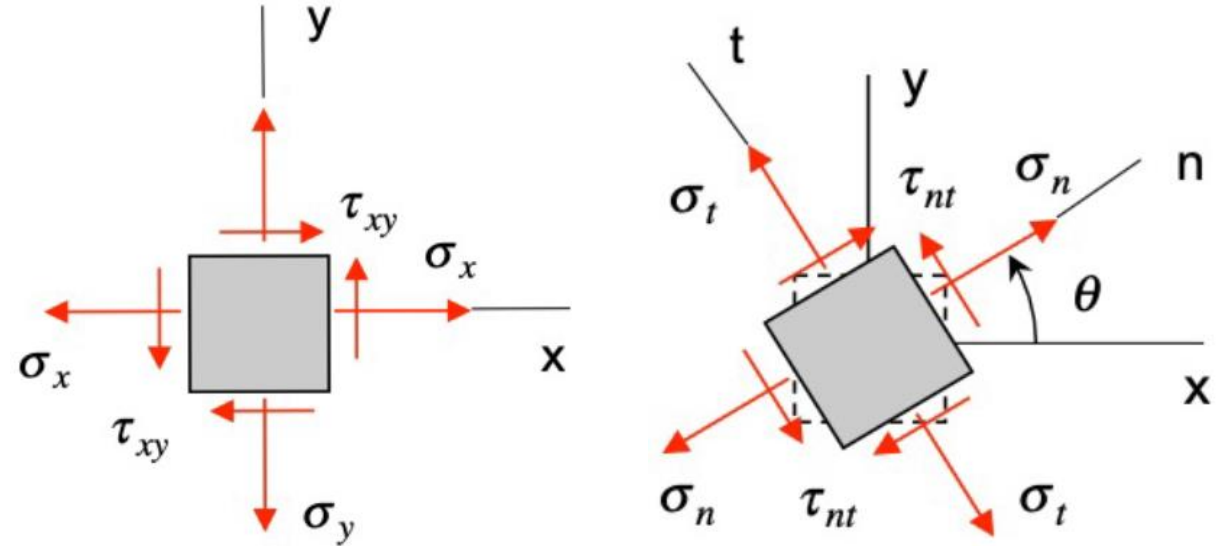
Stress transformation for plane stress

Stress transformation equations for plane stress

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_t = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{nt} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$



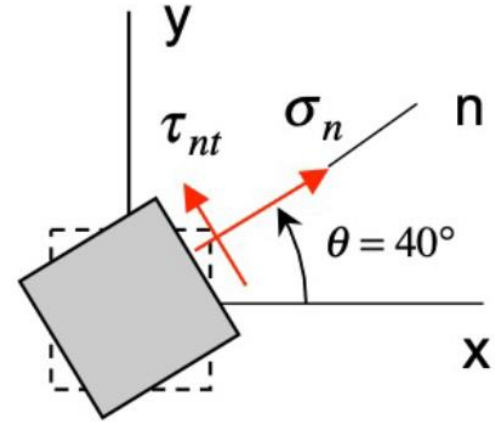
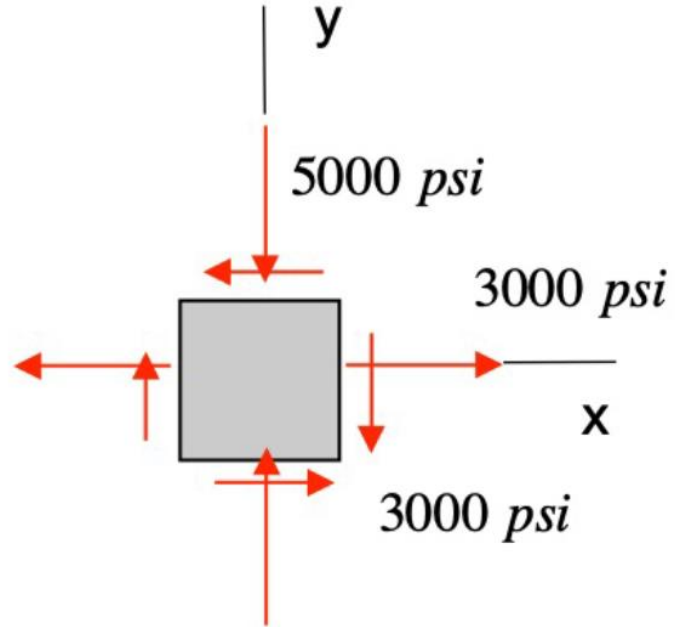
θ measured CCW from x axis

$\sigma > 0$ for tension

$\tau_{ij} > 0$ for shear stress on the positive i face in the positive j direction

Example 13.1

Determine the stresses on a plane whose orientation is a 40° CCW rotation from the x-axis

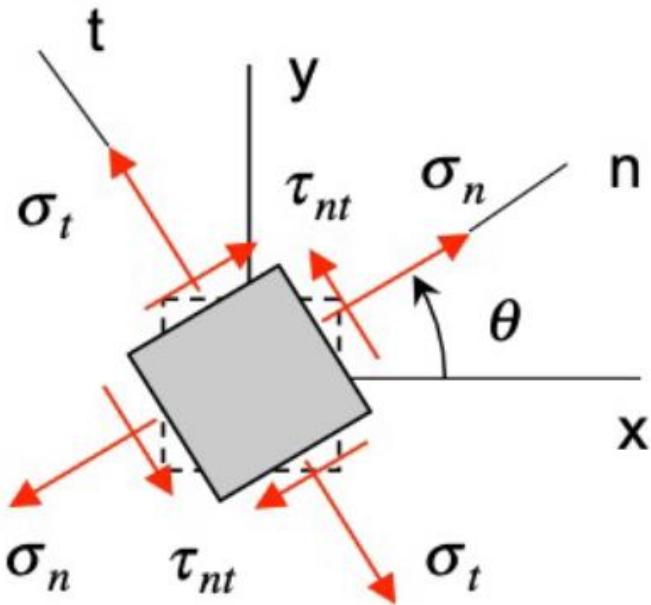


Special states of stress

We want to predict *failure* of a component

→ we are often interested in the *maximum normal stresses* and *maximum shear stress*

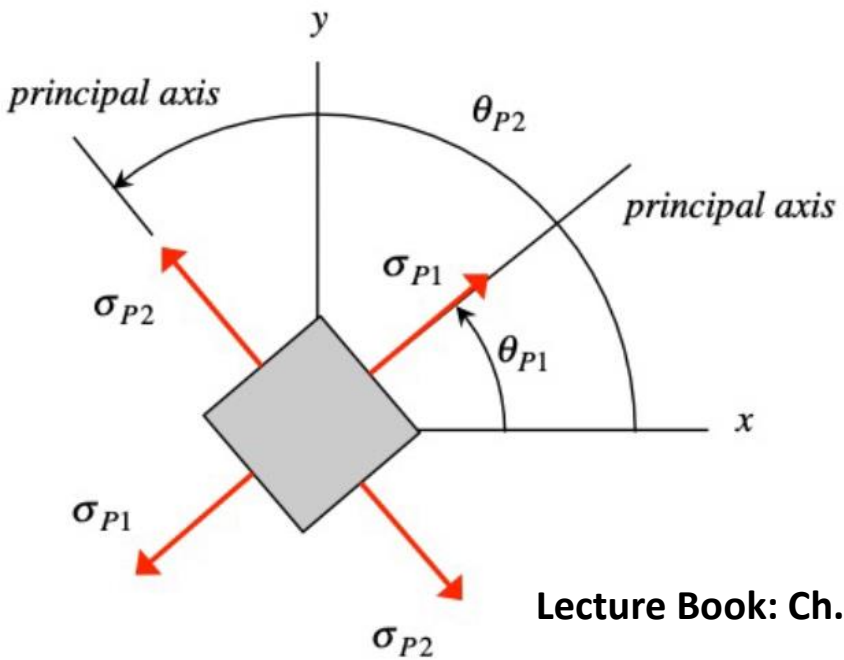
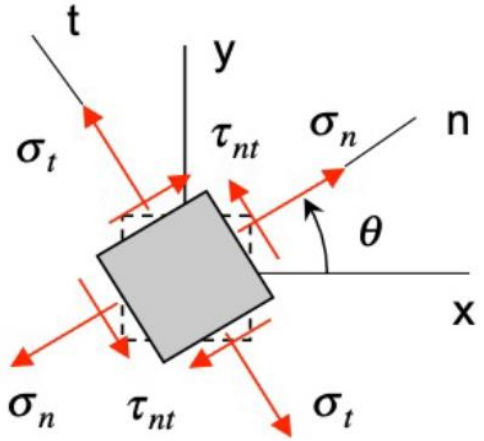
The maximum normal stresses are called principal stresses



From the stress transformation equation, how do we find the extrema of σ_n ?

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

Principal stresses

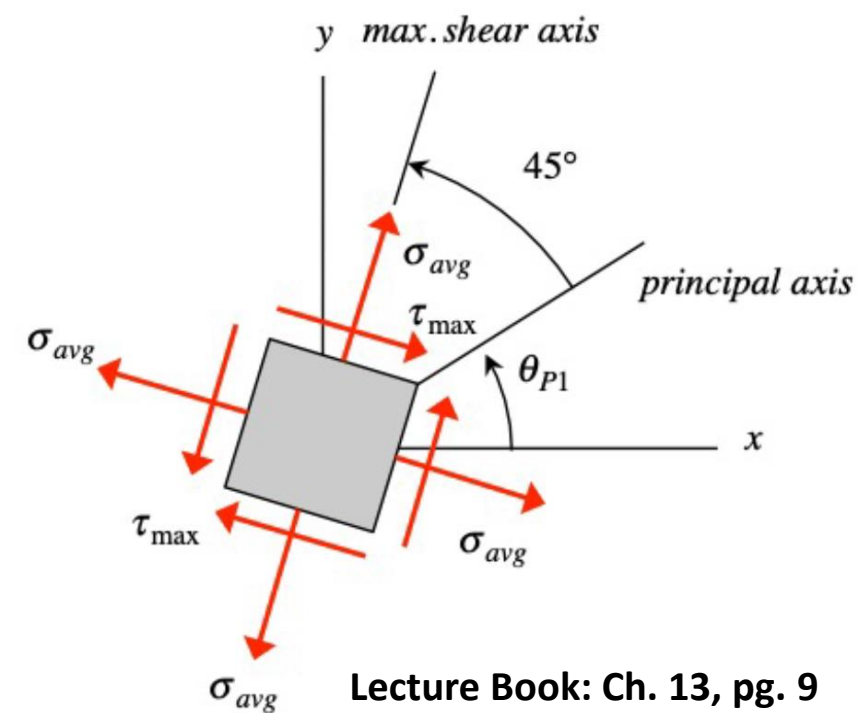
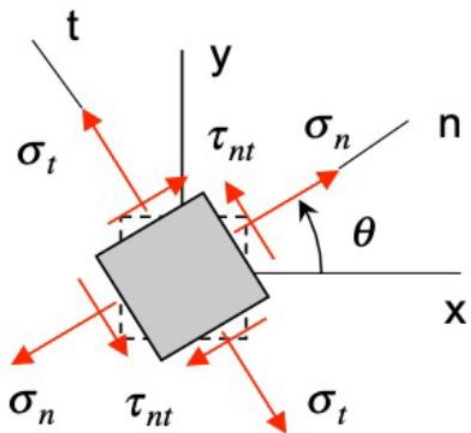


Lecture Book: Ch. 13, pg. 7

Maximum in-plane shear stress

From the stress transformation equation, how do we find the extrema of τ_{nt} ?

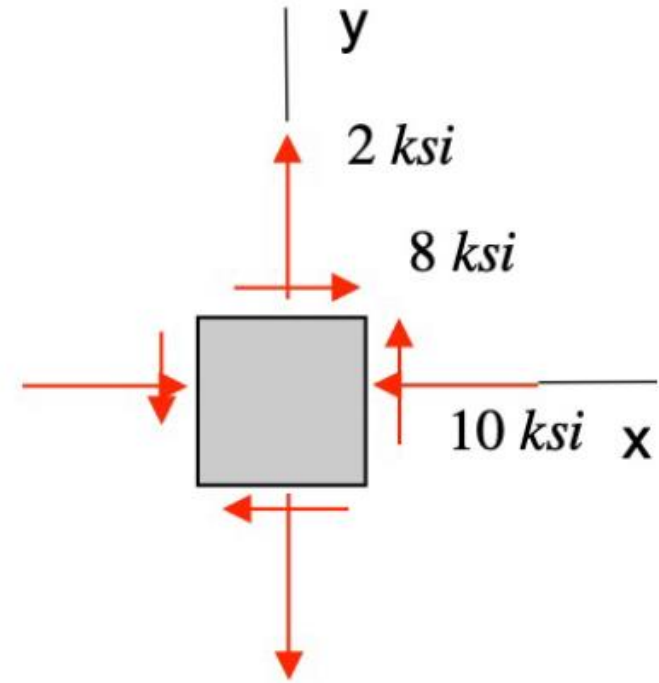
$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$



Example 13.2

For the stress state shown, determine:

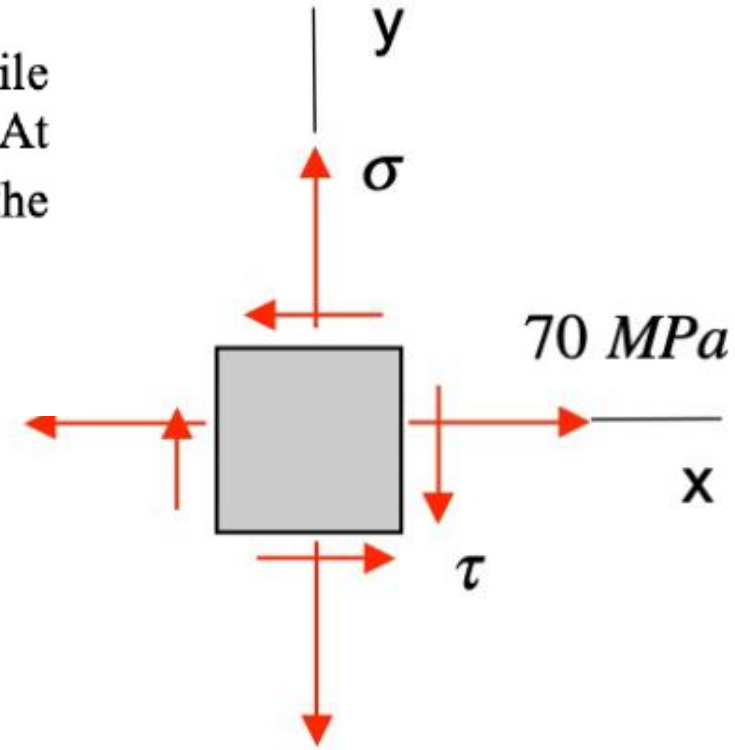
- The in-plane principal stresses and their orientation
- The maximum in-plane shear stress and its orientation



Example 13.3

The state of plane stress at a point shown below can be described by a known tensile stress $\sigma_x = 70 \text{ MPa}$, and unknown tensile stress σ and an unknown shear stress τ . At this point, the maximum in-plane shear stress is known to be 78 MPa , and one of the two in-plane principal stresses is 22 MPa (in tension).

Determine the values of σ and τ , as well as the other in-plane principal stress.



Summary for principal stresses

We can use the following equations to calculate the principal stresses and directions given $\sigma_x, \sigma_y, \tau_{xy}$

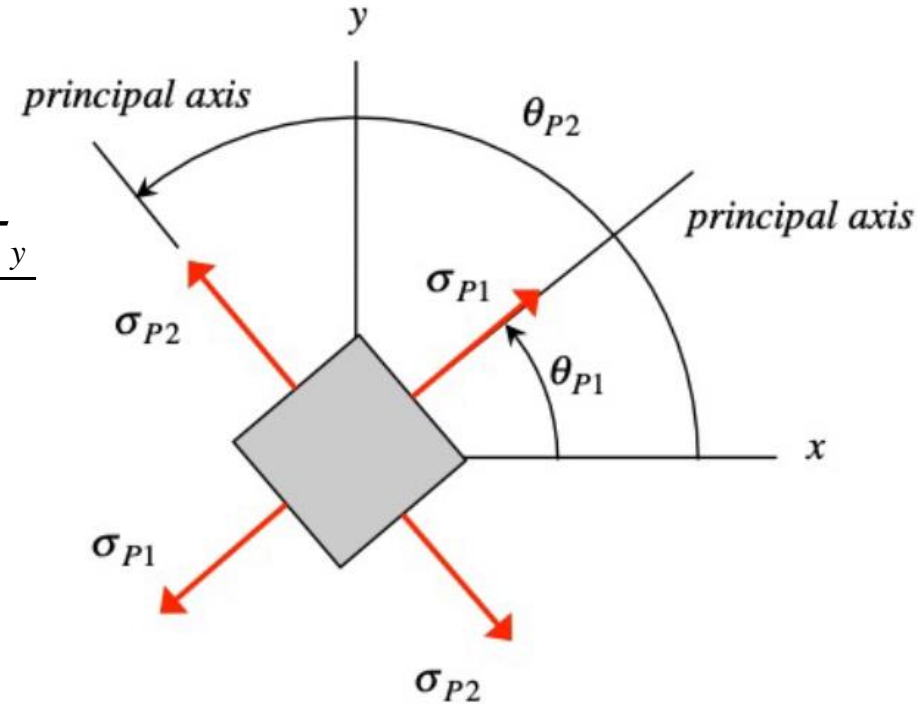
Maximum in-plane normal stress

$$\sigma_{p1} = \sigma_{ave} + R \quad \sin(2\theta_{p1}) = \frac{\tau_{xy}}{R} \quad \cos(2\theta_{p1}) = \frac{\sigma_x - \sigma_y}{2R}$$

Minimum in-plane normal stress

$$\sigma_{p2} = \sigma_{ave} - R \quad \theta_{p2} = \theta_{p1} + 90^\circ$$

where $\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$ $R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$



The shear stress is zero on the principal planes! $\tau_{nt}(\theta_{p1}) = \tau_{nt}(\theta_{p2}) = 0$

Summary for maximum in-plane shear stress

We can use the following equations to calculate the maximum and minimum in-plane shear stress given $\sigma_x, \sigma_y, \tau_{xy}$

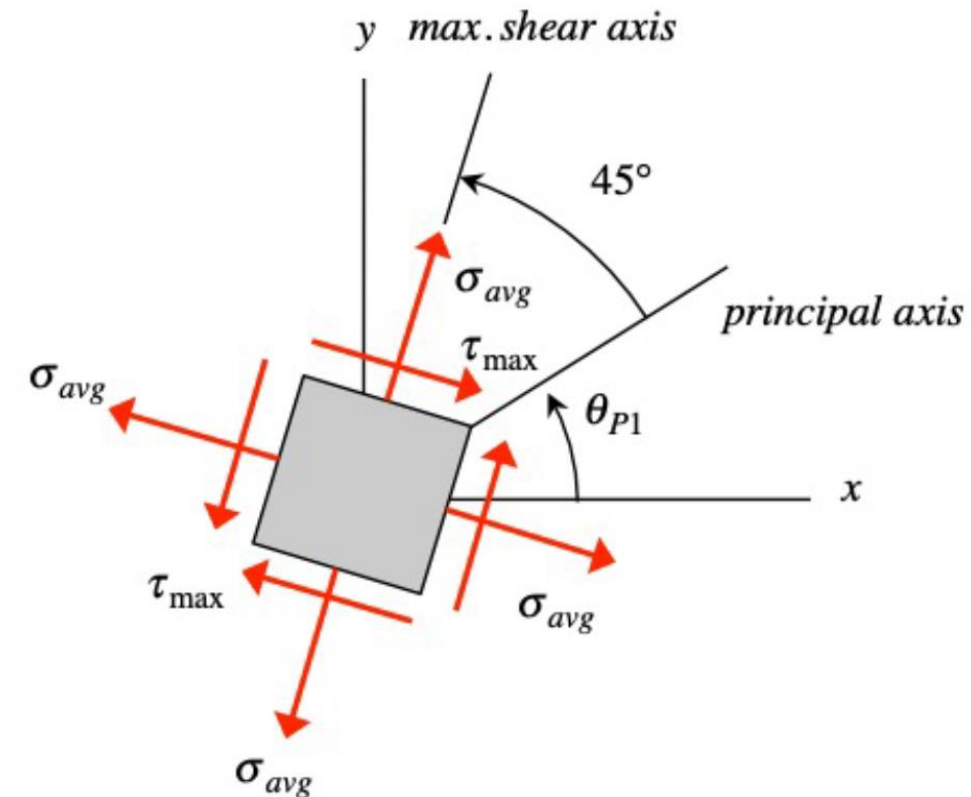
Maximum in-plane shear stress

$$\tau_{max} = \tau_{s1} = R \quad \theta_{s1} = \theta_{p1} + 45^\circ \quad \sin(2\theta_{s1}) = -\frac{\sigma_x - \sigma_y}{2R} \quad \cos(2\theta_{s1}) = \frac{\tau_{xy}}{R}$$

Minimum in-plane shear stress

$$\tau_{min} = \tau_{s2} = -R = -\tau_{max} \quad \theta_{s2} = \theta_{s1} + 90^\circ$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

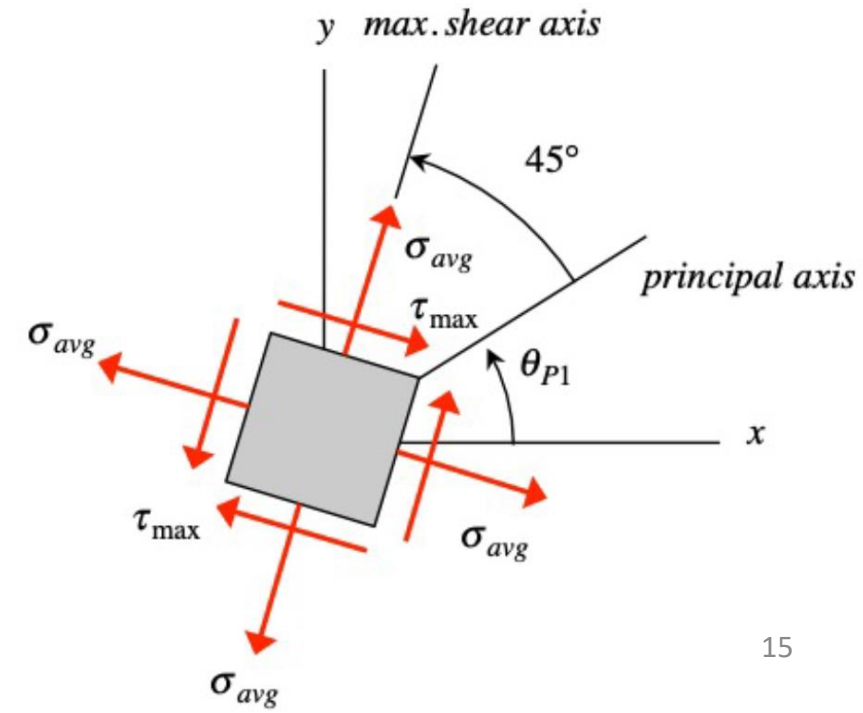
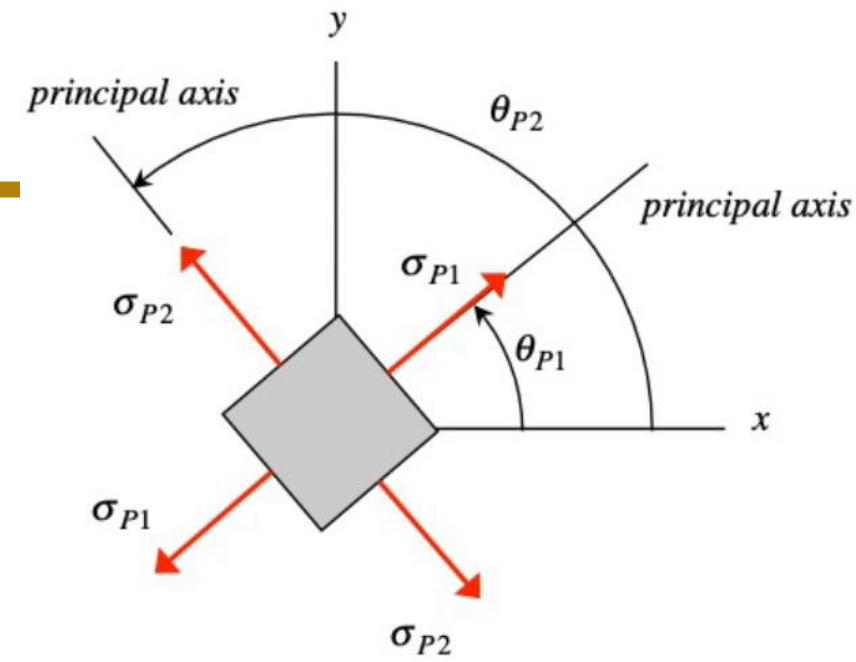


Both normal stresses are equal to the *average* normal stress in a maximum or minimum shear stress state

$$\sigma_n(\theta_{s1,s2}) = \sigma_t(\theta_{s1,s2}) = \sigma_{avg}$$

Summary

- The in-plane principal stresses are:
 - The max and min in-plane normal stresses
 - Found at orientations with no shear stress
 - Always on planes 90° apart
- The max and min in-plane shear stresses are:
 - Equal in magnitude
 - Found at orientations where the normal stresses are both equal to $\sigma_{ave} = (\sigma_x + \sigma_y) / 2$
 - Always on planes 45° away from principal stress planes



Visualizing stress transformations

Home
Chap 1. Stress
Chap 2. Strain
Chap 3. Mech Properties
Chap 4. Design Concepts
Chap 5. Axial Deform
Chap 6. Torsion
Chap 7. Equil of Beam
Chap 8. Bending
Chap 9. Shear Stress Beams
Chap 10. Beam Deflect
Chap 11. Indet Beams
Chap 12. Stress Transform
...Sec 12.6-12.8 Transform Eqns
...Sec 12.9 Mohr's Circle
Chap 13. Strain Transform
Chap 14. Pressure Vessels
Chap 15. Combined Loads
Appendix A
Assignments

M12.1 The Amazing Stress Camera
Discovery example
Illustrates the reason for stress transformations.

To better understand what is happening in the pipes, we're going to use a new instrument that will let us look into the microscopic structure of the pipe material. This new instrument is called the amazing stress camera.

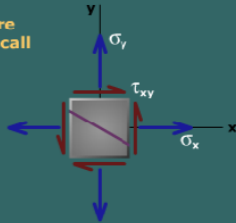
M12.2 Correct angle for stress transformations
Theory; Concept checkpoints
Easy method for finding the proper value of θ for use in the stress transformation equations.

Finding the correct value of θ for use in the stress transformation equations can be a source of confusion for students.

We're going to learn a simple method for finding θ that will always give the correct value.


To help you remember this method, we're even going to give it a corny name. We call this method:

Top-Drop-Sweep the Clock



Chapter 12: Stress Transformations
Sections 12.6-12.8: Stress Transformation Equations

MecMovies 2.00
MecMovies © Timothy A. Philpot 2001-2017

more movies 

“Mec movies” – see M12.1 for help visualizing stress transformations, principal stresses, and maximum shear stress