**PROBLEM #1 (25 Points):** ***SOLUTION***

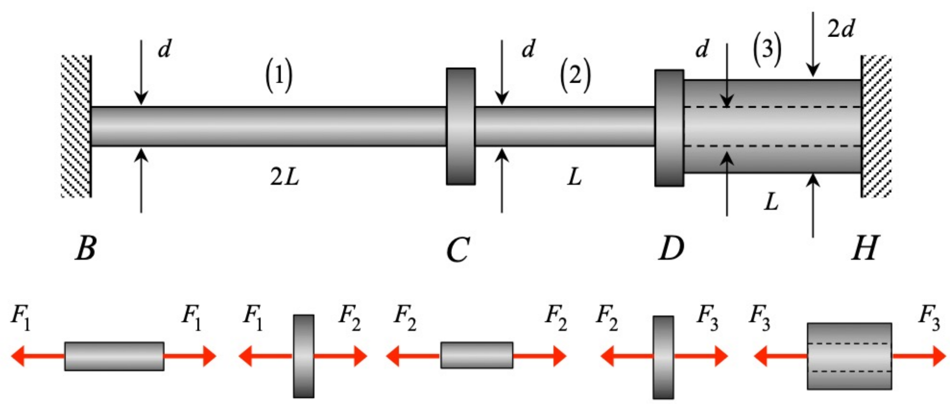
A rod is made up of elastic members (1), (2) and (3), with the material makeup of each member having a Young's modulus of $E$ and a coefficient of thermal expansion of $\alpha$. Members (1), (2) and (3) have lengths of $2L$, $L$ and $L$, respectively. Members (1) and (2) have solid cross-sections with a diameter of $d$, whereas member (3) has a tubular cross-section with inner and outer diameters of $d$ and $2d$, respectively. With the members being initially unstressed, the temperatures of (1) and (2) are increased by amounts of $2\Delta T$ and $\Delta T$, respectively, while the temperature of (3) is held constant.

As a result of the temperature changes described above:

a) Determine the axial load (force) carried by each member. State whether each member is experiencing a compressive or tensile load.

b) Determine the axial strain in each member. Include an appropriate sign with each strain.

Leave your answers in terms of, at most, $E$, $\alpha$, $L$, $d$ and $\Delta T$.




***1. Equilibrium***

$$(1)\ (C):\ \sum F = -F_1 + F_2 = 0 \ \Rightarrow\ F_1 = F_2$$

$$(2)\ (D):\ \sum F = -F_2 + F_3 = 0 \ \Rightarrow\ F_3 = F_2$$

***2. Force/elongation***

$$(3)\ e_1 = \frac{F_1(2L)}{EA_1} + \alpha(2\Delta T)(2L) \ ;\ A_1 = \pi\left(\frac{d}{2}\right)^2$$

$$(4)\ e_2 = \frac{F_2 L}{EA_2} + \alpha\Delta T L \ ;\ A_2 = \pi\left(\frac{d}{2}\right)^2$$

$$(5)\ e_3 = \frac{F_3 L}{EA_3} \ ;\ A_3 = \pi\left(\frac{2d}{2}\right)^2 - \pi\left(\frac{d}{2}\right)^2 = \frac{3}{4}\pi d^2$$

***3. Compatibility***

$$u_C = u_B + e_1 = e_1$$

$$u_D = u_C + e_2 = e_1 + e_2$$

$$(6)\ u_H = u_D + e_3 = e_1 + e_2 + e_3 = 0$$

***4. Solve***

$$(3)-(6)\ \Rightarrow\ 2\frac{F_1L}{E\pi\left(d^2/4\right)}+4\alpha\Delta TL+\frac{F_2L}{E\left(d^2/4\right)}+\alpha\Delta TL+\frac{F_3L}{E\left(3d^2/4\right)}=0\ \Rightarrow$$

$$8\frac{F_1}{E\pi d^2}+5\alpha\Delta TL+4\frac{F_2}{Ed^2}+\frac{4}{3}\frac{F_3}{Ed^2}=0\ \Rightarrow$$

$$8F_1+4F_2+\frac{4}{3}F_3=-5\pi\alpha\Delta TLEd^2$$

Combining with (1) gives:

$$\left(8+4+\frac{4}{3}\right)F_1=-5\pi\alpha\Delta TLEd^2\quad\Rightarrow\quad F_1=F_2=F_3=-\frac{3}{8}\pi\alpha\Delta TLEd^2\ \ (compression)$$

***Strains***

$$\varepsilon_1=\frac{e_1}{2L}=\frac{4F_1}{E\pi d^2}+2\alpha\Delta T=\frac{4F_1}{E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)+2\alpha\Delta T=\frac{1}{2}\alpha\Delta T$$

$$\varepsilon_2=\frac{e_2}{L}=\frac{4F_2}{E\pi d^2}+\alpha\Delta T=\frac{4}{E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)+\alpha\Delta T=-\frac{1}{2}\alpha\Delta T$$

$$\varepsilon_3=\frac{e_3}{L}=\frac{4F_3}{3E\pi d^2}=\frac{4}{3E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)=-\frac{1}{2}\alpha\Delta T$$

ME 323 – Mechanics of Materials
Examination #1
October 1st, 2020

PURDUE UNIVERSITY | Mechanical Engineering

Name (Print) SOLUTION
(Last) (First)

**PROBLEM # 2 (25 points)**

The critical components for the design of the planar truss in Figure 2a are considered to be member AB and the pin at C (shown in Figure 2b). The truss is subjected to a single downward force P at A. All members of the truss have a cross-sectional area of $A=1\ in^2$. The cross-sectional area of the pin at C is $A_C=0.5\ in^2$. The factor of safety (FS) against failure of AB by yielding is $FS_{AB}=3$. The factor of safety against ultimate shear failure of the double-sided pin at C is $FS_C=4$.

For member AB, $\sigma_Y=36$ ksi and for the pin material $\tau_U=48$ ksi.

Find the largest P that can be applied without failure of the member AB and pin C.

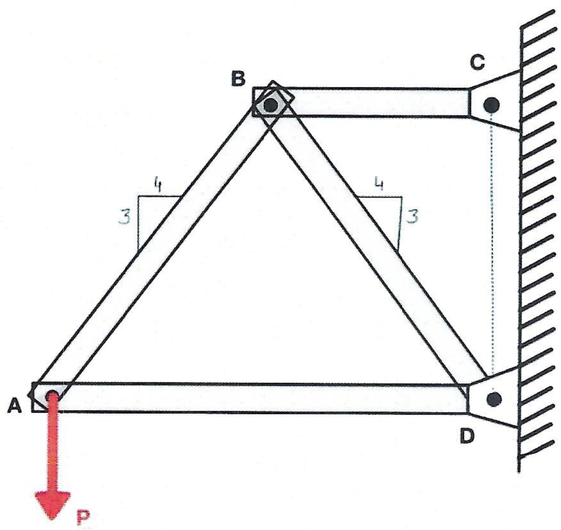



Figure 2(a)

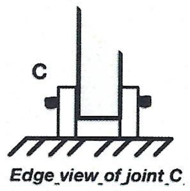
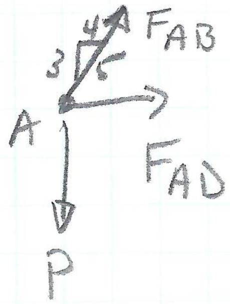



Figure 2(b)

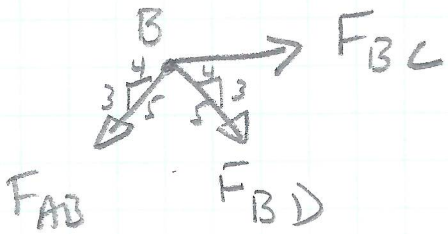
# PROBLEM #2

## Pin A




$$\Sigma F_y = \frac{3}{5} F_{AB} - P = 0 \qquad \boxed{F_{AB} = \frac{5P}{3}}$$

## Pin B




$$\Sigma F_x = F_{BC} + \frac{4}{5} F_{BD} - \frac{4}{5} F_{AB} = 0$$

$$\Sigma F_y = -\frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} = 0$$

$$F_{BD} = -F_{AB} = -\frac{5P}{3}$$

$$F_{BC} = \frac{4}{5} F_{AB} - \frac{4}{5} F_{BD}$$

$$= \frac{4}{\cancel{5}}\left[\frac{\cancel{5}}{3} P\right] - \frac{4}{\cancel{5}}\left[-\frac{\cancel{5}P}{3}\right]$$

$$\boxed{F_{BC} = \frac{8}{3} P}$$

For AB

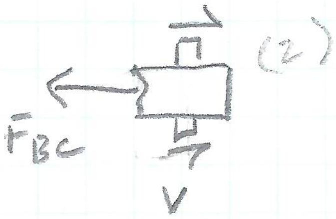
$$\sigma_{AB} = \frac{F_{AB}}{A} = \boxed{\frac{5P}{3} = \sigma_{AB}}$$

$$(FS)_{AB} = \frac{\sigma_y}{\sigma_{AB}} = 3 \Rightarrow \sigma_{AB} = \frac{\sigma_y}{3}$$

$$\boxed{\frac{5P}{3} = \frac{36\text{ ksi}}{3} = 12\text{ ksi}}$$

$$\boxed{P_{AB} = \frac{3}{5}(12) = 7200\text{ lb.}} \quad = \frac{36}{5}$$

For pin C




$$V_{pin} = \frac{F_{BC}}{2} = \frac{\cancel{8}\,4}{(3)\cancel{(2)}} P$$

$$V_{pin} = \frac{4}{3}P$$

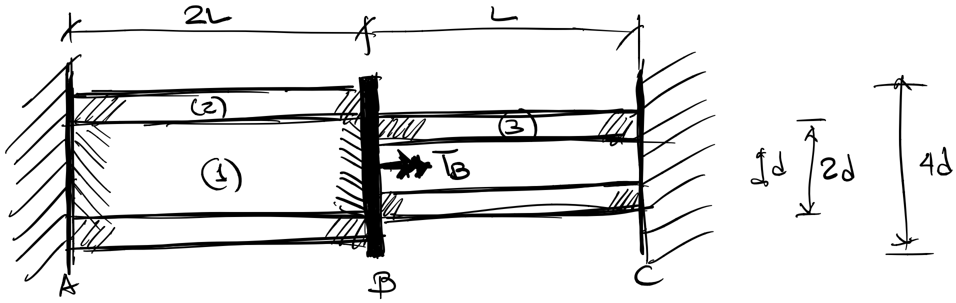
$$\tau_{pin} = \frac{V_{pin}}{A_p} = \frac{\frac{4}{3}P}{0.5} = \boxed{\frac{8}{3}P = \tau_{pin}}$$

$$(FS)_{pin} = \frac{\tau_U}{\tau_{pin}} \Rightarrow \tau_{pin} = \frac{\tau_U}{(FS)_{pin}} = \frac{48}{4} = 12\text{ ksi}$$

$$\frac{8}{3}P = 12\text{ ksi} \quad (.1)\ \boxed{P_{pin} = \frac{36\text{ ksi}}{8} = 4{,}500\text{ lb}}$$

$$\boxed{P_{allow} = 4{,}500\text{ lb}}$$

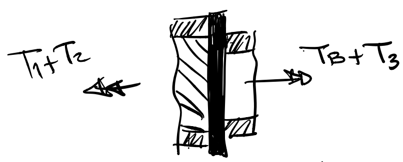
Smallest




- **FBD & Equilibrium**

→ 1 equation & 3 unknowns

$\boxed{\text{STATICALLY INDETERMINATE}}$




$$T_B + T_3 - T_1 - T_2 = 0 \qquad (*3)$$

- **Compatibility conditions**

$\phi_A = 0$ & $\phi_C = 0$

$$\left.\begin{array}{l} \phi_1 = \phi_B \\ \phi_2 = \phi_B \\ \phi_3 = -\phi_B \end{array}\right\} \quad \begin{array}{l} \phi_1 = \phi_2 \\ \phi_2 = -\phi_3 \end{array} \Rightarrow$$

- **Torque-twist behavior**

$\Rightarrow$

$$T_1 \frac{L_1}{G_1 I_{p1}} = T_2 \frac{L_1}{G_2 I_{p2}} \qquad (*1)$$

$$T_2 \frac{L_1}{G_2 I_{p2}} = -T_3 \frac{L_3}{G_3 I_{p3}} \qquad (*2)$$

with ... $\quad I_{p1} = \frac{\pi (2d/2)^4}{2} = \frac{\pi d^4}{2}$

$$I_{p2} = \frac{\pi}{2}\left[(4d/2)^4 - (2d/2)^4\right] = \frac{\pi}{2}\left[16d^4 - d^4\right] = \frac{\pi}{2} 15 d^4$$

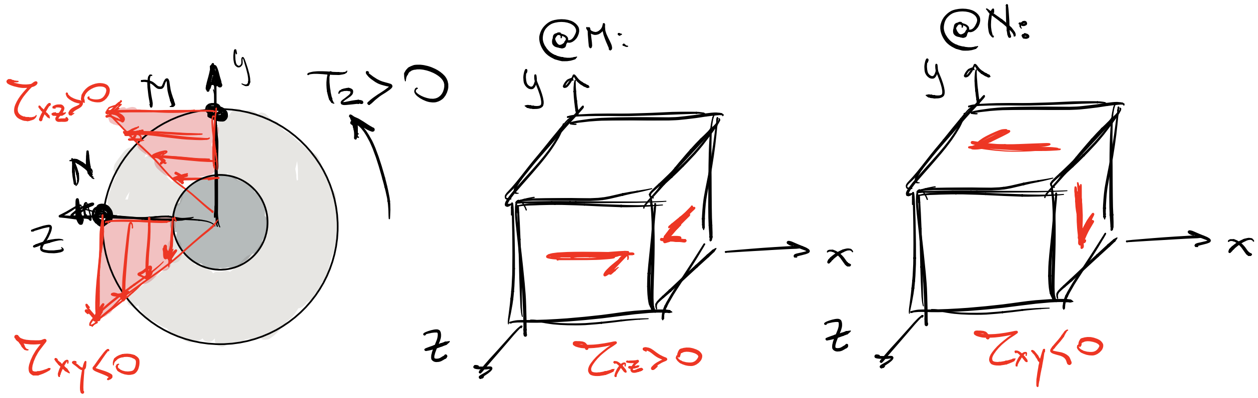
$$I_{p3} = \frac{\pi}{2}\left[(2d/2)^4 - (d/2)^4\right] = \frac{\pi}{2}\left[d^4 - \frac{1}{16}d^4\right] = \frac{\pi}{32}\left[16d^4 - d^4\right] = \frac{\pi}{32} 15 d^4$$

- **Solve**

$$(*1) \Rightarrow T_1 = T_2 \frac{G_1}{G_2 15}$$

$$(*2) \Rightarrow T_3 = -T_2 \frac{L_1}{L_3} \frac{G_3}{G_2} \frac{1}{16}$$

$$(*3) \Rightarrow T_B = T_2 \left[\frac{G_3 L_1}{16 G_2 L_3} + \frac{G_1}{15 G_2} + 1\right] \Rightarrow \boxed{\begin{array}{l} T_2 = \frac{1}{3} T_B \\ T_1 = \frac{1}{3} T_B \\ T_3 = -\frac{1}{3} T_B \end{array}}$$



τxz>0
M
y
N
z
τxy<0
Tz>0
@M:
y
x
z
τxz>0
@N:
y
x
z
τxy<0

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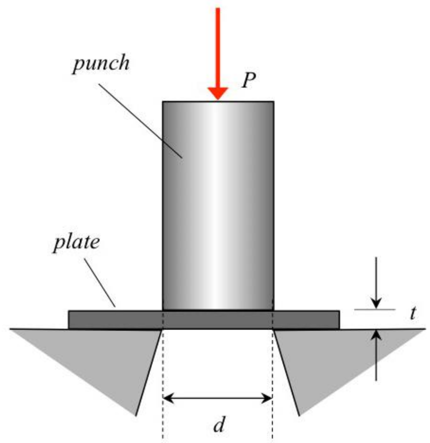
**(Print) (Last) (First)**

**<u>PROBLEM #4 (25 Points. Partial credits may not be granted)</u>:**

## *PART A – 3 points*

A hydraulic punch of diameter $d$ is used to punch circular holes in a plate of thickness $t$. Upon applying a punch force of $P$, what is the shear resistance $\tau$ of the plate?




Justify your answer.

(a) $\boldsymbol{\tau} = \frac{\boldsymbol{P}}{\frac{\pi}{4}\boldsymbol{d}^2}$

(b) $\boldsymbol{\tau} = \frac{\boldsymbol{P}}{\pi \boldsymbol{d}\boldsymbol{t}}$

(c) $\boldsymbol{\tau} = \frac{\boldsymbol{P}/2}{\pi \boldsymbol{d}\boldsymbol{t}}$

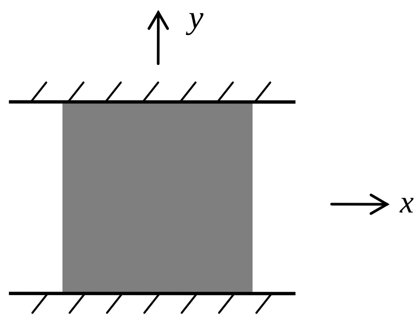
(d) $\boldsymbol{\tau} = \frac{\boldsymbol{P}/2}{\frac{\pi}{4}\boldsymbol{d}^2}$

(e) None of the above

**<u>PROBLEM #4 (cont.)</u>:**

***PART B – 3 points***

A block is fully constrained in the *y* direction and is free to expand in the *x* and *z* (out of paper) directions. The block is free of stress at the initial temperature *T*. When the temperature is increased by $\Delta T$, which of the following statements about stresses and strains is correct? The coeffiecient of thermal expansion of the block is $\alpha$. Young's modulus and Poisson's ratio are *E* and $\upsilon$, respectively. Justify your answer.




(a) $\boldsymbol{\sigma_x = -\alpha E \Delta T, \varepsilon_x = 0, \sigma_y = 0, \varepsilon_y = \alpha \Delta T}$

(b) $\boldsymbol{\sigma_x = 0, \varepsilon_x = \alpha \Delta T, \sigma_y = -\alpha E \Delta T, \varepsilon_y = -v \varepsilon_x}$

(c) $\boldsymbol{\sigma_x = 0, \varepsilon_x = (1+v)\alpha \Delta T, \sigma_y = -\alpha E \Delta T, \varepsilon_y = 0}$

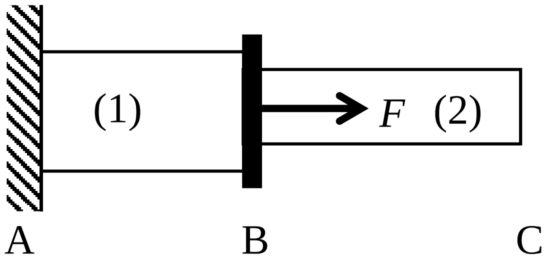
(d) $\boldsymbol{\sigma_x = -\alpha E \Delta T, \varepsilon_x = 0, \sigma_y = 0, \varepsilon_y = -v \varepsilon_x}$

(e) None of the above

**PROBLEM #4 (cont.):**

***PART C – 7 points***

A stepped shaft is composed of two members (1) and (2). The two members are connected by a rigid connector at B on which an external force $F$ is applied. The shaft is fixed to the wall at A and has a free end at C. Circle TRUE or FALSE for the following statements (No need to justify your answers):




(a) **TRUE** or **FALSE**: The two members have the same internal force.

(b) **TRUE** or **FALSE**: The member (2) is free of stress.

(c) **TRUE** or **FALSE**: The internal forces in the two members are of equal magnitude but different signs.

(d) **TRUE** or **FALSE**: The shaft is a statically indeterminate structure.

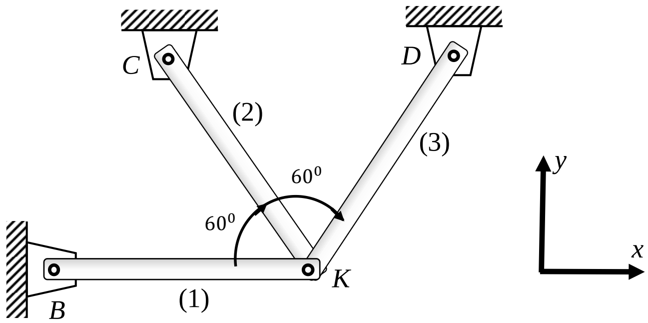
(e) **TRUE** or **FALSE**: The two members have the same elongation.

(f) **TRUE** or **FALSE**: The sum of elongations of the two members is 0.

(g) **TRUE** or **FALSE**: Elongation of member (1) is equal to the elongation of the shaft.

***PART D – 3 points***

The geometry of deformation in planar truss is given by $e = u\cos\theta + v\sin\theta$, where $e$ represents the elongation of a truss member, $u$ and $v$ are the displacement of the joint K in the $x$ and $y$ directions, respectively. Determine the $\theta$ value, in radian, for the members (1), (2), and (3) in the following truss for an arbitrary force applied at the joint K:



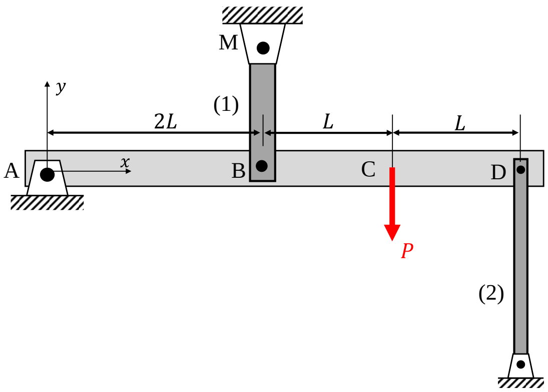

$\boldsymbol{\theta_1} =$ _____**0**_________

$\boldsymbol{\theta_2} =$ _______$\boldsymbol{5\pi/3}$________

$\boldsymbol{\theta_3} =$ ________$\boldsymbol{4\pi/3}$_______

***PART E – 3 points***

A rigid bar ABCD is supported by a pin at A, and two rods (1) and (2) as follows. A force $P$ is applied at C. By equilibrium analysis, this structure is found to be statically indeterminate. Which of the following statements represents the correct compatibility condition between the elongation of rod (1) $e_1$ and elongation of rod (2) $e_2$?




Make a sketch to justify your answer.

(a) $\boldsymbol{e_1 = e_2}$

(b) $\boldsymbol{e_1 = -e_2}$

(c) $\boldsymbol{e_1 = e_2/2}$

(d) $\boldsymbol{e_1 + e_2 = 0}$

(e) None of the above

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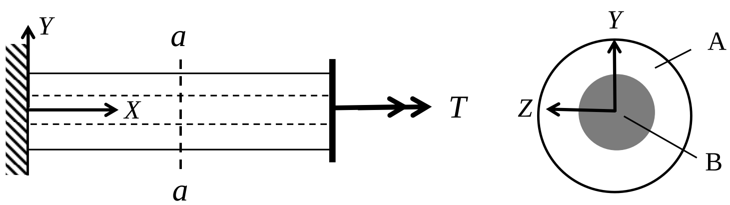
***PART F – 6 points***

A circular bimetallic bar consists of a tubular shell A and a core B. The bimetallic bar is subject to a torque $T$. The shear moduli of the core and shell are known to be $G_A = 2G_B$, and polar moment of inertia $I_{PA} = 8I_{PB}$. In the cross section *aa*, circle TRUE or FALSE for the following statements (No need to justify your answers):




(a) **TRUE** or **FALSE**: The two members experience the same internal torque.

(b) **TRUE** or **FALSE**: The two members experience the same twist angle within the cross section *aa*.

(c) **TRUE** or **FALSE**: The twist angle $\varphi$ within the cross section *aa* is a linear function of the radial distance.

(d) **TRUE** or **FALSE**: The shear strain distribution (i.e., shear strain as a function of the radial distance) within the cross section *aa* is continuous across the boundary between A and B.

(e) **TRUE** or **FALSE**: The shear stress distribution (i.e., shear stress as a function of the radial distance) within the cross section *aa* is continuous across the boundary between A and B.

(f) **TRUE** or **FALSE**: The shear stress distribution (i.e., shear stress as a function of the radial distance) in A and B on the cross section *aa* has the same slop.