Name *(Print)* ***SOLUTION***

(Last) *(First)*

ME 323 - Mechanics of Materials

Exam # 2

November 4, 2015

6:30 – 7:30 PM

Instructions:

Circle your lecturer's name and your class meeting time.

Krousgrill	Gonzalez	Ghosh	Zhao
11:30AM-12:20PM	12:30-1:20PM	2:30-3:20PM	4:30-5:20PM

Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Write your name on every sheet of the exam.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution does not follow a logical thought process, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

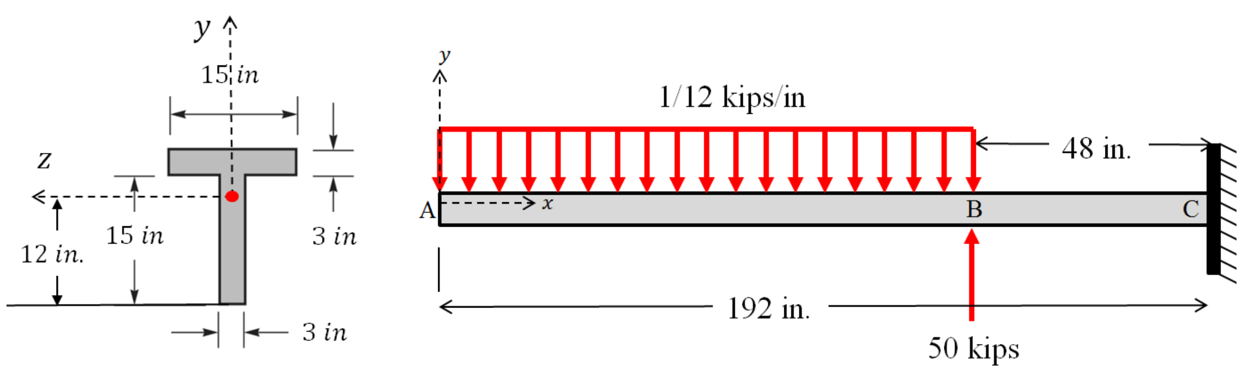
SOLUTION

Problem 1.

A structural 'T' section is used as a cantilever beam (AC) to support a distributed load of 1 kip/ft and a point load of 50 kips as shown in Fig. 1.

(a) Draw the Shear Force and Bending Moment Diagrams for the beam
(b) Determine the location (x,y) and magnitude of maximum Tensile Flexural stress.
(c) Determine the location (x,y) and magnitude of maximum Compressive Flexural stress.

Please use the coordinate system already provided in the figures. The centroid and the area moment of inertia are shown in the figure.

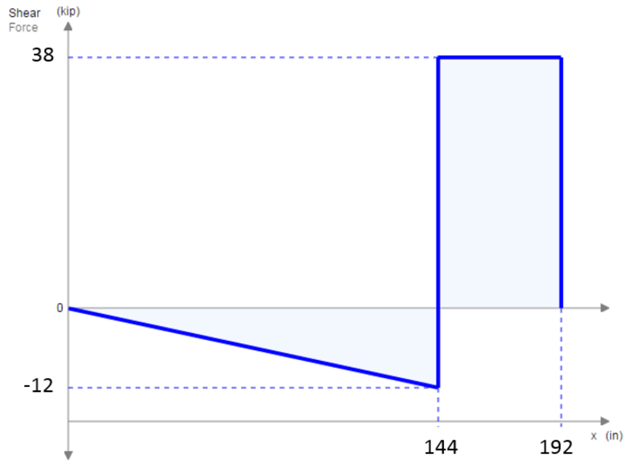


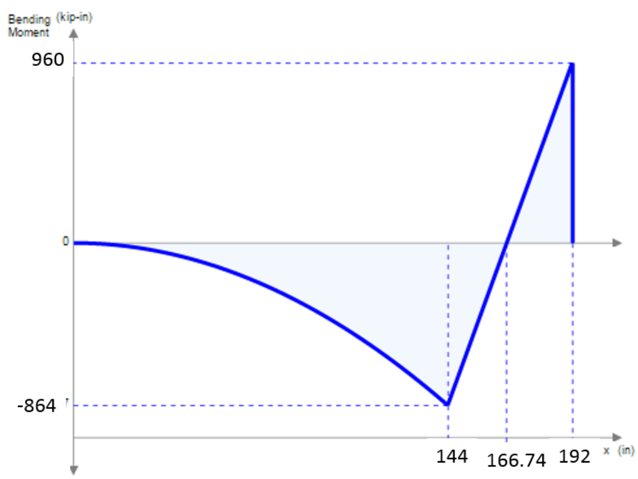
$I_z = 2700\ in^4$

Fig. 1

Solution:

(a) Shear Force and Bending Moment Diagrams:





From the Bending Moment Diagram, it can be observed that there is a positive maximum and negative maximum moment. The cross section is unsymmetrical, therefore both the points ($x = 144\ in. and\ x = 192\ in.$) of maximum moment need to be investigated.

At x=144 in, y=-12in, M=-864 kips in.

$$\sigma = -\frac{(-864)(-12)}{2700} = -3.84\ in.(C)$$

At x=144 in, y=6in, M=-864 kips in.

$$\sigma = -\frac{(-864)(6)}{2700} = 1.92\ ksi\ (T)$$

At x=192 in, y=-12in, M=960 kips in.

$$\sigma = -\frac{(960)(-12)}{2700} = 4.26ksi\ (T)$$

At x=192 in, y=6in, M=960 kips in.

$$\sigma = -\frac{(960)(6)}{2700} = -2.13\ ksi\ (T)$$

(b) Location of maximum Compressive Stress: x=144 in., y=-12 in.

Magnitude of maximum Compressive stress: 3.84 ksi

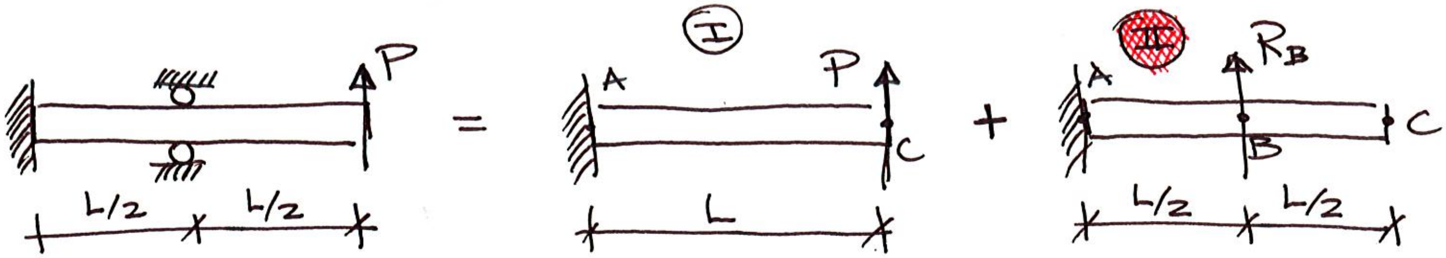
(c) Location of maximum Tensile Stress: x=192 in. , y=-12 in.

Magnitude of maximum Tensile stress: 4.26 ksi

ME 323 Examination #2 Name SOLUTION

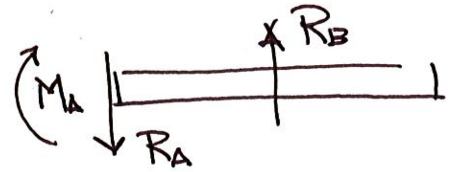
November 4, 2015

PROBLEM NO. 2 (continued)



- <u>Subproblem I</u>: $v_I(x) = \dfrac{Px^2}{6EI}(3L - x)$ for $x \in [0, L]$

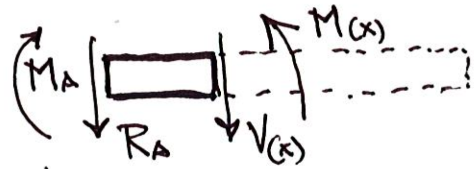
- <u>Subproblem II</u>:



- Equilibrium $\sum F_y = 0 \Rightarrow R_A = R_B$

$$\left(\sum M\right)_A = 0 \Rightarrow M_A = R_B L/2$$

- <u>For $x \in [0, L/2]$</u>



$\Rightarrow M(x) = \dfrac{R_B L}{2} - R_B \cdot x$

Using 2nd-order method

$$EI\, v_{II}'' = M(x) = \frac{R_B L}{2} - R_B x \quad \text{for } x \in [0, L/2]$$

$$EI\, v_{II}' = \frac{R_B L}{2} x - \frac{1}{2} R_B x^2 + C_1$$

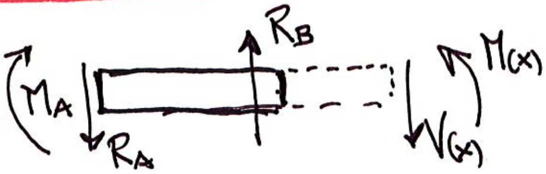
$$EI\, v_{II} = \frac{R_B L}{4} x^2 - \frac{R_B}{6} x^3 + C_1 x + C_2$$

Boundary conditions: $v_{II}(x=0) = 0$; $v_{II}'(x=0) = 0$

$$\Rightarrow C_1 = 0 \text{ and } C_2 = 0$$

Therefore, $\boxed{v_{II}(x) = \dfrac{R_B x^2}{6EI}\left(\dfrac{3L}{2} - x\right) \text{ for } x \in [0, L/2]}$

- <u>For $x \in [L/2, L]$</u>



$\Rightarrow M(x) = \dfrac{R_B L}{2} + R_B\left(x - \dfrac{L}{2}\right) - R_B x$

$M(x) = 0$ for $x \in [\frac{L}{2}, L]$

ME 323 Examination #2 Name SOLUTION

November 4, 2015

PROBLEM NO. 2 (continued)

Using 2nd-order method

$$EI v_{II}'' = M(x) = 0 \quad \text{for } x \in [L/2, L]$$

$$EI v_{II}' = C_1 \Rightarrow EI v_{II} = C_1 x + C_2 \quad \text{for } x \in [L/2, L]$$

Continuity conditions:

$$v_{I}\left(\frac{L}{2}^-\right) = v_{II}\left(\frac{L}{2}^+\right) \Rightarrow \frac{R_B L^3}{24} = C_1 \frac{L}{2} + C_2$$

$$v_{I}'\left(\frac{L}{2}^-\right) = v_{II}'\left(\frac{L}{2}^+\right) \Rightarrow \frac{R_B L^2}{8} = C_1$$

Therefore, $\boxed{v_{II}(x) = \frac{R_B L^2}{24EI}\left(3x - \frac{L}{2}\right) \quad \text{for } x \in [L/2, L]}$

- Using superposition method

$$v(x) = v_I(x) + v_{II}(x) \Rightarrow \text{compatibility equation is } \boxed{v(L/2) = 0}$$

- The reaction at B is:

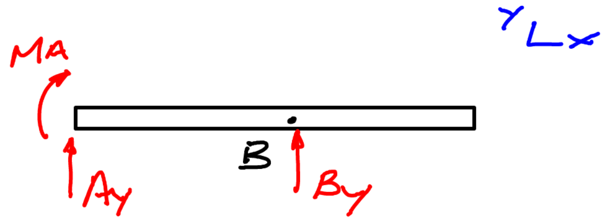
$$v\left(\frac{L}{2}\right) = 0 = \frac{P(L/2)^2}{6EI}\left(3L - L/2\right) + \frac{R_B (L/2)^2}{6EI}\left(\frac{3L}{2} - \frac{L}{2}\right)$$

$$\Rightarrow \boxed{R_B = -\frac{5}{2}P}$$

Alternate solution for Parts (c)–(e)

SOLUTION

Equilibrium



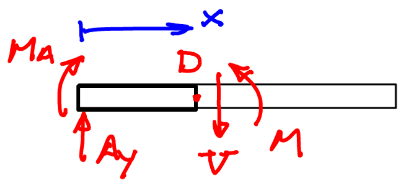
$$\Sigma M_A = -M_A + B_y\left(\tfrac{L}{2}\right) = 0$$

$$\hookrightarrow \; M_A = \frac{B_y L}{2}$$

$$\Sigma F_y = A_y + B_y = 0$$

$$\hookrightarrow \; A_y = -B_y$$

AB



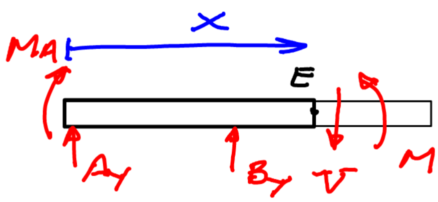
$$\Sigma M_D = -M_A - A_y x + M = 0$$

$$\hookrightarrow \; M(x) = M_A + A_y x = \frac{B_y L}{2} - B_y x$$

- $$\theta_{II}(x) = \cancelto{0}{\theta_{II}(0)} + \frac{1}{EI}\int_0^x M(x)\,dx = \frac{1}{EI}\left[\frac{B_y L}{2}x - \frac{1}{2}B_y x^2\right]$$
- $$v_{II}(x) = \cancelto{0}{v_{II}(0)} + \int_0^x \theta_{II}(x)\,dx = \frac{1}{EI}\left[\frac{B_y L}{4}x^2 - \frac{1}{6}B_y x^3\right]$$

$$\therefore \begin{cases} \theta_{II}\left(\tfrac{L}{2}\right) = \dfrac{1}{EI}\left[\dfrac{B_y L^2}{4} - \dfrac{B_y L^2}{8}\right] = \dfrac{1}{EI}\dfrac{B_y L^2}{8} \\[2ex] v_{II}\left(\tfrac{L}{2}\right) = \dfrac{1}{EI}\left[\dfrac{B_y L^3}{16} - \dfrac{B_y L^3}{48}\right] = \dfrac{1}{EI}\dfrac{B_y L^3}{24} \end{cases}$$

BC



$$\Sigma M_E = -M_A - A_y x - B_y\left(x - \tfrac{L}{2}\right) + M = 0$$

$$\hookrightarrow \; M(x) = \cancel{M_A} + \cancelto{0}{(A_y + B_y)}x - \cancel{\frac{B_y L}{2}} = 0$$

- $$\theta_{II}(x) = \theta_{II}\left(\tfrac{L}{2}\right) = \frac{1}{EI}\frac{B_y L^2}{8}$$
- $$v_{II}(x) = v_{II}\left(\tfrac{L}{2}\right) + \int_{L/2}^{x}\theta(x)\,dx = \frac{1}{EI}\left[\frac{B_y L^3}{24} + \frac{B_y L^2}{8}\left(x - \tfrac{L}{2}\right)\right]$$

Using superposition

$$v(x) = v_I(x) + v_{II}(x) \quad ; \quad v_I(x) = \frac{Px^2}{6EI}(3L - x)$$

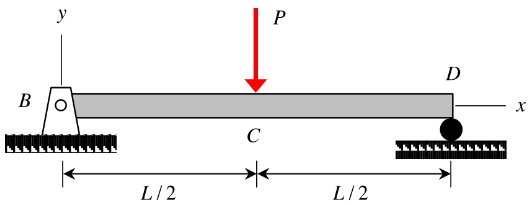
Enforce BC @ B

$$v\left(\tfrac{L}{2}\right) = v_I\left(\tfrac{L}{2}\right) + v_{II}\left(\tfrac{L}{2}\right) = \frac{5P\cancel{L^3}}{48\cancel{EI}} + \frac{1}{\cancel{EI}}\frac{B_y \cancel{L^3}}{24} = 0 \Rightarrow B_y = -\frac{5}{2}P$$

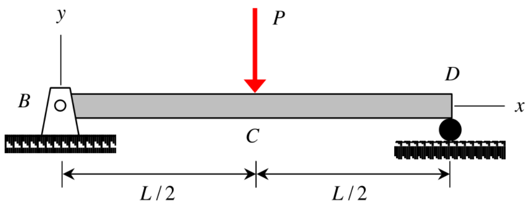
ME 323 Examination #2 **Name** Solution

November 4, 2015

PROBLEM NO. 3 – Part A (6 points max.)



Beam (i) - STEEL



Beam (ii) - ALUMINUM

Beams (i) and (ii) shown above are identical, except that Beam (i) is made up of steel, and Beam (ii) is made up of aluminum. Note that $E_{steel} > E_{aluminum}$.

Let $\left(|\sigma|_{max}\right)_i$ and $\left(|\sigma|_{max}\right)_{ii}$ represent the maximum magnitude of flexural stress in Beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $\left(|\sigma|_{max}\right)_i > \left(|\sigma|_{max}\right)_{ii}$

b) $\boxed{\left(|\sigma|_{max}\right)_i = \left(|\sigma|_{max}\right)_{ii}}$

c) $\left(|\sigma|_{max}\right)_i < \left(|\sigma|_{max}\right)_{ii}$

For determinant beams, internal reactions are found directly from equilibrium analysis. Equilibrium is not material dependent.

Let $\left(|\tau|_{max}\right)_i$ and $\left(|\tau|_{max}\right)_{ii}$ represent the maximum magnitude of the xy-component of shear stress in Beams (i) and (ii), respectively. Circle the correct relationship between these two stresses:

a) $\left(|\tau|_{max}\right)_i > \left(|\tau|_{max}\right)_{ii}$

b) $\boxed{\left(|\tau|_{max}\right)_i = \left(|\tau|_{max}\right)_{ii}}$

c) $\left(|\tau|_{max}\right)_i < \left(|\tau|_{max}\right)_{ii}$

Same argument as for above.

Let $\left(|\delta|_{max}\right)_i$ and $\left(|\delta|_{max}\right)_{ii}$ represent the maximum magnitude of deflection in Beams (i) and (ii), respectively. Circle the correct relationship between these two deflections:

a) $\left(|\delta|_{max}\right)_i > \left(|\delta|_{max}\right)_{ii}$

b) $\left(|\delta|_{max}\right)_i = \left(|\delta|_{max}\right)_{ii}$

c) $\boxed{\left(|\delta|_{max}\right)_i < \left(|\delta|_{max}\right)_{ii}}$

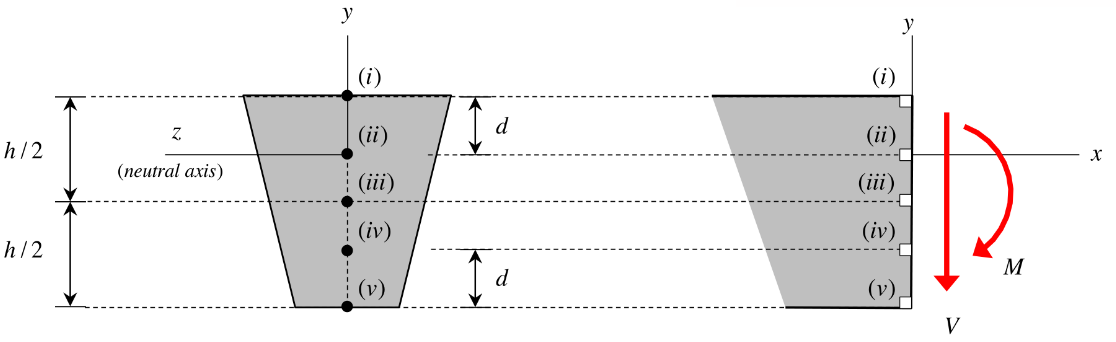
Deflections go as $\frac{1}{EI}$. Since $E_{steel} > E_{aluminum} \Rightarrow$

$(\delta_{max})_{aluminum} > (\delta_{max})_{steel}$

ME 323 Examination #2 **Name** SOLUTION

November 4, 2015

PROBLEM NO. 3 – Part B (10 points max.)



cross section of beam ***SIDE view of beam***

A shear force V and bending moment M act at a cross section of a trapezoidal cross-sectioned beam. Consider the five points (i), (ii), (iii), (iv) and (v) on the beam cross section, as shown above. *Match up the state of stress at each of these five points with the stress elements (a) through (o) shown below.* If you choose "(o) NONE of the above", provide a sketch of the correct state of stress for your answer.

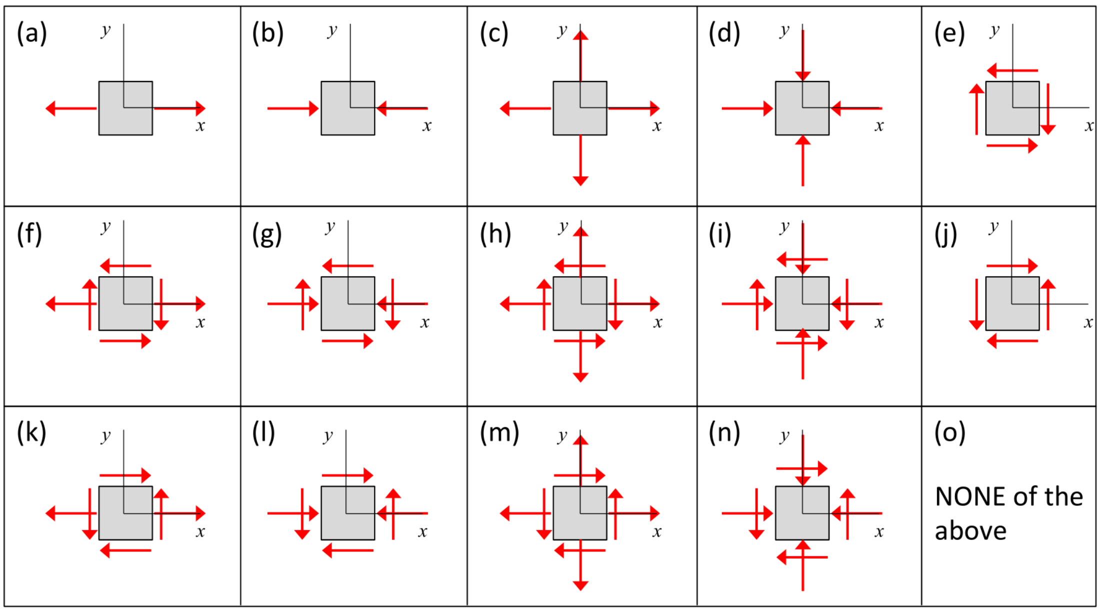
The state of stress at point (i) is a {comp. normal stress & zero shear stress at free surface

The state of stress at point (ii) is e zero normal stress @ neutral axis

The state of stress at point (iii) is g {both tensile normal and downward shear

The state of stress at point (iv) is g Same as for (iii)

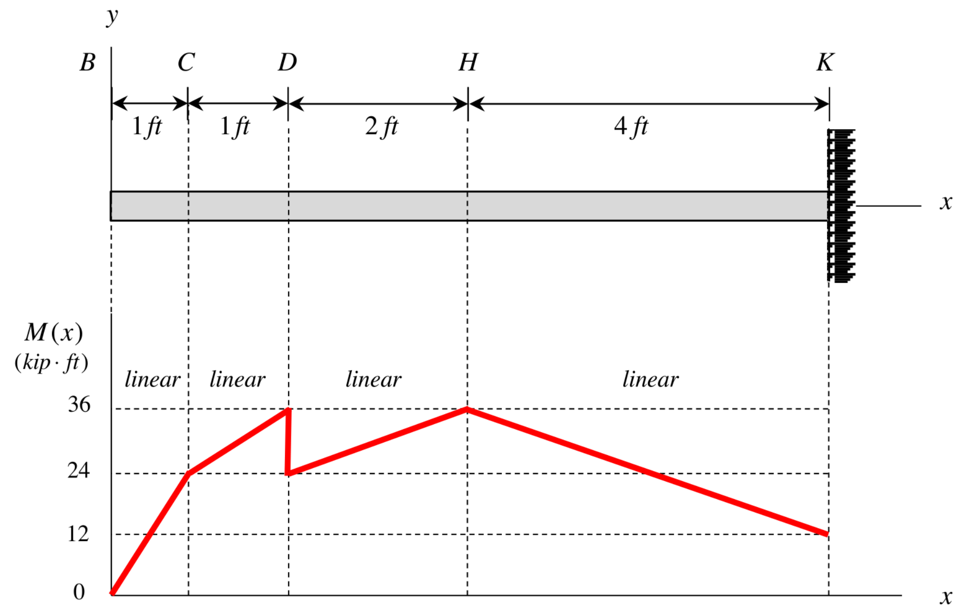
The state of stress at point (v) is b {tensile normal stress & zero shear stress at free surface



ME 323 Examination #2 **Name** SOLUTION

November 4, 2015

PROBLEM NO. 3 – Part C (7 points max.)



The cantilevered beam is loaded with concentrated moments and concentrated forces. This loading is unknown; however, the bending moment diagram for the beam is known and provided above.

a) Determine the maximum value of the magnitude of *internal shear force* V in the beam.

b) If the beam has a square cross section with a cross-sectional area of $A = 2\,ft^2$, determine the maximum value of the magnitude of *shear stress* τ_{xy} in the beam?

(a) Since $V = \frac{dM}{dx}$, the shear force will be maximum where the magnitude of the slope for M is maximum. This occurs between B & C:

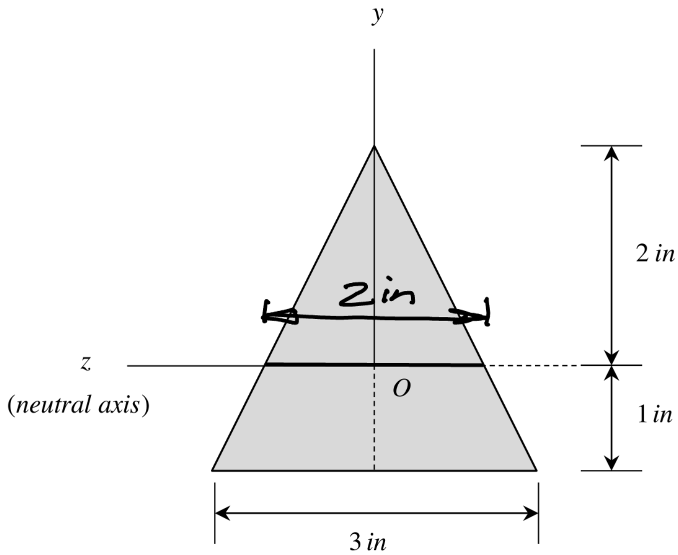
$$V_{BC} = 24 \text{ kips}$$

(b) $|\tau|_{max} = \frac{3}{2}\frac{V_{BC}}{A} = \frac{3}{2}\frac{(24)}{2} = 18 \frac{\text{kips}}{ft^2}$ ← $|\tau|_{max}$

ME 323 Examination #2 **Name** Solution

November 4, 2015

PROBLEM NO. 3 – Part D (7 points max.)



At a given location along a beam, it is known that a shear force of $V = 4$ *kips* acts in the y-direction on the beam's triangular cross section. Determine the *shear stress* at O on the cross section of the beam.

$$\tau = \frac{V A^* \bar{y}^*}{I t}$$

w/ $A^* = \frac{1}{2}(2)(2) = 2\ in^2$

$\bar{y}^* = \frac{1}{3}(2) = \frac{2}{3}\ in$

$I = \frac{1}{36}(3)(3)^3 = 2.25\ in^2$

$t = 2\ in$

$$\therefore \quad \tau = \frac{(4)(2)(2/3)}{(2.25)(2)} = \frac{32}{27}\ \frac{kips}{in^2}$$

τ