ME 323: Mechanics of Materials
Exam 03 - Equation sheet

USEFUL EQUATIONS

$$\sigma_{avge} = \frac{F_N}{A} \qquad FS = \frac{\sigma_{fail}}{\sigma_{allow,member}}$$

$$\tau_{avge} = \frac{V}{A} \qquad FS = \frac{\tau_{fail}}{\tau_{allow,member}}$$

Generalized Hooke's Law

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z) - (1+\nu)\alpha\Delta T\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_y + \nu(\epsilon_x + \epsilon_z) - (1+\nu)\alpha\Delta T\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_z + \nu(\epsilon_x + \epsilon_y) - (1+\nu)\alpha\Delta T\right]$$

Axial Deformations

$$e_{AB} = u_B - u_A \qquad e = \int_0^L \frac{F}{A\,E}dx + \int_0^L \alpha\,\Delta T\;dx, \qquad e = \frac{F\,L}{A\,E} + \alpha\,\Delta T\,L$$

$$e = u\cos(\theta) + v\sin(\theta)$$

Torsional Deformations

$$\phi_{AB} = \phi_B - \phi_A \qquad \phi = \int_0^L \frac{T(x)}{G(x)\,I_p(x)}dx \qquad \phi = \frac{T\,L}{G\,I_P}$$

$$\gamma = \rho\frac{d\phi}{dx} \qquad \tau = G\,\rho\frac{d\phi}{dx} \qquad \gamma = \frac{\rho\,T}{G\,I_p} \qquad \tau = \frac{\rho\,T}{I_p}$$

with $I_p = \int_A \rho^2\,dA,$ $\quad I_p = \dfrac{\pi r^4}{2}$ (solid), $\quad I_p = \dfrac{\pi}{2}\left(r_o^4 - r_i^4\right)$ (hollow)

Bending Deformations

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x) \qquad M = EIv'' \qquad \Delta V = P \qquad \Delta M = -M_0$$

$$\sigma(x,y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}} \qquad I_{zz} = \frac{bh^3}{12}\ \text{(rectangle)},\ I_{zz} = \frac{\pi r^4}{4}\ \text{(circle)}$$

$$\tau(x,y) = \frac{VQ}{I_{zz}t} = \frac{VA^* y^*}{I_{zz}t}, \qquad \tau_{\max} = \frac{3V}{2A}\ \text{(rectangle)},\ \tau_{\max} = \frac{4V}{3A}\ \text{(circle)}$$

Finite Element Method

$$k = (EA)_{\text{avg}} / L$$

Transformation of stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{p1} = \sigma_{ave} + R,\ \sigma_{p2} = \sigma_{ave} - R$$

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \sin 2\theta_{P1} = \frac{\tau_{xy}}{R}, \quad \cos 2\theta_{P1} = \frac{\sigma_x - \sigma_y}{2R}$$

$$\sin 2\theta_{P2} = -\frac{\tau_{xy}}{R}, \quad \cos 2\theta_{P2} = -\frac{\sigma_x - \sigma_y}{2R}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max-inplane} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tan 2\theta_S = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \quad \sin 2\theta_{S1} = -\frac{\sigma_x - \sigma_y}{2R}, \quad \cos 2\theta_{S1} = \frac{\tau_{xy}}{R}$$

$$\sin 2\theta_{S2} = \frac{\sigma_x - \sigma_y}{2R}, \quad \cos 2\theta_{S2} = -\frac{\tau_{xy}}{R}$$

Strain energy density

$$\bar{u} = \frac{1}{2}[\sigma_x(\varepsilon_x - \alpha\Delta T) + \sigma_y(\varepsilon_y - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods

$$U=\frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx \qquad U=\frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx \qquad U=\frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U=\frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}dx$$

Work-energy principle: $U=W$

Castigliano's 2[nd] theorem:

$$\delta_{P_i}=\frac{\partial U}{\partial P_i} \qquad \theta_{M_i}=\frac{\partial U}{\partial M_i} \qquad \phi_{T_i}=\frac{\partial U}{\partial T_i}$$

$$\delta_{P_i}=\int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial P_i}dx+\int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial P_i}dx+\int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial P_i}dx+\int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial P_i}dx$$

$$\theta_{M_i}=\int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial M_i}dx+\int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial M_i}dx+\int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial M_i}dx+\int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial M_i}dx$$

$$\phi_{T_i}=\int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial T_i}dx+\int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial T_i}dx+\int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial T_i}dx+\int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial T_i}dx$$

f_s=6/5 (rectangular cross section), f_s=10/9 (circular cross section)

Thin wall pressure vessels

Cylindrical: $\sigma_h=\dfrac{pR}{t}$ $\qquad \sigma_a=\dfrac{pR}{2t}$

Spherical: $\sigma_s=\dfrac{pR}{2t}$

Failure theories

Maximum distortional energy failure theory for ductile materials:

$$\sigma_M=\sigma_Y$$

$$\begin{aligned}\sigma_M&=\sqrt{\sigma_{P1}^2-\sigma_{P1}\sigma_{P2}+\sigma_{P2}^2}\\&=\frac{\sqrt{2}}{2}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}\\&=\frac{\sqrt{2}}{2}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_x-\sigma_z\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right]^{1/2}\end{aligned}$$

Maximum shear stress failure theory for ductile materials:

$$\tau_{\max}^{abs} = \frac{\sigma_Y}{2}$$

$$\tau_{\max}^{abs} = \frac{\sigma_1 - \sigma_3}{2}$$

Maximum normal stress failure theory for brittle materials:

$|\sigma_{P1}| = \sigma_U$ or $|\sigma_{P2}| = \sigma_U$

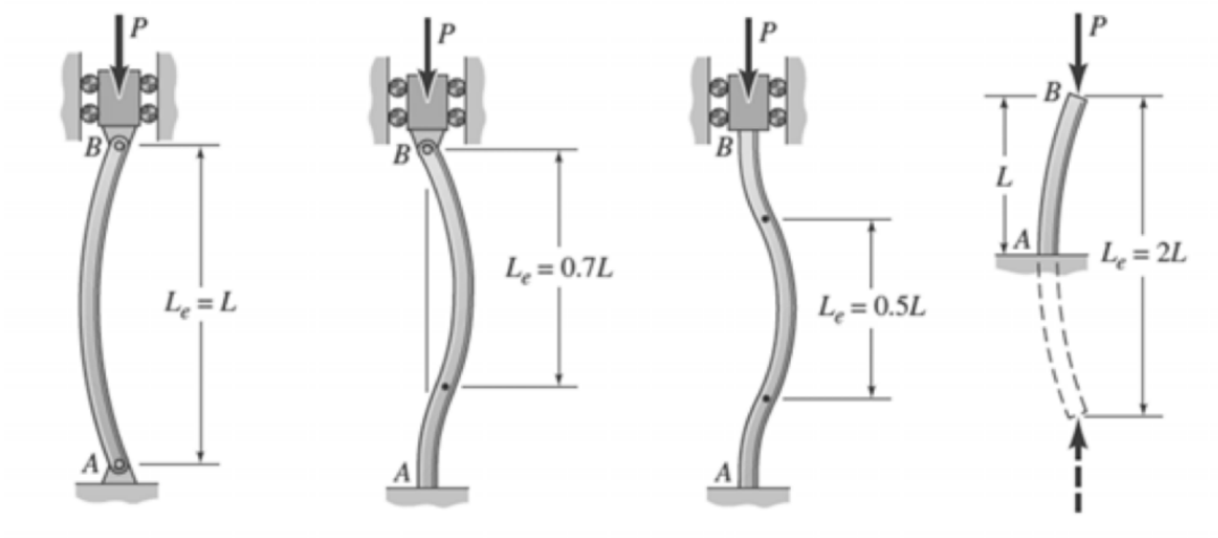
Mohr's failure theory for brittle materials:

If σ_{P1} and σ_{P2} are of the same sign: $\sigma_{P1} = \sigma_{TU}$ or $\sigma_{P2} = -\sigma_{CU}$

If σ_{P1} and σ_{P2} are of different signs: $\dfrac{\sigma_{P1}}{\sigma_{TU}} = \dfrac{\sigma_{P2}}{\sigma_{CU}} + 1$

Buckling of columns

Euler Column $$\frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(L_e / \sqrt{I/A}\right)^2}$$



(a) Pinned-pinned column, $K = 1$. (b) Fixed-pinned column, $K = 0.7$. (c) Fixed-fixed column, $K = 0.5$. (e) Fixed-free column, $K = 2$.