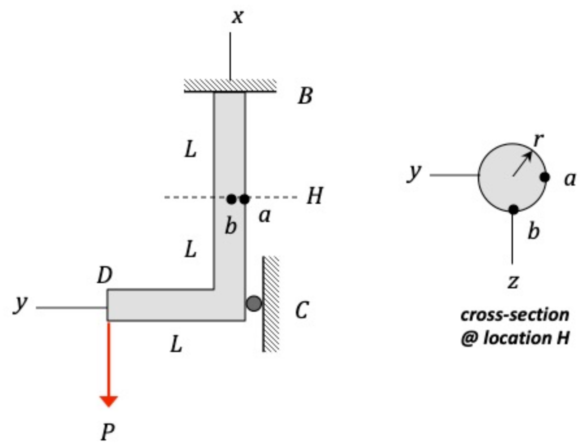
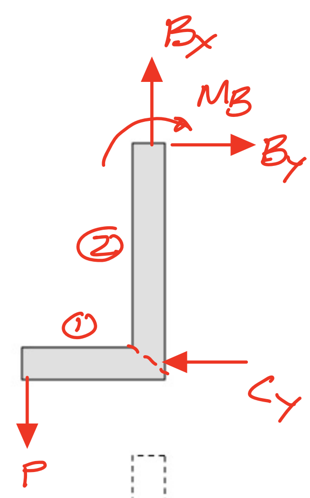
Quiet Week Example No. 2



The L-shaped structural member above has a circular cross-section with a radius of *r*. The member is supported by a fixed connection to ground at B and a roller at C as it carries a downward vertical load of *P* at D. What is the absolute maximum shear stress in the member at points "a" and "b" on the cross-section at H? Expect the following steps in the analysis.

i. Equilibrium analysis
ii. Deflection analysis (for finding external reactions in indeterminate structures)
iii. Internal resultant analysis
iv. Description of the states of stress
v. Mohr's circles for the two states of stress
vi. Absolute maximum shear stress



SOLUTION

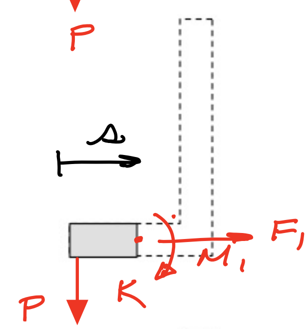
i) Equilibrium

$$\Sigma F_x = -P + B_x = 0$$

$$\Sigma F_y = B_y - C_y = 0$$

$$\Sigma M_B = -M_B - C_y(2L) + PL = 0$$

INDETERMINATE ; Choose C_y as redundant



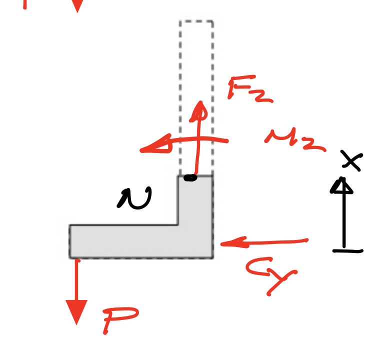
ii) Castigliano

① $\Sigma F_y = -F_1 = 0$

$$\Sigma M_K = -M_1 + P\Delta = 0 \Rightarrow M_1 = P\Delta$$

$$\therefore U_1 = \frac{1}{2}\cancel{\frac{F_1^2(2L)}{EA}}^{\,0} + \frac{1}{2}\int_0^{2L} \frac{M_1^2}{EI}\,ds$$

& independent of C_y



② $\Sigma F_x = F_2 - P = 0 \Rightarrow F_2 = P$

$$\Sigma M_N = -C_y x + PL + M_2 = 0 \Rightarrow M_2 = PL + C_y x$$

$$\therefore \quad U_2 = \frac{1}{2}\frac{F_2^2 L}{EA} + \frac{1}{2}\int_0^L \frac{M_2^2}{EI}\,dx$$

$$U = U_1 + U_2$$

Since C_y = redundant load (by choice):

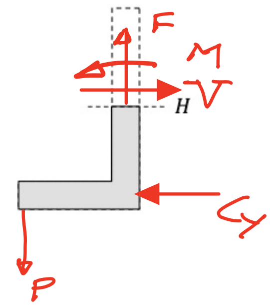
$$0 = \frac{\partial U}{\partial C_y} =$$

$$= \cancel{\frac{\partial U_1}{\partial C_y}}^{\,0} + \frac{L}{EA} F_2 \cancel{\frac{\partial F_2}{\partial C_y}}^{\,0} + \frac{1}{EI}\int_0^L M_2 \frac{\partial M_2}{\partial C_y}\,dx$$

$$= \frac{1}{EI}\int_0^L (-PL + C_y x)(+x)\,dx$$

$$= \frac{1}{EI}\left[-\frac{1}{2}PL^3 + \frac{1}{3}C_y L^3\right]$$

$$\hookrightarrow \quad C_y = \frac{3}{2}P$$

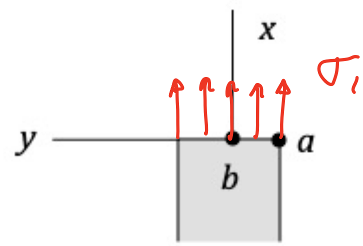


(iii) $\Sigma F_x = 0 \Rightarrow F = P$

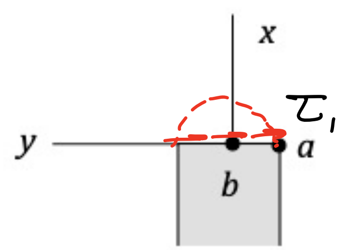
$\Sigma F_y = 0 \Rightarrow V = C_y$

$\Sigma M_H = M - C_y L + PL = 0$

$\hookrightarrow \quad M = \frac{PL}{2}$



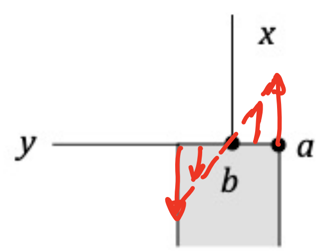
front view
@ H



front view
@ H

(iv)

internal resultant	stress @ a	stress @ b
F	$\sigma_1 = \frac{P}{A}$	$\sigma_1 = \frac{P}{A}$
V	0	$\tau_1 = \frac{4}{3}\frac{V}{A}$
M	$\sigma_2 = \frac{Mr}{I}$	0



front view
@ H

(v) <u>Point "a"</u>

$$\begin{cases} \sigma_{ave} = \dfrac{\sigma_1 + \sigma_2}{2} \\[2ex] R = \dfrac{\sigma_1 + \sigma_2}{2} = |\sigma_{ave}| \end{cases}$$

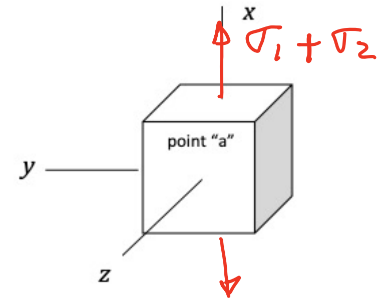
$$|\tau|_{max,abs} = R$$

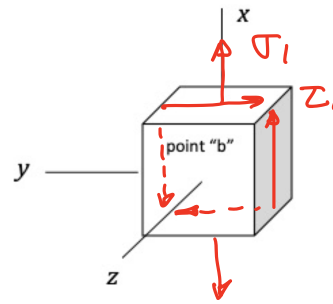
<u>Point "b"</u>

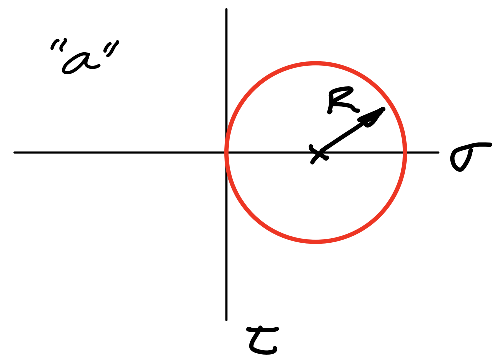
$$\sigma_{ave} = \frac{\sigma_1}{2}$$

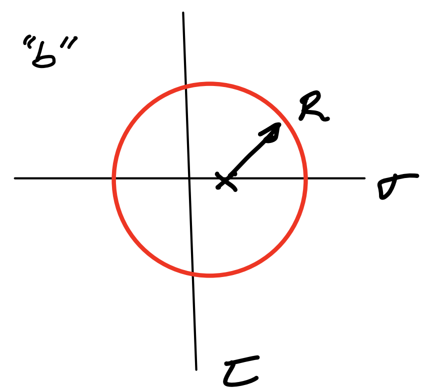
$$R = \sqrt{\left(\frac{\sigma_1}{2}\right)^2 + \tau_1^2} > |\sigma_{ave}|$$

Again, $|\tau|_{max,abs} = R$









<u>Note</u>: Had we not neglected shear

$$C_y = \frac{P}{2\left[f_c \dfrac{EI}{GAL^2} + \dfrac{1}{3}\right]} \approx \frac{3}{2}P$$

$$f_c \frac{EI}{GAL^2} = \frac{\overset{10/9}{f_c}\,\cancel{E}\,\frac{\cancel{\pi}}{4}\,r^{\cancel{4}\,2}}{2(1+\nu)\,\cancel{E}\,(\cancel{\pi}\,\cancel{r^2})\,L^2}$$

$$= \frac{10}{72}\left(\frac{1}{1+\nu}\right)\left(\frac{r}{L}\right)^2 \ll 1$$