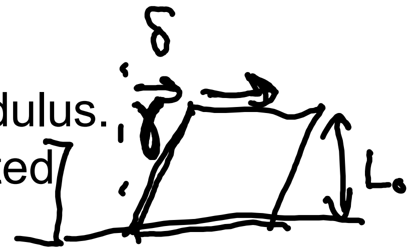
Lecture 3 Review Questions

$\frac{E}{G}$

The shear strain is defined as:

- The elastic modulus divided by the shear modulus.
- The change in angle of a face originally oriented perpendicular to the applied force.
- The displacement in the direction of the applied force divided by the length of the structure perpendicular to the applied force.
- The change in angle between two forces.



$\frac{\delta}{L_0}$

We have discussed three materials parameters (Young's modulus, Poisson's ratio, and shear modulus). These materials parameters are independent of each other:

- True
- $\boxed{\text{False}}$

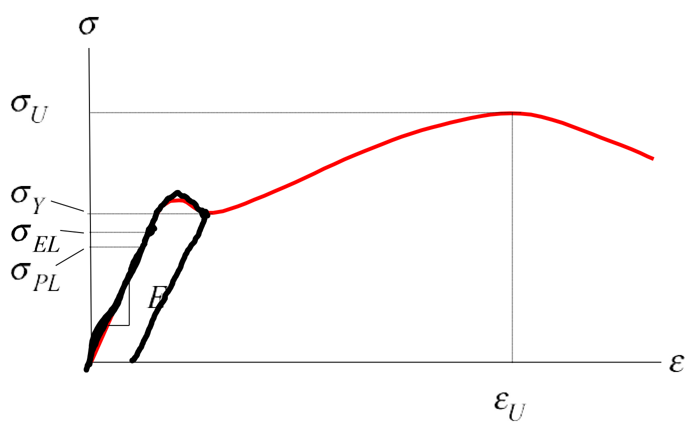
E
ν
G

$$G = \frac{E}{2(1+\nu)}$$

4. Introduction to design of deformable bodies

Objectives:
To study the relationships between design factors of a structure and the allowable loads on the structure in order to be able to bring about a design of the structure to avoid failure.

Background:
Axial stress-strain relationships for steel:

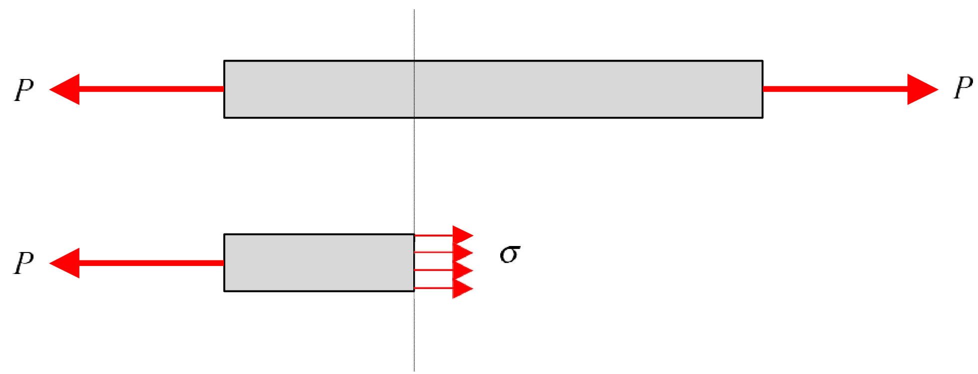


- σ_{PL} = *proportional limit of material*: For $0<\sigma<\sigma_{PL}$, a linear relationship between normal stress and strain exists; that is, we can write: $\sigma = E\varepsilon$. Unloading on this curve follows the reverse of the loading curve, and has $\varepsilon = 0$ at $\sigma = 0$ after complete unloading.
- σ_{EL} = *elastic limit of material*: For $\sigma_{PL}<\sigma<\sigma_{EL}$, a nonlinear relationship between normal stress and strain exists. However, unloading on this curve still follows the reverse of the loading curve.
- σ_Y = *yield point of material*: For $\sigma > \sigma_Y$, yielding occurs in the material. Unloading on this curve does NOT follow the reverse of the loading curve. Importantly, a "permanent set" results with $\varepsilon > 0$ at $\sigma = 0$.
- σ_U = *ultimate stress of material*: For $\varepsilon > \varepsilon_U$, the material begins to "neck down" resulting in a significant local reduction in specimen cross section, a reduction that is not recovered on unloading.

Lecture

a) Failure of member experiencing tensile axial load

Consider a straight structural member having a cross sectional area of A loading with a tensile axial load P, as shown below.



This loading creates a tensile normal stress σ given by:

$$\sigma = \frac{P}{A}$$

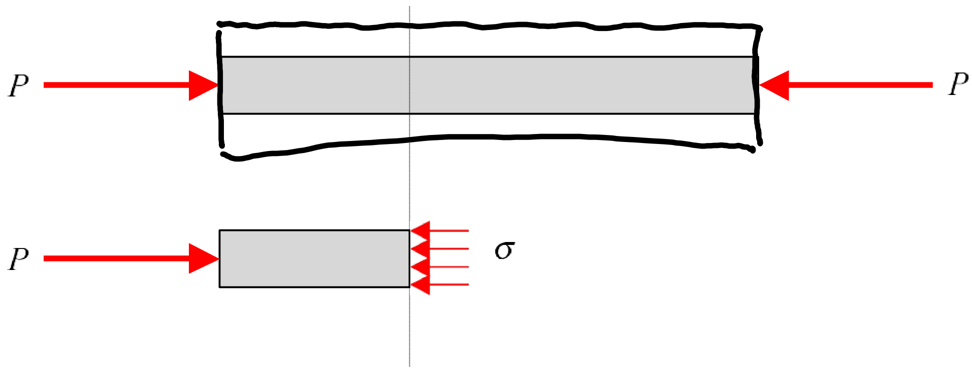
Recall from our earlier discussion:

- If $\sigma > \sigma_Y$, plastic (yielding) deformations will occur. Importantly, the member will experience a permanent set (extended length) after unloading. For many applications where length of structure member is critical (e.g., member in a precision mechanism), a loading of $P > P_Y = \sigma_Y A$ will produce a "failure" of the member. For this situation, P_Y will be known as the failure load, P_{fail}, and $\sigma_{fail} = \sigma_Y$. Parts in car engine.

- If $\varepsilon > \varepsilon_U$ (see figure on preceding page for notation), a dramatic permanent localized reduction in structural member cross section will result. A loading of $P > P_U = \sigma_U A$ can be considered to be a failure of the member since the member has a reduced structural integrity for future loadings. For this situation, P_U will be known as the failure load, P_{fail}, and $\sigma_{fail} = \sigma_U$ Structural components (bridges).

b) <u>Failure of member experiencing compressive axial load</u>

Consider a straight structural member having a cross sectional area of A loading with a compressive axial load P, as shown below.



This loading creates a compressive normal stress σ given by:

$$\sigma = -\frac{P}{A}$$

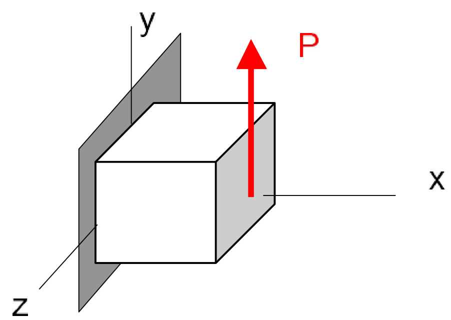
Recall from our earlier discussion:

- For designs where yielding is critical, then a loading of $P > P_Y = \sigma_Y A$ will produce a failure of the member. For this situation, P_Y will be known as the failure load, P_{fail}, and $\sigma_{fail} = \sigma_Y$.
- For designs where structural integrity is critical, a loading of $P > P_U = \sigma_U A$ can be considered to be a failure of the member. For this situation, P_U will be known as the failure load, P_{fail}, and $\sigma_{fail} = \sigma_U$

For long slender members, failure due to compressive loads may not result from the normal stresses exceeding either the yield or ultimate stresses. Instead, the member will fail due to "buckling". As we will learn later on in the course, buckling is in instability developed in the member that can produce large bending stresses. Buckling will occur when $P > P_{buckling}$, where $P_{buckling}$ depends on the length of the member, some measure of the member cross section and Young's modulus of the material. For this mode of failure, $P_{fail} = P_{buckling}$. → Lecture 39.

c) *Failure of member experiencing shear load*

Consider a straight structural member having a cross sectional area of A loading with a shear load P, as shown below.



This loading creates a direct shear stress τ given by:

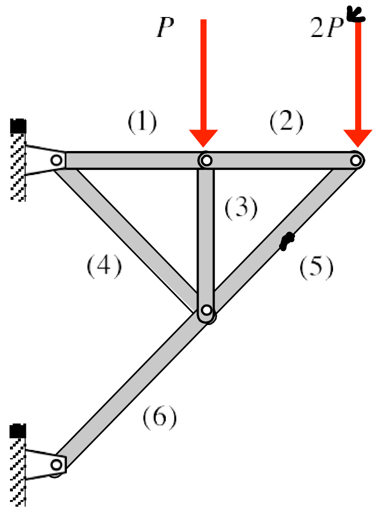
$$\tau = \frac{P}{A}$$

Recall from our earlier discussion:

- For designs where yielding is critical, then a loading of $P > P_Y = \tau_Y A$ will produce a failure of the member, when τ_Y is the shear yield strength of the material. For this situation, P_Y will be known as the failure load, P_{fail}, and $\tau_{fail} = \tau_Y$.
- For designs where structural integrity is critical, a loading of $P > P_U = \tau_U A$ (where τ_U is the ultimate shear strength of the material) can be considered to result in a failure of the member. For this situation, P_U will be known as the failure load, P_{fail}, and $\tau_{fail} = \sigma_U$.

d) Design for strength using factors of safety

In the preceding, we have discussed different mechanisms of failure due to axial and direct shear loadings. The failure load in each case depends on strength properties of the material and the dimensions of the structural member. Note, also, that the loading P is often not known directly, but rather results from internal force analysis of a larger structure containing the structure member of interest. For example, suppose we are interested in the axial stress in member (4) of the truss shown below.



For the stress analysis of member (4), we need to know the external loadings on the truss, the geometry of the problem (member lengths and angles) and good models for the joints of the members and their members' connection to ground. Any uncertainties in the material properties, loadings, member geometries, or pin joint connections will lead to uncertainties in predicting the failure load of member CD. To account for this, we will introduce a "factor of safety" into our design, where factor of safety is defined as:

$$FS = \frac{\text{failure load} \leftarrow \sigma_Y}{\text{allowable}(\text{applied})\ \text{load} \leftarrow} > 1$$

where "*allowable* (*applied*) *load*" is a measure of the loading that is predicted from analysis, and "*failure load*" is a measure of the loading that will lead to failure. For example, we might specify the yield strength of the material σ_Y for "*failure load*", and use σ_{CD} in terms of the external loading P for "*allowable* (*applied*) *load*".

The actual size of *FS* is determined by the designer based on the level on uncertainty in the structural properties as well as the perceived level of risk to failure (for a nuclear reactor design one might choose FS > 3 for critical components, whereas for the closing mechanism on the lecture room door for our class might be closer to FS = 1).

$$FS = \frac{failure\ load}{allowable\ load} > 1$$

Tolerances of Design

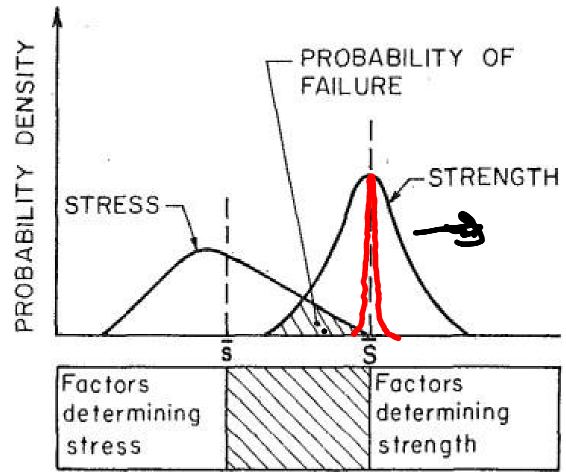
Consequences of Failure

Variation in Environment/Use

1 10

Factor of Safety

https://www.engineeringtoolbox.com/factors-safety-fos-d_1624.html



Thien and Massoud, J Eng Industry, 853, 1974.

Tolerances of Design

Moderate.

Consequences of Failure

High.

Variation in Environment/Use

High

Wind
#Cars.

What FoS you would expect for:

1. Aircraft components 1.1 → 1.5 ; 1.25 landing gear; 2.0 cabin.
2. Boilers 2.0 → Indiana laws ⇒ 4.5; 6.0 for second hand.
3. Stairs and elevators 10,3 → National law ⇒ 4.0
4. Roller coaster 8-9, 3

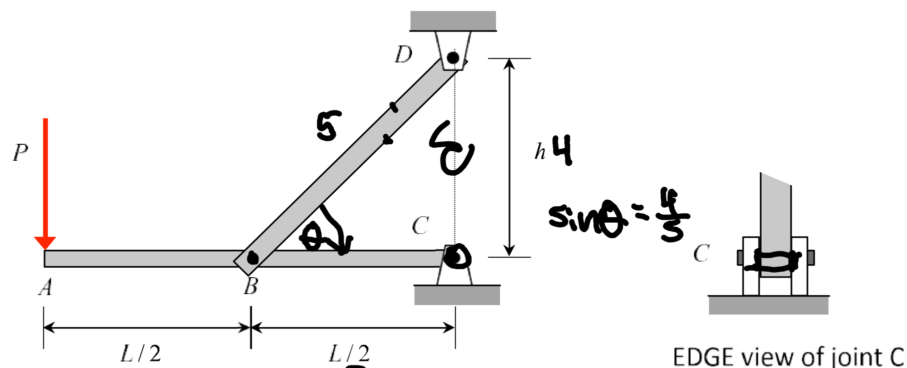
Example 4.1

The critical components for the design of the frame shown below are assumed to be member BD and the pin at C.

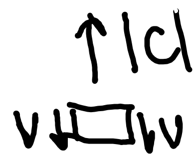
a) Determine the required thickness t (into the page) of member BD (whose width is b) to avoid yielding failure with a factor of safety $FS = 3.0$.

b) Determine the required diameter d of pin C to avoid ultimate shear failure with a factor of safety $FS = 3.0$.

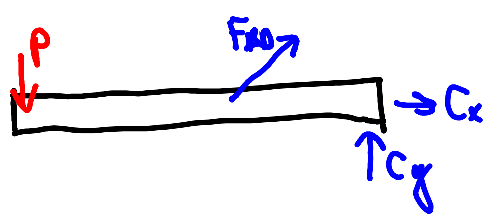
Use the following parameters in your analysis: $P = 2400 lb$, $L = 6 ft$, $h = 4 ft$, $b = 1 in$, $\sigma_Y = 36 ksi$ (for member BD) and $\tau_U = 60\ ksi$ (for pin C).



$\sin\theta = \frac{4}{5}$



a)



$$(\Sigma M)_B = P\left(\frac{L}{2}\right) + C_y\left(\frac{L}{2}\right) = 0 \Rightarrow C_y = -P$$

$$\Sigma F_y = -P + C_y + F_{BD}\sin\theta = 0$$

$$F_{BD} = \frac{P - C_y}{\sin\theta} = 2P\left(\frac{5}{4}\right) = \frac{5}{2}P$$

$$\Sigma F_x = C_x + F_{BD}\cos\theta = 0$$

$$C_x = -\frac{5}{2}P\left(\frac{3}{5}\right) = -\frac{3}{2}P$$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{\left(\frac{5}{2}P\right)}{bt}$$

$$FS = \frac{\sigma_Y}{\sigma_{BD}}$$

$$FS = \sigma_Y\left(\frac{bt}{\frac{5}{2}P}\right)$$

$$t = \frac{\frac{5}{2}P \cdot FS}{\sigma_Y \cdot b}$$

$$t = \frac{\frac{5}{2}(2400)(3)}{(36000)(1)} = 0.5 \text{ in.}$$

b) $|C| = \sqrt{C_x^2 + C_y^2}$

$$C = \sqrt{\left(\frac{3}{2}P\right)^2 + P^2} = \frac{\sqrt{13}}{2}P$$

$$\tau_c = \frac{V}{A} = \frac{|C|/2}{A} = \frac{\left(\frac{\sqrt{13}}{2}P\right)/2}{\pi(d/2)^2} = \frac{\sqrt{13}\,P}{\pi d^2}$$

$$FS = \frac{\tau_U}{\tau_c}$$

$$FS = \tau_U\left(\frac{\pi d^2}{\sqrt{13}\,P}\right)$$

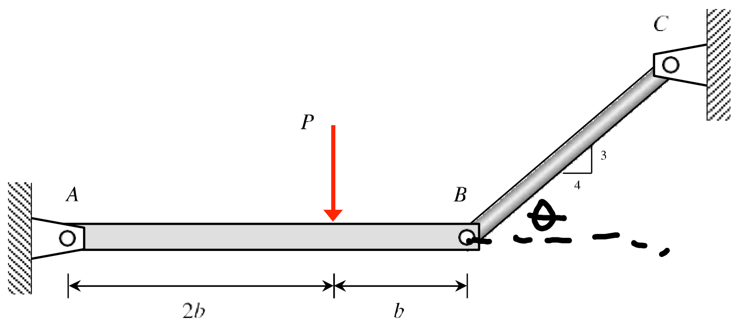
$$d = \sqrt{\frac{(3)\sqrt{13}\,P}{\pi \tau_U}}$$

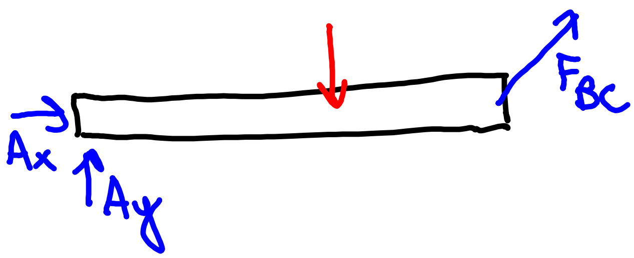
$$d = 0.37\text{ in}$$

$$FS = \frac{\tau_u}{\tau_{allow}}.$$

Example 4.3

If the pins have an allowable shear stress of $\tau_{allow} = 12.5\,ksi$ and the allowable tensile stress of rod CB is $\sigma_{allow} = 16.2\,ksi$ determine to the nearest 1/16 in. the smallest diameter of pins A and B and diameter of rod CB necessary. Use: $b = 2\,ft$ and $P = 3kip$. The pin connections at A and C are double-sided, whereas the pin connection at B is single-sided.





$$(\Sigma M)_A = -P(2b) + F_{BC}\sin\theta\,(3b) = 0$$

$$F_{BC} = \frac{2}{3\sin\theta}P = \frac{2}{3}\left(\frac{5}{3}\right)P = \frac{10}{9}P$$

$$\Sigma F_x = A_x + F_{BC}\cos\theta = 0 \Rightarrow A_x = -\frac{8}{9}P$$

$$\Sigma F_y = A_y + F_{BC}\sin\theta - P = 0$$

$$A_y = \frac{P}{3}$$

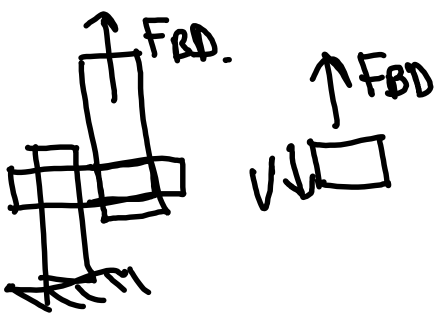
$$\tau_{BC} = \frac{F_{BC}}{\pi (d_{BC}/2)^2} = \frac{4F_{BC}}{\pi d_{BC}^2} \leq \tau_{allow}$$

$$d_{BC} \geq \sqrt{\frac{4F_{BC}}{\pi \tau_{allow}}}$$

$$d_{BC} \geq \sqrt{\frac{4\left(\frac{10}{9}\right)(5\ \text{kips})}{\pi (16.2\ \text{ksi})}}$$

$$d_{BC} \geq 0.512\ \text{in.} \Rightarrow d_{BC} = 9/16 = 0.5625.$$

Pin B



$$V = F_{BC} = \frac{10}{9} P$$

$$\tau_B = \frac{V_B}{A} = \frac{V_B}{\pi (d_B/2)^2} = \frac{4F_{BC}}{\pi d_B^2} < \tau_{allow}$$

$$d_B > \sqrt{\frac{4F_{BC}}{\pi \tau_{allow}}}$$