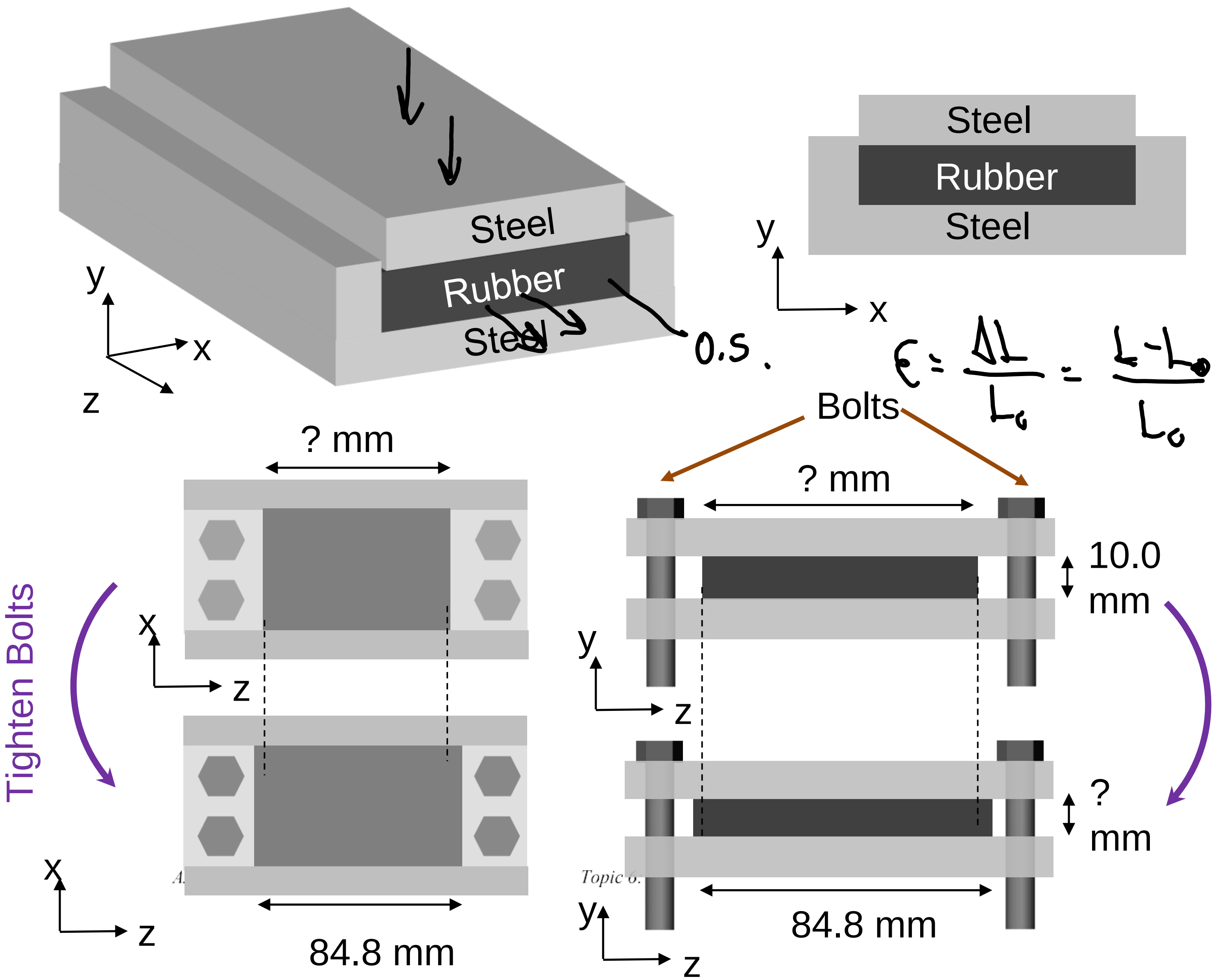
Rubber is often used to reduce the vibration transmission between parts. A rectangular rubber vibration isolator fits into a slot that is the same width as the rubber block in the x-direction. It is bolted between two pieces of steel so that the steel uniformly applies a compressive stress of 1 MPa on the rubber block in the y-direction. There are no constraints in the z-direction. The Poisson's ratio of rubber is 0.5 and the Young's modulus is 12.5 MPa.

(a) What is the stress in the x-direction? (4 points)
(b) The rubber starts with a thickness of 10 mm in the y direction. What is the thickness after applying the 1 MPa stress? (3 points)
(c) In the compressed state, the block has a length of 127.2 mm in the z-direction. What is the original length in the z-direction? (3 points)

You can assume that the deformation of the steel is negligible compared to the deformation of the rubber.



$\epsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$

c) Stress analysis of statically indeterminate structures with externally applied loads

In some structures, the internal force resultants *cannot* be calculated from the static equilibrium of sections. One has to consider material properties and use compatibility relations for this

How much load does each tire carry on the automobile above? How much weight does each leg of the millipede support?

Recall that for a statically *determinate* structure:

# of unknown forces = # of equations of static equilibrium

For a statically *indeterminate* problem:

# of unknown forces > # of equations of static equilibrium

To identify determinate or indeterminate problems: *redundant supports* (if removed, structure remains equilibrium).

unknowns − equations = degree of indeter.

General approach:

1. Force balance. -
2. Force-deformation $e = \frac{FL}{EA}$
3. Compatibility eqns. $e_1 = a e_2$.
4. Solve.

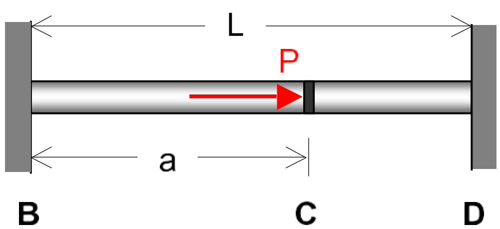
Motivating Example

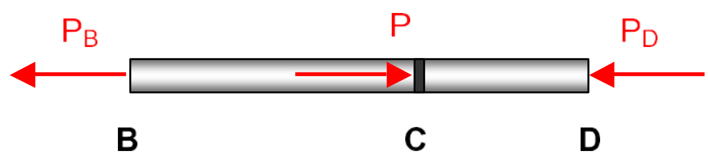
An axial load P acts at point C on a rod, with $a > L/2$. Determine the reactions at the fixed ends B and D. The rod has a cross-sectional area of A with a Young's modulus of E. Answer the following questions:

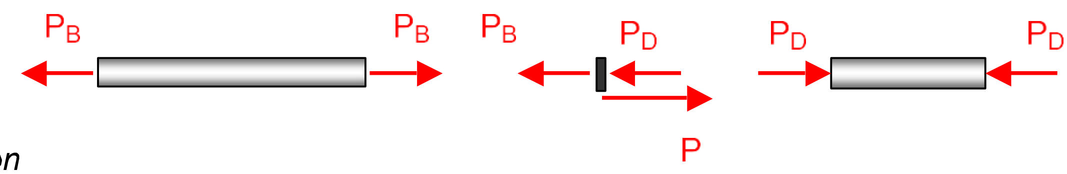
Q1: Why is this problem indeterminate if considering the rod as a rigid member?

Q2: How does a consideration of strain (deformation) allow you to solve for the reactions?

Q3: Which end (B or D) carries the largest reaction load? Defend your answer with a physical argument.







<u>*Solution*</u>

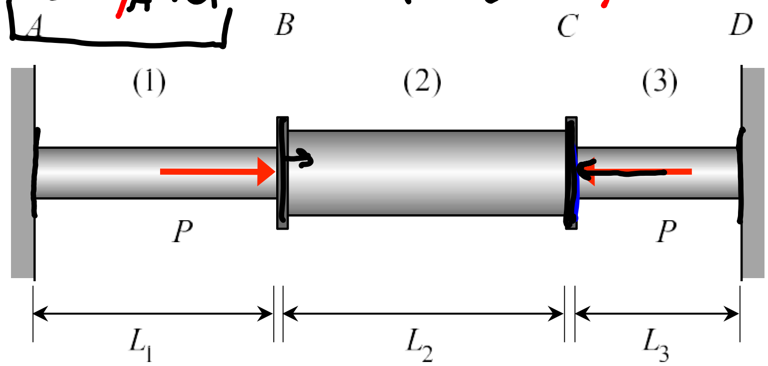
Example 6.7

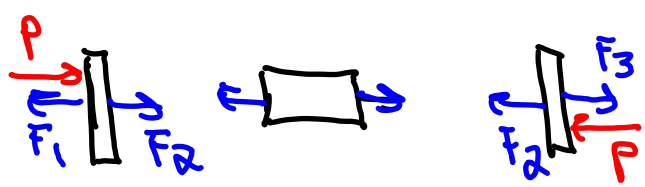
A three-segment rod that is initially stress-free is attached to rigid supports at ends A and D and is subjected to equal and opposite external loads P at nodes B and C as shown in the figure below. The rod is homogeneous and linearly elastic, having a Young's modulus of E. For this analysis, use the following: $A_1 = A_3 = A$, $A_2 = 2A$, $L_1 = 2L$ and $L_2 = L_3 = L$.

a) Determine the axial stresses in the three elements: σ_1, σ_2 and σ_3

b) Determine the horizontal displacements u_B and u_C at nodes B and C, respectively.

$u_B = \cancel{u_A}^{0} + e_1$ $\qquad u_C = u_B + e_2$ $\qquad \cancel{u_D}^{0} = u_C + e_3$





1. $(\Sigma F_x)_B = F_2 - F_1 + P = 0$
   $(\Sigma F_x)_C = F_3 - F_2 - P = 0$

3 unknowns
2 eqns
1 indeterminate.

2.) $e_1 = \dfrac{F_1 L_1}{E_1 A_1}$ $\qquad e_2 = \dfrac{F_2 L_2}{E_2 A_2}$ $\qquad e_3 = \dfrac{F_3 L_3}{E_3 A_3}$

3.) $e_1 + e_2 + e_3 = 0 \leftarrow$

4.) $$\frac{F_1 2L}{EA} + \frac{F_2 L}{2EA} + \frac{F_3 L}{EA} = 0$$

$$2(P + F_2) + \frac{F_2}{2} + (P + F_2) = 0$$

$$\frac{7}{2}F_2 = -3P$$

$$F_2 = -\frac{6}{7}P$$

$$F_1 = P - \frac{6}{7}P = \frac{1}{7}P \qquad F_3 = P - \frac{6}{7}P = \frac{1}{7}P$$

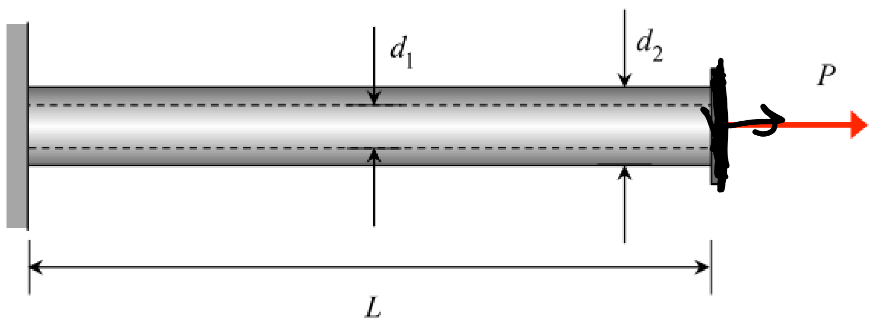
a) $\sigma_1 = \frac{F_1}{A}$

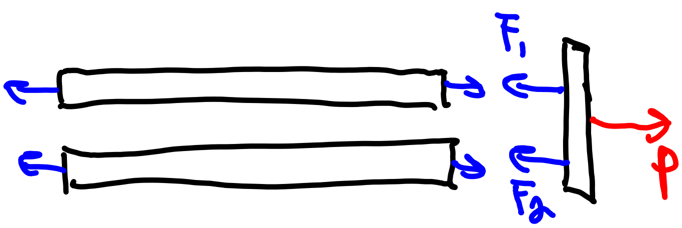
b) $u_B = \frac{F_1 2L}{EA} = \frac{2PL}{7EA}$

Example 6.3

A magnesium-alloy rod ($E_1 = 8x\ 10^3$ksi), having a diameter of $d_1 = 1$in, is encased in a brass tube ($E_2 = 16x\ 10^3$ksi), having an outer diameter of $d_2 = 2$ in. The rod and tube both have a length of L = 30 in. An axial load P = 20 kips is applied to the free end, as shown below.

a) *Determine the normal stresses a_1 and a_2 in the two materials.*
b) *Determine the elongation of the bimetallic rod.*





1. $\Sigma F_x = P - F_1 - F_2 = 0$

2 unknowns
1 eqn.
1.

2.) $e_1 = \dfrac{F_1 L_1}{E_1 A_1}$ $\qquad e_2 = \dfrac{F_2 L_2}{E_2 A_2}$

3.) Compatibility.

$$e_1 = e_2$$

$$\frac{F_1 L_1}{E_1 A_1} = \frac{(P - F_1) L_2}{E_2 A_2}$$

$$F_1 \left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2} \right) = \frac{P}{E_2 A_2}$$

$$F_1 (E_2 A_2 + E_1 A_1) = P E_1 A_1$$

$$\sigma_1 = \frac{F_1}{A_1} = \left[\frac{E_1}{E_1 A_1 + E_2 A_2} \right] P = 3.64 \text{ ksi}$$

$$\sigma_2 = 7.28 \text{ ksi}$$

b) $e = \dfrac{PL}{E_1 A_1 + E_2 A_2}$ $\qquad e_1 = \dfrac{F_1 L_1}{E_1 A_1} = \left(\dfrac{P \cancel{E_1 A_1}}{E_1 A_1 + E_2 A_2} \right) \dfrac{L_1}{\cancel{E_1 A_1}}$

$$\sigma_{U,concrete} = 0.33\ ksi$$

$$\sigma_{U,steel} = \underline{60\ ksi}$$

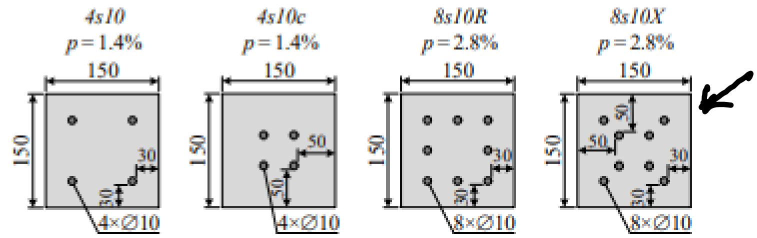
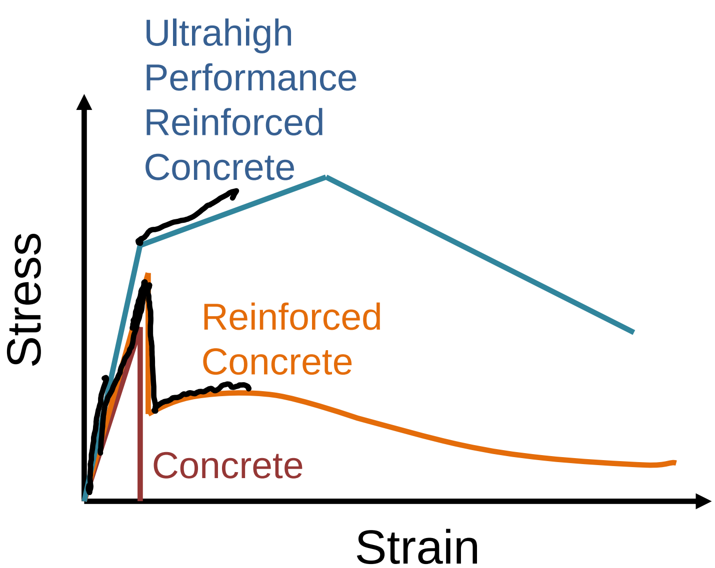


Figure 5. Cross-sections of the test specimens



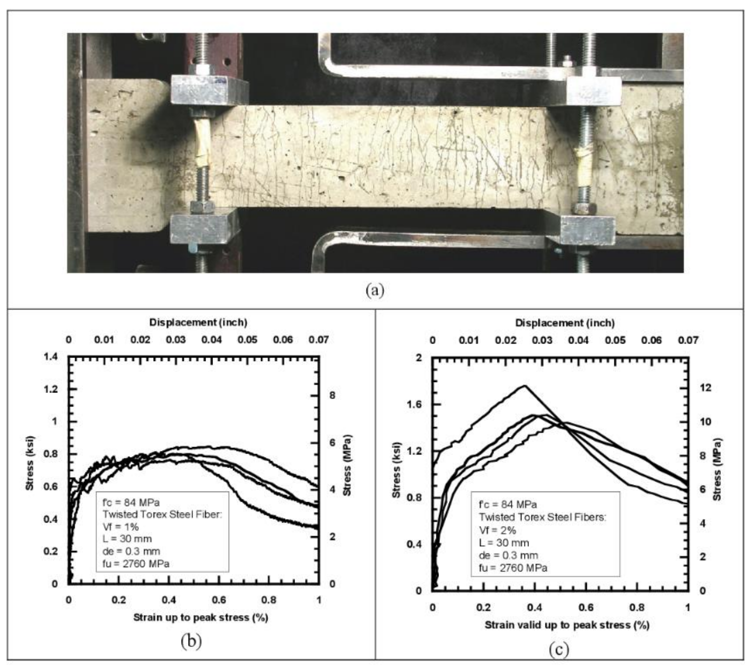
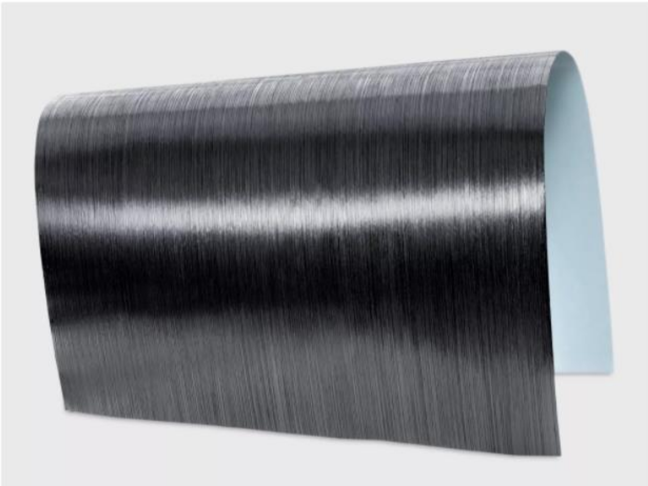
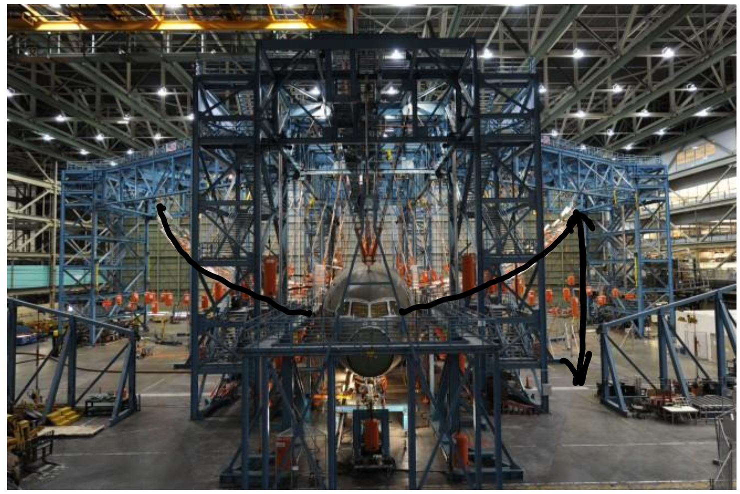


Figure 6: Typical stress-strain curves in tension of HPFRC composite for two series of tests

A. E. Naaman. *Proceedings of the 2nd International RILEM Conference*, pp. 17-26. 2011

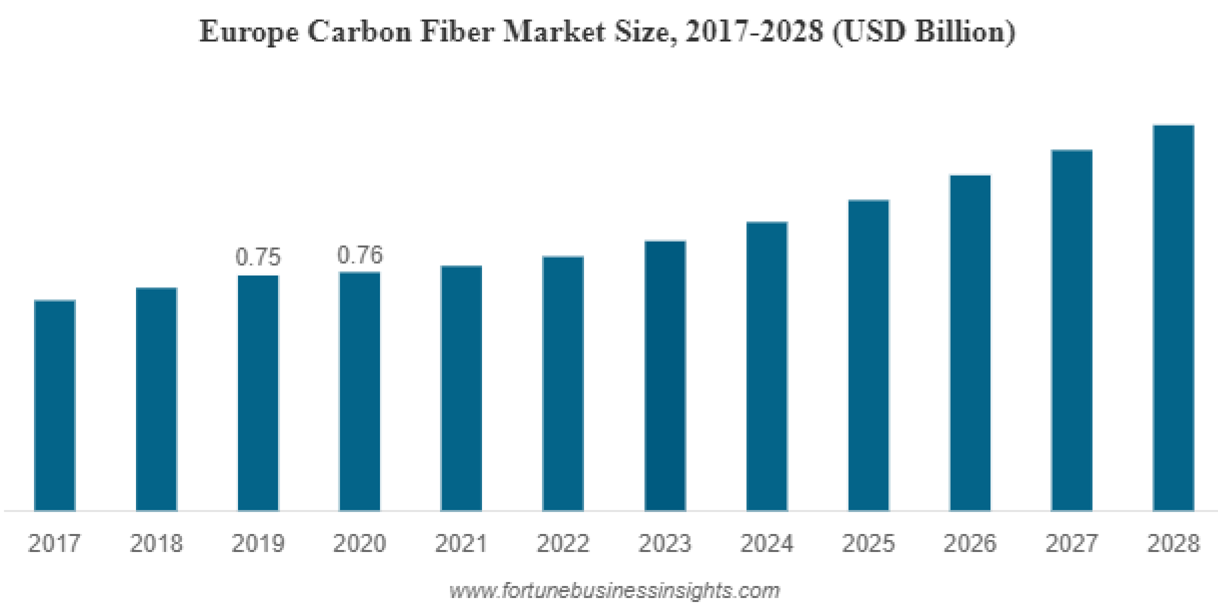


https://www.sglcarbon.com/en/markets-solutions/material/sigrapreg-pre-impregnated-materials/



https://www.wired.com/2010/03/boeing-787-passes-incredible-wing-flex-test/

Carbon fiber market will grow from $2.33B in 2021 to $4.01B in 2028



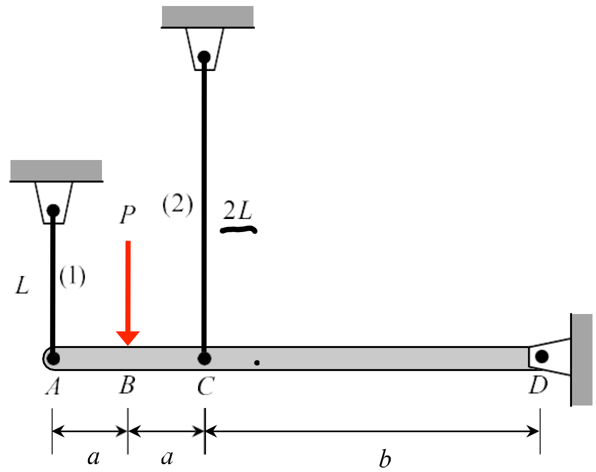
Axial deformation *Topic 6:*

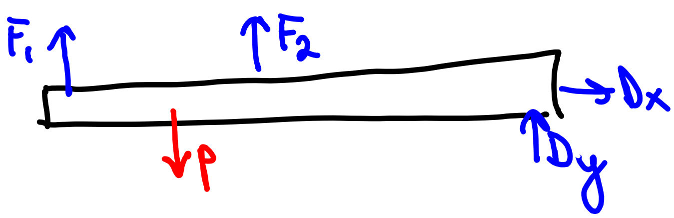
Example 6.8

Load P is applied to rigid beam AD. Members (1) and (2) are made up of a material having a Young's modulus of E, and each have a cross-sectional area of A.

a) Determine the axial stresses in support rods (1) and (2) after the load P is applied.

b) Determine the resulting rotation angle θ in beam AD. Assume θ to be small.





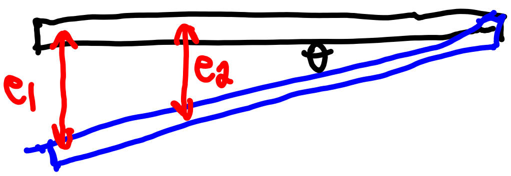
1.) $(\Sigma M)_D = -F_2 b + P(a+b) - F_1(b+2a) = 0$] 2 unknowns / 1 eqn / 1.

$\cancel{\Sigma F_y = F_1 + F_2 - P + D_y}$

$$F_2 = \frac{(a+b)}{b}P - F_1\frac{(b+2a)}{b} \quad (1)$$

2.) $e_1 = \dfrac{F_1 L_1}{E_1 A_1}$ $\qquad e_2 = \dfrac{F_2 L_2}{E_2 A_2}$.

3.)



$$\tan\theta \approx \theta = \frac{e_2}{b} = \frac{e_1}{b+2a}$$

$E_1 = E_2$

$A_1 = A_2$

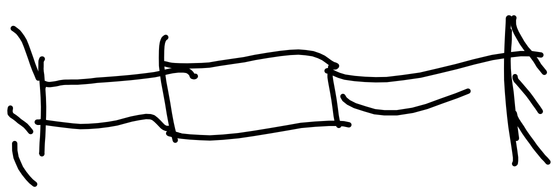
$L_2 = 2L_1$

$$e_2 = \left(\frac{b}{b+2a}\right)e_1 \qquad \frac{F_2L_2}{E_2A_2} = \left(\frac{b}{b+2a}\right)\frac{F_1L_1}{E_1A_1}$$

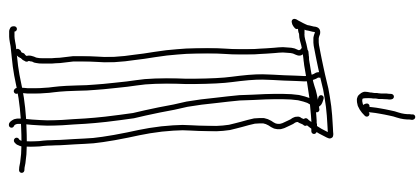
(2) $$2F_2 = F_1\left(\frac{b}{b+2a}\right)$$

$$F_1(b+2a) + \frac{F_1}{2}\left(\frac{b^2}{b+2a}\right) = P(a+b)$$

$$F_1 = \frac{P(a+b)}{\left(b+2a+\frac{1}{2}\left(\frac{b^2}{b+2a}\right)\right)}$$



$$e_1 + e_2 + e_3 = 0$$



$$\underline{e_1 = e_2}.$$

$$\tan\theta = \frac{e_1}{a} = \frac{e_2}{b}$$

(ΣM) 1eqn 3 unknowns