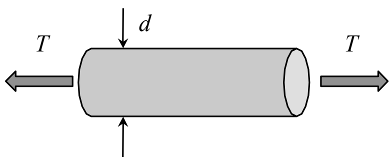
Conceptual question 8.1

Shaft (a) has a solid cross section with outer radius d. Shaft (b) has a tubular cross section with an outer radius of d. Each shaft has the same length and the same shear modulus G. Let $\tau_{a,max}$ and $\tau_{b,max}$ represent the maximum shear stress in shafts (a) and (b), respectively, due to the torque T applied at the shafts' ends. Circle the correct answer:

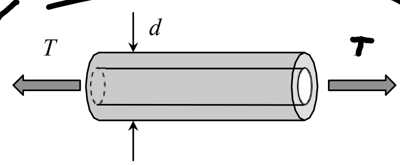
a) $|\tau_{a,max}| > |\tau_{b,max}|$

b) $|\tau_{a,max}| = |\tau_{b,max}|$

c) $|\tau_{a,max}| < |\tau_{b,max}|$



shaft (*a*)



shaft (*b*)

$$\tau_{max} = \frac{TR}{I_p}$$

$$I_{pa} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4$$

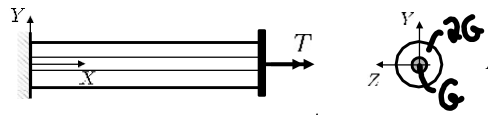
$$I_{pb} = \frac{\pi}{2}\left[\left(\frac{d}{2}\right)^4 - r_i^4\right]$$

Conceptual question 8.2

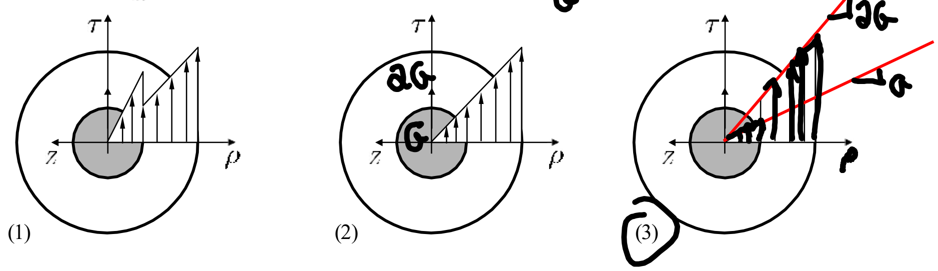
For each loading configuration shown below, indicate the correct stress distribution over a cross section perpendicular to the x-axis.

(a) A bimetallic bar with circular cross section comprised of two elastic materials is subjected to a torque *T*. Material A, depicted using white, is stiffer than material B, depicted using gray. Specifically, the Young's modulus of material A is two times larger than the Young's modulus of material B, and both materials have the same Poisson's ratio.

(a)

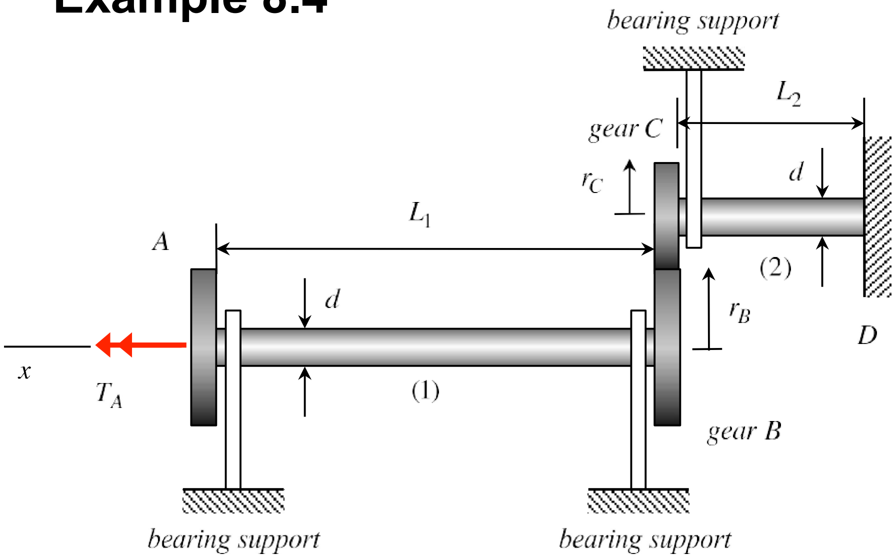


Answer: ______



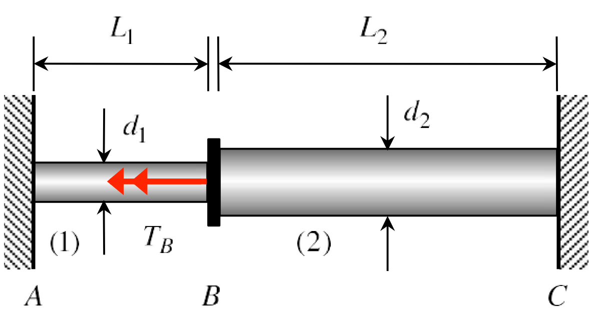
$$\tau = \frac{T\rho}{I_p} = G\rho\left(\frac{\Delta\phi}{L}\right)$$

Example 8.4



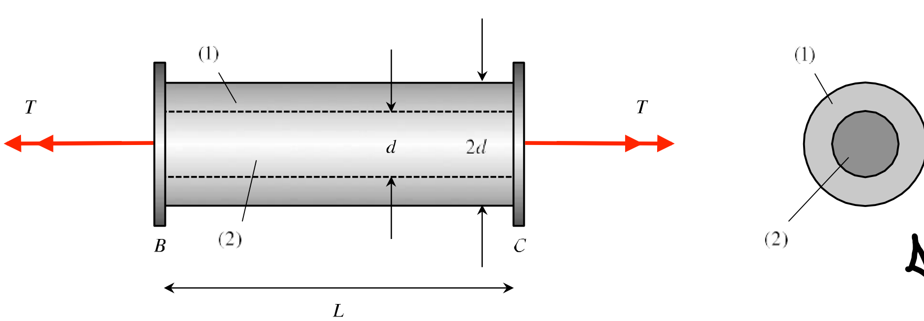
$$\phi_C = -\frac{r_B}{r_C}\phi_B$$

Example 8.5



$$\Delta\phi_1 + \Delta\phi_2 = 0$$

Example 8.7

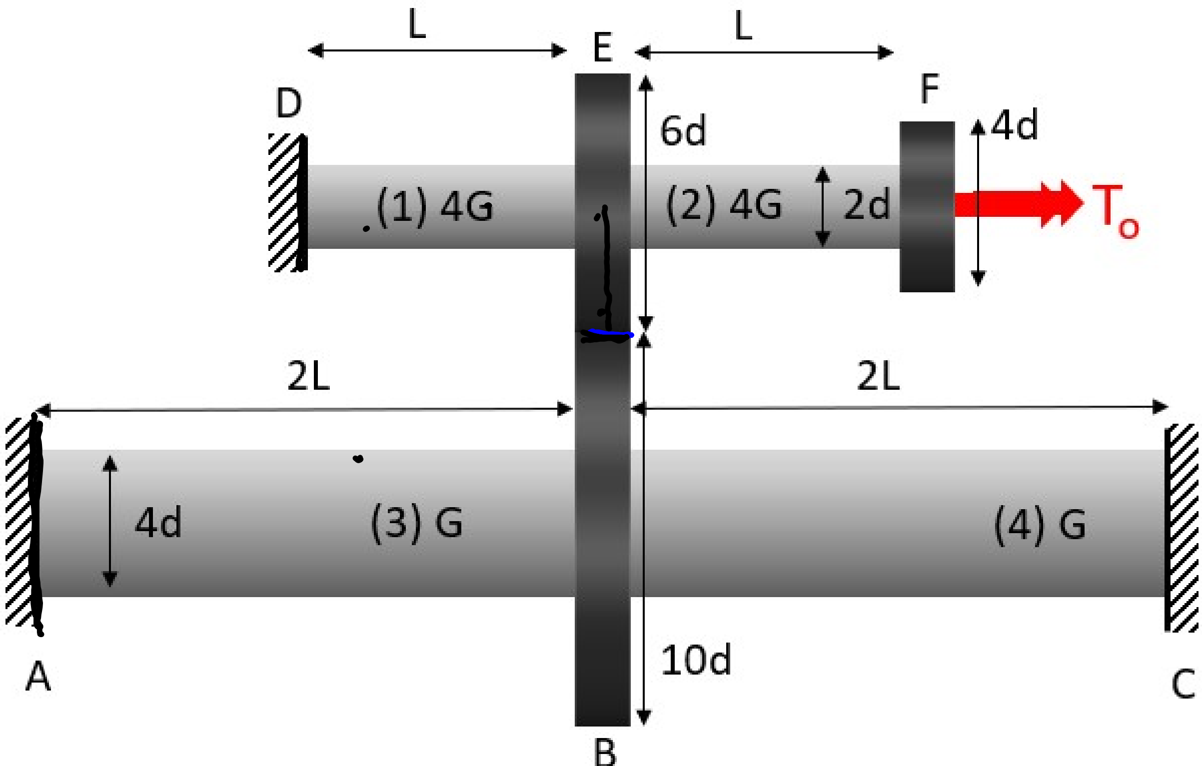


$$\Delta\phi_1 = \Delta\phi_2.$$

Stress analysis of members in torsion

Additional Example: Complex assembly

(a) Set up the equilibrium equations.
(b) Identify the key compatibility equations.



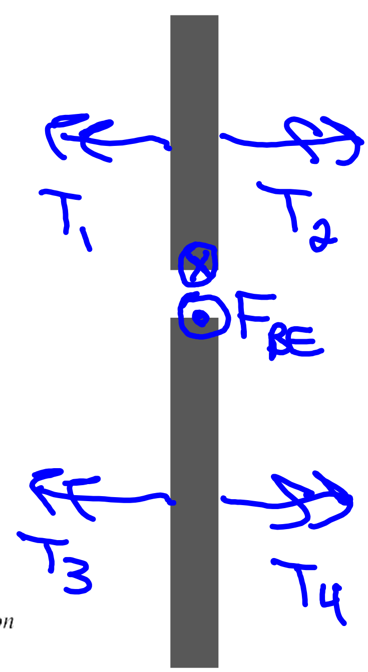
$$\boxed{\Delta\phi_3 + \Delta\phi_4 = 0} \quad (4)$$

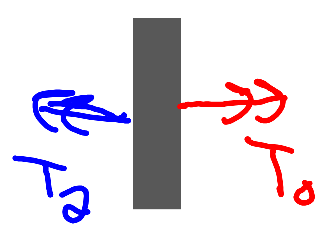
$$\phi_E = -\frac{r_B}{r_E}\phi_B$$

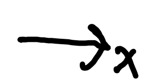
$$\phi_B = \cancel{\phi_A}^{0} + \Delta\phi_3$$

$$\phi_E = \cancel{\phi_D}^{0} + \Delta\phi_1$$

$$\boxed{\Delta\phi_1 = -\frac{r_B}{r_E}\Delta\phi_3.} \quad (5)$$







1.)

$$\left.\begin{aligned}(\Sigma M)_F &= T_0 - T_2 = 0 \Rightarrow T_2 = T_0\\ (\Sigma M)_E &= T_2 - T_1 + F_{BE}(3d) = 0\\ (\Sigma M)_B &= T_4 - T_3 + F_{BE}(5d) = 0\end{aligned}\right\}$$

5 unknowns
3 eqns.
2

2.)

$$\Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{p1}} = \frac{T_1 L}{4G I_{p1}}$$

$$\Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{p2}} = \frac{T_2 L}{4G I_{p2}}$$

$$\Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{p3}} = \frac{T_3 (2L)}{G I_{p3}}$$

$$\Delta\phi_4 = \frac{T_4 L_4}{G_4 I_{p4}} = \frac{T_4 (2L)}{G I_{p4}}$$

3.) Compatibility.

$$\Delta\phi_3 + \Delta\phi_4 = 0$$

$$\Delta\phi_1 = -\frac{r_B}{r_E}\Delta\phi_3$$

4.) Solve

$$I_{p1} = I_{p2} = \frac{\pi}{2}(d)^4 = \frac{\pi}{2}d^4$$

$$I_{p3} = I_{p4} = \frac{\pi}{2}(2d)^4 = 8\pi d^4$$

$$-\frac{r_B}{r_E}\left(\frac{T_3(2L)}{G I_{p3}}\right) = \frac{T_1 L}{4 G I_{p1}} \quad (5)$$

$$-\left(\frac{5}{3}\right)(8)\left(\frac{I_{p1}}{I_{p3}}\right)T_3 = T_1$$

$$-\left(\frac{5}{6}\right)T_3 = T_1$$

$$0 = \frac{T_3(2L)}{G I_{p3}} + \frac{T_4(2L)}{G I_{p4}}$$

$$0 = T_3 + T_4 \quad (4)$$