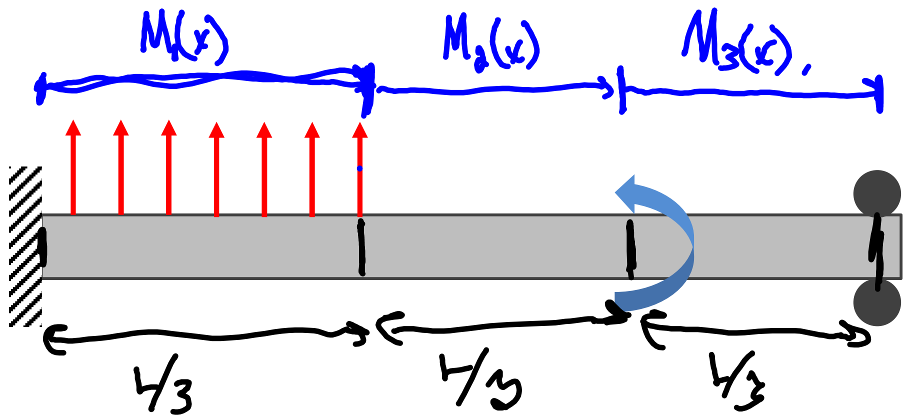
Lecture 19 Review

1. In the beam below, how many sections will we need to use? 3



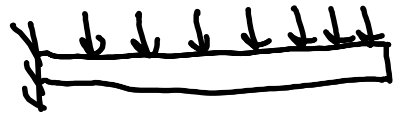
2. This beam has __3__ boundary conditions and __4__ continuity conditions.

- $v(0)=0$
- $\theta(0)=0$
- $v(L)=0$

$\Big\}\ 3$

$M(L)=0$

$\theta_1(L/3) = \theta_2(L/3)$ $\quad \theta_2(2L/3) = \theta_3(2L/3)$



M(x)

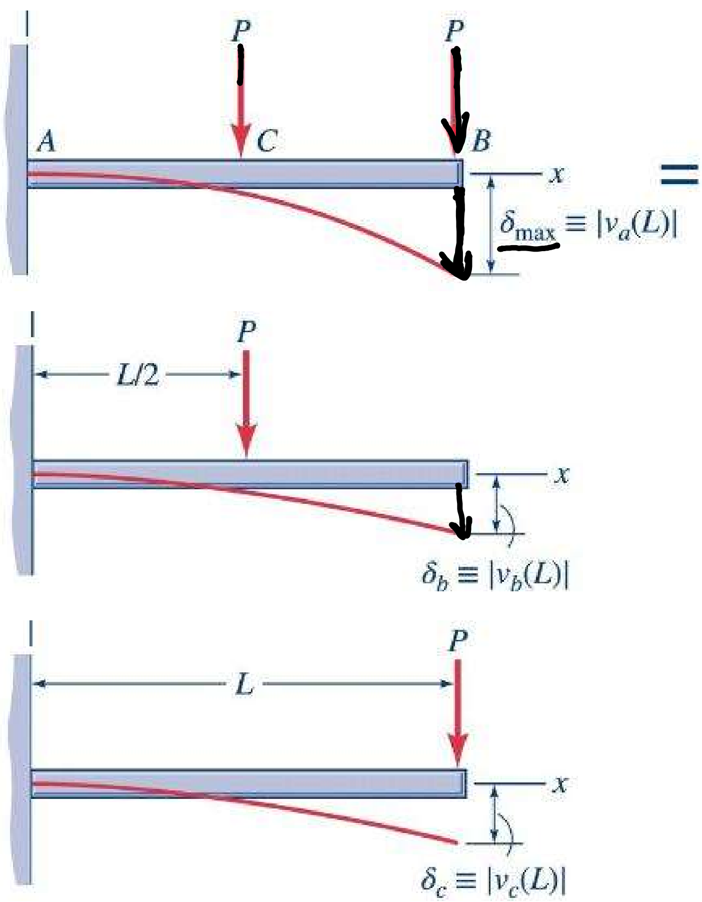
Calculation of beam deflections using superposition

In many situations, the loadings on beams can be recognized as the combination of simple loadings. You might discover that the beam deflection due to each of the simple loadings individually have already been computed by others before you, with some of these results appearing in the Appendix of the textbook (with these results repeated here in these notes). Since the Euler-Bernoulli theory used for the deflection of beams here is *linear*, the deflection of the beam due the combination of simpler loads is a "superposition" of the deflections due of the beam as a result of each simpler load acting individually.

For determinate beams, we need only to find the deflection due to each loading individually, and then add together these deflections.

For indeterminate beams, we will first need to recognize which loadings (both applied and reactive) combine together to give the indeterminate problem. We next find the deflections due to the simple loadings individually, and add together through superposition. Prescribed displacements/rotations are then imposed on the combined deflection relations to determine the unknown reactions.

Please note that linear superposition is a generic and widely applicable method in linear elasticity. This method originates from the assumed linear relationship between applied loads and deformations. Although we will use linear superposition here in combining known deflection relationships of simple problems in determining the deflection of more complicated loadings, the concept of "superposition" extends well beyond this limited application.

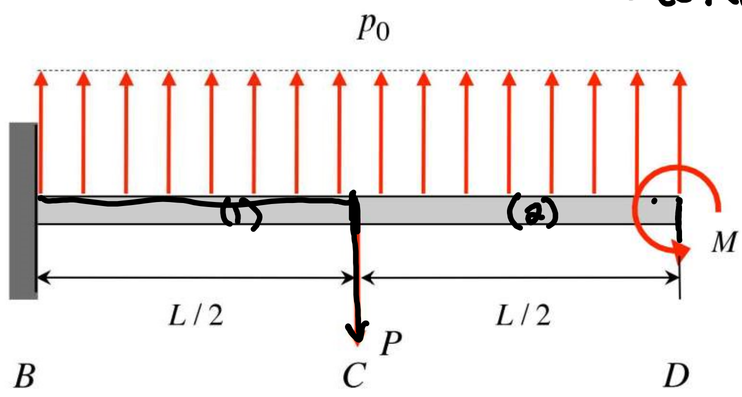


$$v_a(x) = v_b(x) + v_c(x)$$

Example 11.17

Determine the deflection of the beam loaded as below using superposition.

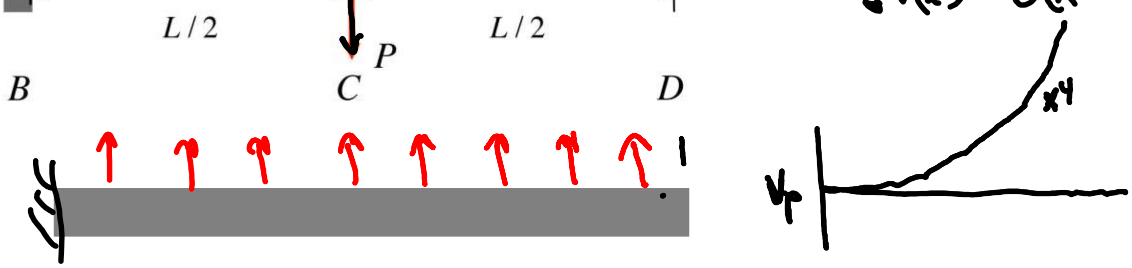
2 sections.

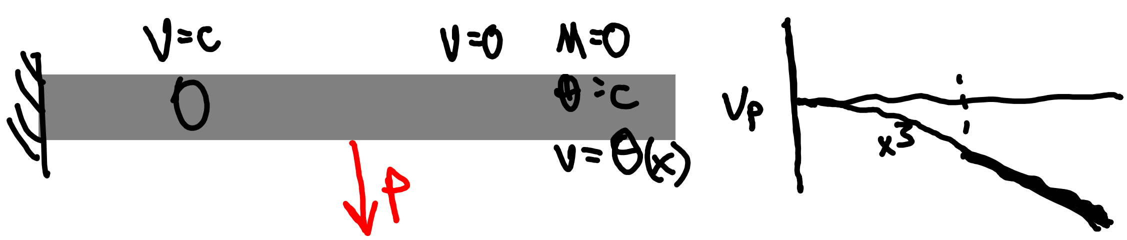


$p(x) = C$

$M(x) = C$

$V(x) = O(x^4)$



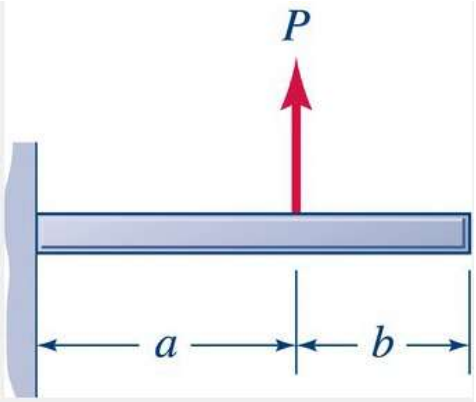


$$v_{p0} = \frac{1}{24}\frac{p_0}{EI}\left[x^2\left(6L^2 - 4Lx + x^2\right)\right]$$

$$v_M = \frac{1}{2}\frac{M}{EI}x^2$$

$$v_P = -\frac{1}{6}\frac{P}{EI}\left[x^2\left(\frac{3L}{2} - x\right)\right] \qquad 0 < x < L/2$$

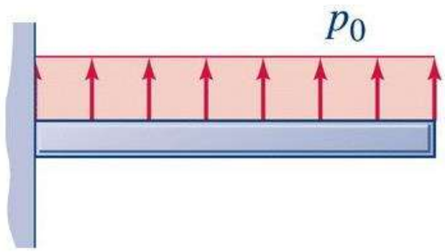
$$v_P = -\frac{1}{6}\frac{P}{EI}\left[\frac{L^2}{4}\left(3x - \frac{L}{2}\right)\right] \qquad L/2 < x < L.$$



$$v = \frac{Px^2}{6EI}(3a - x) \quad v' = \frac{Px}{2EI}(2a - x) \quad 0 \leq x \leq a$$

$$v = \frac{Pa^2}{6EI}(3x - a) \quad v' = \frac{Pa^2}{2EI} \quad a \leq x \leq L$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \quad \theta_B = \frac{Pa^2}{2EI}$$



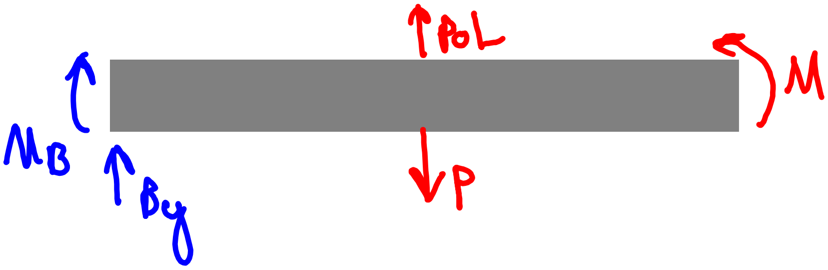
$$v = \frac{p_0 x^2}{24EI}\left(6L^2 - 4Lx + x^2\right)$$

$$v' = \frac{p_0 x}{6EI}\left(3L^2 - 3Lx + x^2\right)$$

$$\delta_B = \frac{p_0 L^4}{8EI} \quad \theta_B = \frac{p_0 L^3}{6EI}$$

$$v = \frac{M_0 x^2}{2EI} \quad v' = \frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 L^2}{2EI} \quad \theta_B = \frac{M_0 L}{EI}$$

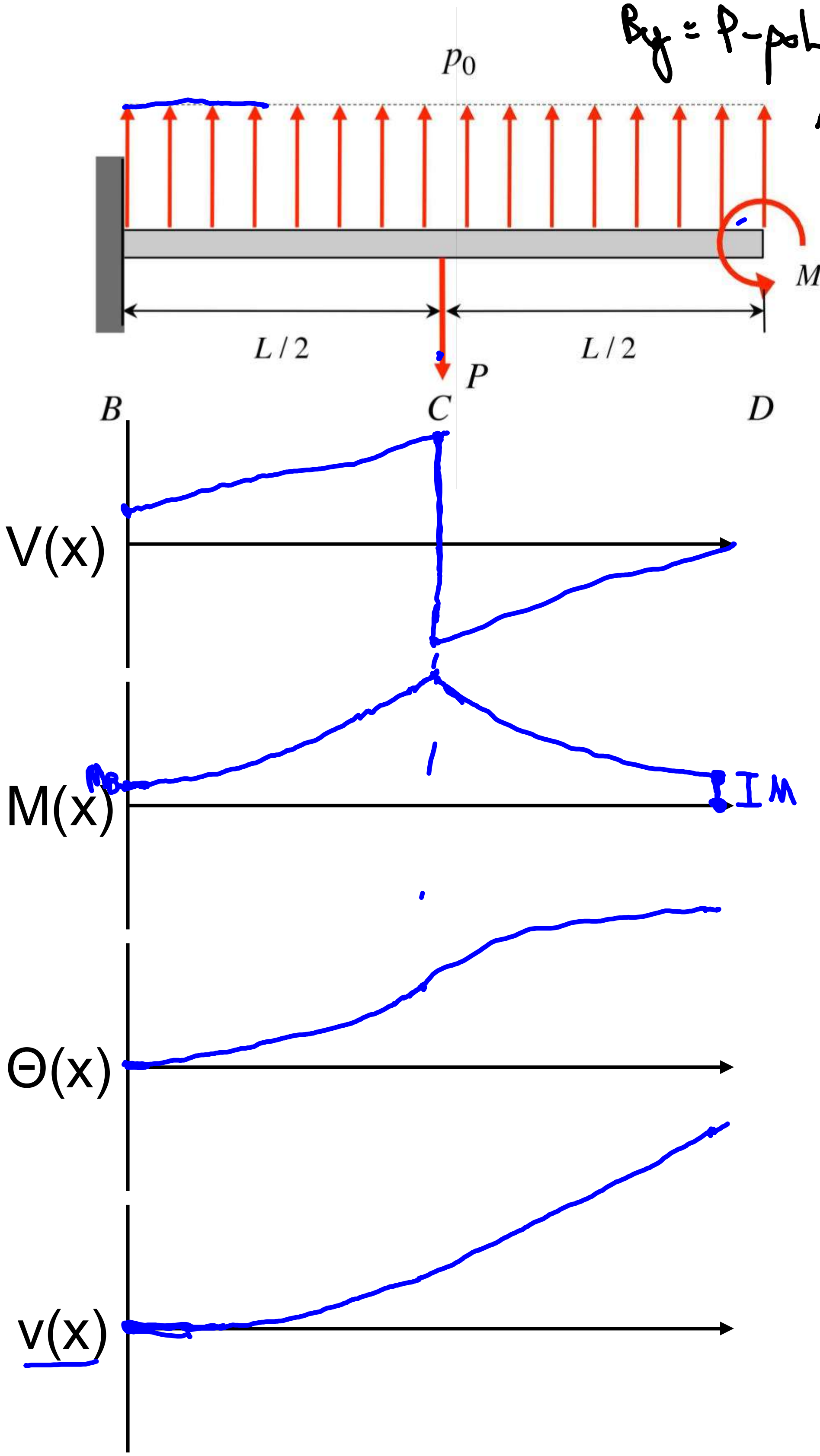


$$\Sigma F_y = B_y + p_0 L - P = 0$$

$$(\Sigma M)_B = -M_B + M + p_0 L\left(\frac{L}{2}\right) - P\left(\frac{L}{2}\right) = 0$$

$$B_y = P - p_0 L$$

$$M_B = M + p_0 \frac{L^2}{2} - P\left(\frac{L}{2}\right)$$



1.) Approximate as a determinate and add reactions as applied loads

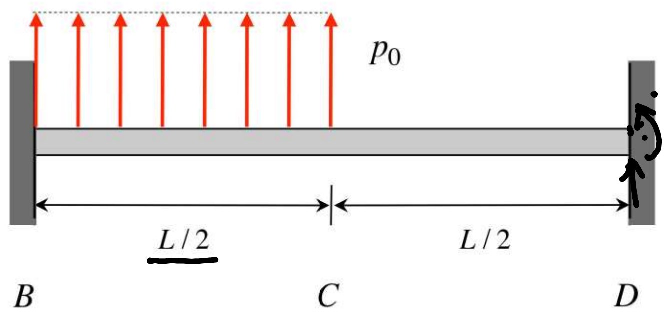
Example 11.20 2.) Enforce the BCs.

Determine the reactions acting on the beam loaded as below using superposition.

B.C.s.

$\theta(0)=0$ ✓ $\theta(L)=0$

$v(0)=0$ ✓ $v(L)=0$

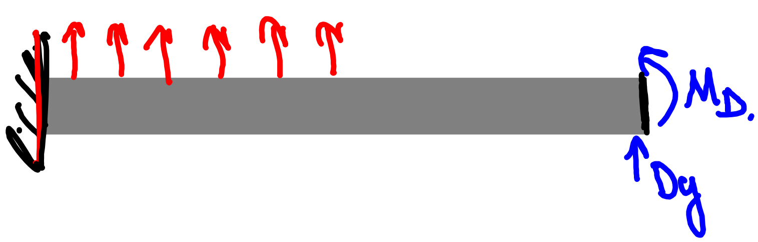


1.)

$\Sigma F_y = B_y + p_0 \frac{L}{2} + D_y = 0$ (1)

$(\Sigma M)_B = -M_B + M_D + p_0\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + D_y L = 0$ (2)

2.) Force-displacement.

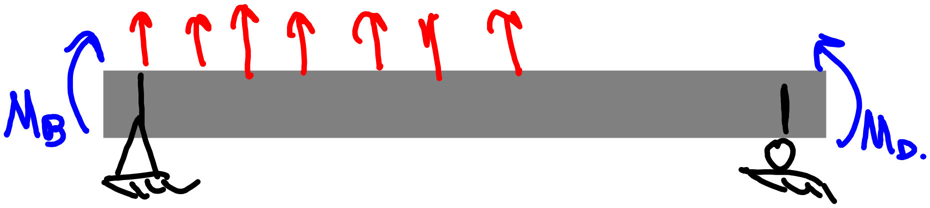


$v(x) = v_{p_0} + v_{D_y} + v_{M_D}.$

$\boxed{\begin{array}{l} v(L)=0 \\ \theta(L)=0 \end{array}}$

$v_2(L) = \frac{p_0}{24EI}\left(\frac{L}{2}\right)^3\left(4L - \frac{L}{2}\right) + \frac{D_y L^3}{3EI} + \frac{M_D L^2}{2EI} = 0$ (3)

$\theta_2(L) = \frac{p_0}{6EI}\left(\frac{L}{2}\right)^3 + \frac{D_y L^2}{2EI} + \frac{M_D L}{EI} = 0$ (4)



$v(0)=0$ ✓ $\quad v(L)=0$ ✓

$$\boxed{\theta(0)=0 \quad \theta(L)=0}$$

$$\theta_1(0) = \theta_{M_B} + \theta_{p0} + \theta_{M_D}$$

$$\theta_2(L) = \theta_{M_B} + \theta_{pc} + \theta_{M_D}$$

Example 11.19

Determine the load carried by rod BD using superposition. The beam has a flexural rigidity of EI. Rod DB is made of a material with a Young's modulus of E, cross-sectional area of A and length L.

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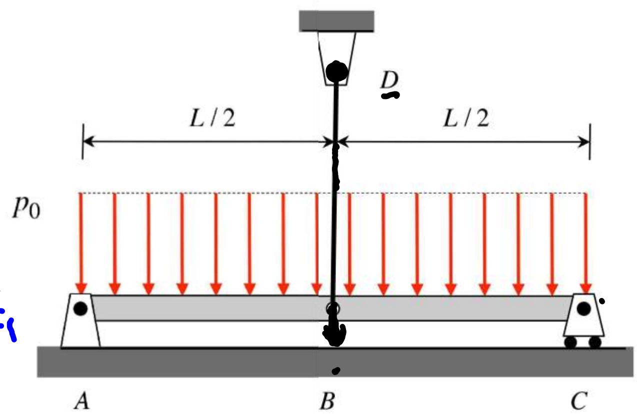


B.C.

$v(0)=0$ ✓

$v(L)=0$ ✓

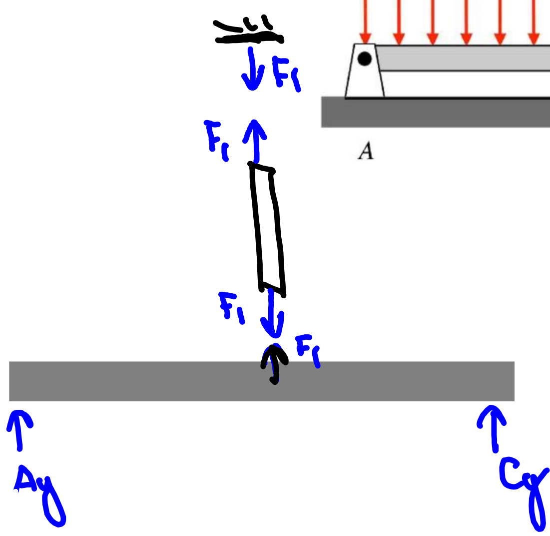
$\boxed{v\left(\frac{L}{2}\right)=-e_1}$

C.E.

$\theta_1\left(\frac{L}{2}\right)=\theta_2\left(\frac{L}{2}\right)$

$v_1\left(\frac{L}{2}\right)=v_2\left(\frac{L}{2}\right)$.

1.)



$\Sigma F_y = A_y + C_y + F_1 = 0$

$\Sigma M = \sim\sim\sim$

3 unknowns
2 eqns

2.) $v_1(x)$ $\quad 0 < x < L/2$

$v_2(x)$ $\quad L/2 < x < L$

3.) Compatibility.

$v\left(\frac{L}{2}\right) = -e_1 \qquad e_1 = \frac{FL}{EA} = \frac{B_y L}{EA}$

Example 11.19

Determine the load carried by rod BD using superposition. The beam has a flexural rigidity of *EI*. Rod DB is made of a material with a Young's modulus of *E*, cross-sectional area of *A* and length *L*.

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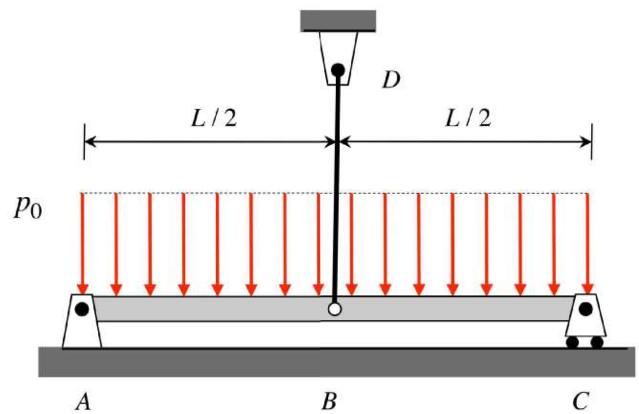
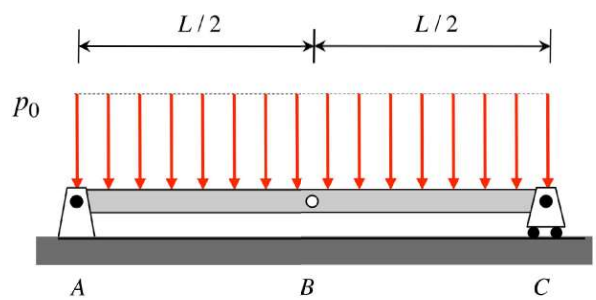


SOLUTION

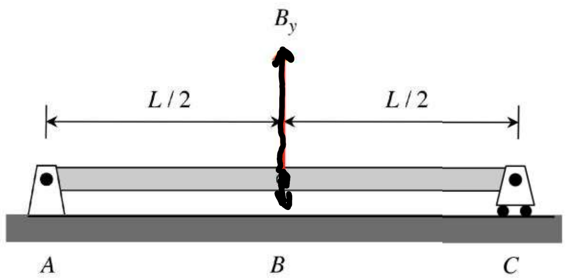
Will break the problem into two loadings, as shown in the following:

<u>*Loading 1*</u>



$$v_1(x) = -\frac{p_0 x}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

Loading 2



$$v_2(x) = \frac{B_y x}{48EI}\left(3L^2 - x^2\right) \quad ; \quad 0 \le x \le L/2$$

where: $e = \dfrac{B_y L}{EA} \Rightarrow \quad B_y = \dfrac{eEA}{L}$ $\qquad -\dfrac{B_y L}{EA}$

Therefore for $0 \le x \le L/2$ we have:

$$v(x) = v_1(x) + v_2(x)$$

$$= -\frac{p_0 x}{24EI}\left(L^3 - 2Lx^2 + x^3\right) + \frac{eAx}{48LI}\left(3L^2 - x^2\right)$$

Enforcing the boundary condition at B:

$$v\left(\frac{L}{2}\right) = -e = -\frac{p_0(L/2)}{24EI}\left[L^3 - 2L\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^3\right] + \frac{eA(L/2)}{48LI}\left[3L^2 - \left(\frac{L}{2}\right)^2\right]$$

$$= -\frac{5p_0L^4}{384EI} + \frac{11eAL^2}{384I} \quad \Rightarrow$$

$$\left(1 + \frac{11AL^2}{384I}\right)e = \frac{5p_0L^4}{384EI} \quad \Rightarrow \quad e = \frac{5p_0L^4/384EI}{1 + 11AL^2/384I} = \frac{5p_0L^4/E}{384I + 11AL^2}$$

Therefore,

$$B_y = \frac{eEA}{L} = \frac{5p_0AL^3}{384I + 11AL^2}$$