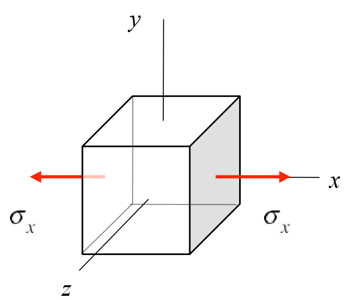
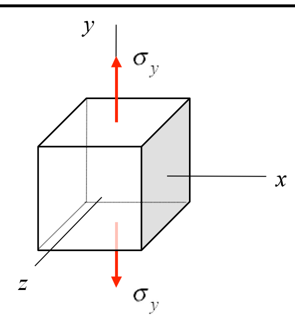
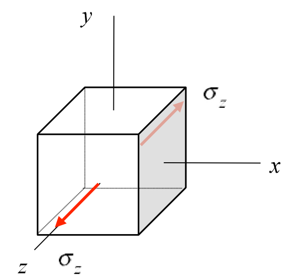
c) *Generalized Hooke's law for normal stresses/strains*

Recall that for uni-axial loading along the x-axis, the normal strains in the x, y and z directions in the body were found to be:

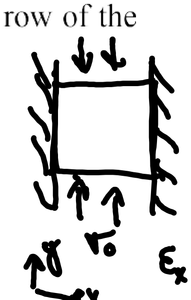
$$\varepsilon_x = \sigma_x / E$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\sigma_x / E$$

where E and ν are the Young's modulus and Poisson's ratio for the material. For a 3-D loading of a body, we have three normal stress components σ_x, σ_y and σ_z acting simultaneously. For this case, we will consider the strains due to each normal component of stress individually and add these together using linear superposition (along with the thermal strains) to determine the resulting three components of strain ε_x, ε_y and ε_z. Consider the individual contributions of the three components of stress shown below:

Strains due to mechanical loading in the x-direction	*Strains due to mechanical loading in the y-direction*	*Strains due to mechanical loading in the z-direction*
		
$\varepsilon_x = \sigma_x / E$	$\varepsilon_x = -\nu\varepsilon_y = -\nu\sigma_y / E$	$\varepsilon_x = -\nu\varepsilon_z = -\nu\sigma_z / E$
$\varepsilon_y = -\nu\varepsilon_x = -\nu\sigma_x / E$	$\varepsilon_y = \sigma_y / E$	$\varepsilon_y = -\nu\varepsilon_z = -\nu\sigma_z / E$
$\varepsilon_z = -\nu\varepsilon_x = -\nu\sigma_x / E$	$\varepsilon_z = -\nu\varepsilon_y = -\nu\sigma_y / E$	$\varepsilon_z = \sigma_z / E$

The total strain in each direction is found through superposition of the individual strains along with the thermal strains. Adding together these components (across each row of the above table) gives:



$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$\varepsilon_x = 0$

$\sigma_y = \sigma_0$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

The above are known as the generalized Hooke's law equations for normal stresses/strains due to 3-D loadings on a body. $\sigma_z = 0$

Summary: Indeterminate axial problems (the four-step method)

1. *Draw FBD's and write down equilibrium equations.*
   a. Break system into "elements". For problems in this course, an element:
      i. has forces acting only at its ends,
      ii. has a constant cross section, and
      iii. has constant material properties.
   b. It is recommended that you always draw end forces on element corresponding to "tension" (if the force actually corresponds to compression, you will get a negative value for the force in the end...trust the math...it works!).
   c. Be sure to abide by Newton's 3rd Law (reactions appear in equal and opposite pairs) when drawing your FBD's

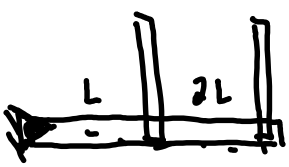
✓

2. *Write down the elemental force-deformation equations.*
   a. For the jth element:

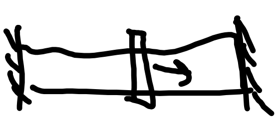
$$e_j = \frac{P_j L_j}{E_j A_j}$$

   b. If you draw all element forces as in tension (as recommended in 1b) above), then P_j has a positive sign in the equation above. If you choose to draw the elemental force as in compression, then P_j has a negative sign.

✓



$\frac{e_1}{L} = \frac{e_2}{2L}$.



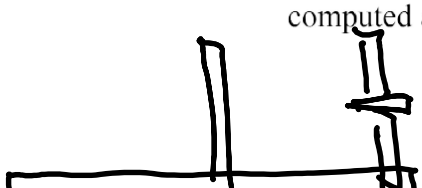
$e_1 + e_2 = 0$.

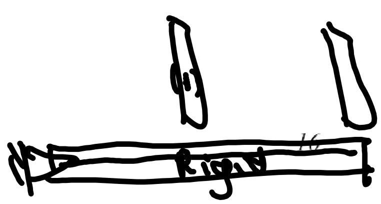
3. *Write down appropriate compatibility equations*
   a. In this step you will write down the constraint equations that exists among the element elongations.
   b. This step is *problem-dependent* (and requires the most thought):
      i. For an axial system constrained between rigid supports, the compatibility equation needed is that the sum of all the elemental elongations is zero.
      ii. For elements attached to a rigid member, the motion of the rigid member dictates the relationship that exists among the elemental elongations.
      iii. For truss elements, a trigometric relationship must be used to relate the elemental elongations.

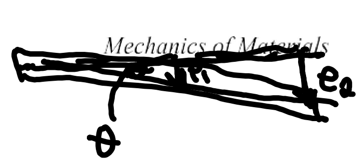
Draw deformed geom!

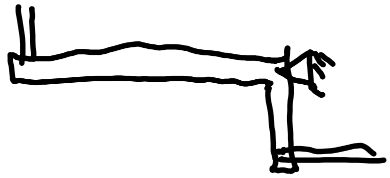
4. *Solve equations derived in Steps 1-3 for the elemental forces* P_j. Count your number of equations and number of unknowns. If you have sufficient equations to solve, then solve. If not, review the first three steps to see if you are missing needed equilibrium, force/deformation or compatibility equations. From these forces determine at this step, the elemental stresses, strains and elongations can be computed as needed.

$\tan\theta \sim \theta = \frac{e_1}{L} = \frac{e_2}{2L}$



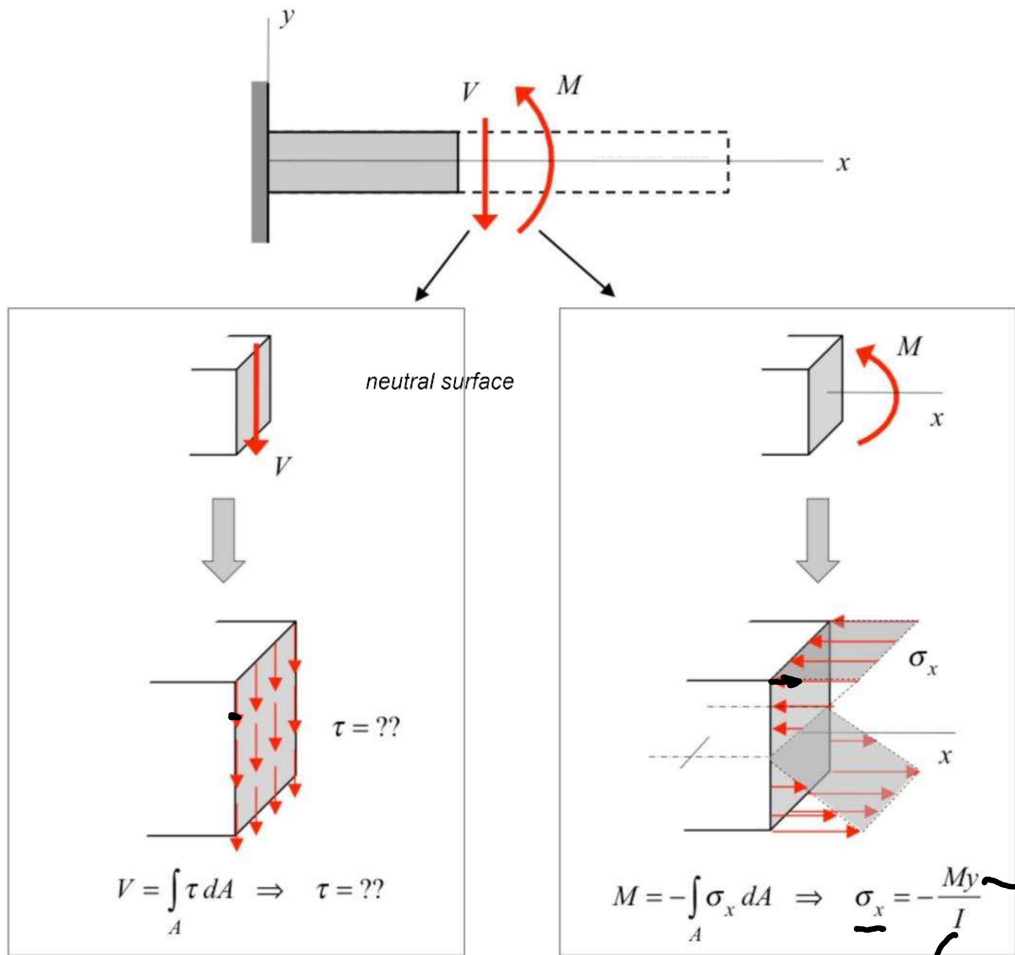






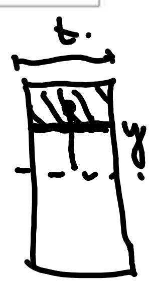
c) Stresses due general transverse force and bending-couple loading of beams

Earlier in the chapter, we considered the normal stress distribution within the cross section of a beam experiencing pure bending (i.e., in the absence of a shear force resultant on the cross-sectional cut). Here we will now consider the more general case of having both shear force and bending moment couples on the cross-sectional cut, as demonstrated by the figure below.



distance from neutral axis.

$$\tau(y) = \frac{VA^*\bar{y}^*}{It.}$$



$$I_{rect} = \frac{bh^3}{12}$$

$$I_{circ} = \frac{\pi}{4}R^4$$

PROBLEM #3 (25 points)

A torque T is applied to a gear-shaft system and is transmitted through rigid gears B and C to a fixed end E as shown in Fig. 3(a). The shafts (1) and (2) are tightly fit to each other. Frictionless bearings are used to support the shafts. The geometry and material property of the shafts and gears are listed in the following table.

	Size	Length	Shear modulus
Shaft (1)	Outer diameter = $2d$ Inner diameter = d	L	$2G$
Shaft (2)	Diameter = d	L	G
Shaft (3)	Diameter = d	$L/2$	G
Shaft (4)	Diameter = d	L	G
Gear B	Diameter = $1.5d$	Negligible	Rigid
Gear C	Diameter = $3d$	Negligible	Rigid

(a) Determine the torque carried by each shaft.
(b) Determine the angle of twist at the free ends A and D.
(c) Consider the cross section *aa*' for the shafts (1) and (2), show the magnitude of the shear stress as a function of the distance from the center on Fig. 3(b). Mark the critical values in the diagram.
(d) Consider the points M and N on the cross section *aa*', shown in Fig. 3(c). Sketch the stress states at M and N on the stress elements on Fig. 3(d).

Express all your answers in terms of d, L, G, T, π.

$\phi_C = \phi_E + \Delta\phi_1$

$\Delta\phi_1 = \phi_C - \phi_E$

$\Delta\phi_1 = \phi_C - \phi_E.$

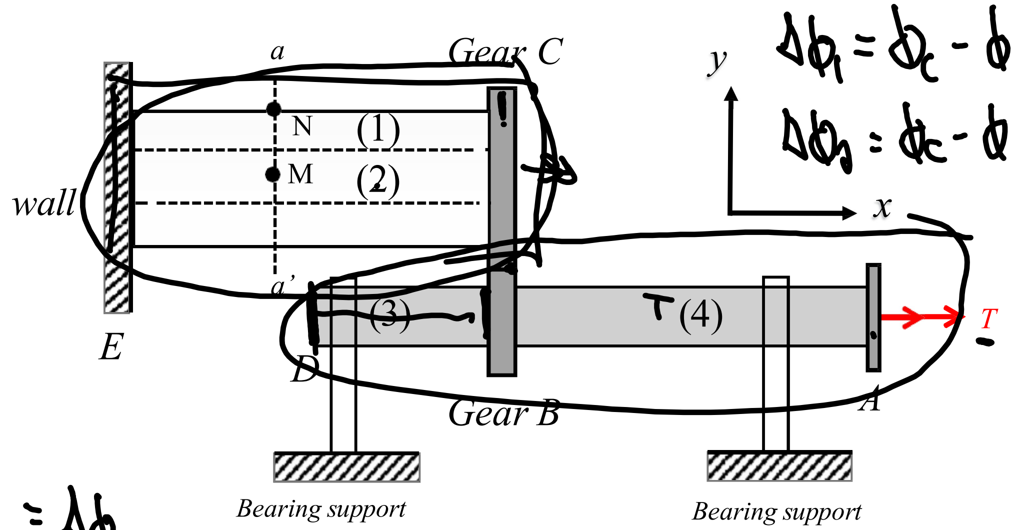


Fig. 3(a)

$\phi_E = 0$

$\phi_C = \phi_E + \Delta\phi_1 = \Delta\phi_1$

$\phi_B = -\left(\frac{r_C}{r_B}\right)\phi_C$

$\phi_D = \phi_B + \cancel{\Delta\phi_3}^{\,0}$

$\phi_A = \phi_B + \Delta\phi_4$

$\phi_A = -\left(\frac{r_C}{r_B}\right)\Delta\phi_1 + \Delta\phi_4.$

ME 323 Examination # 1

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$$\tau = \rho G \frac{d\phi}{dx}$$

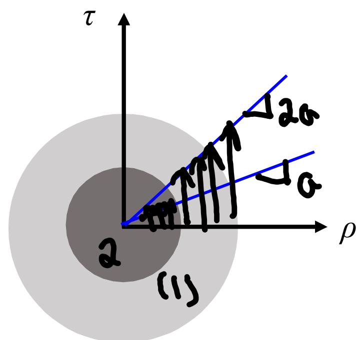


Fig. 3(b)

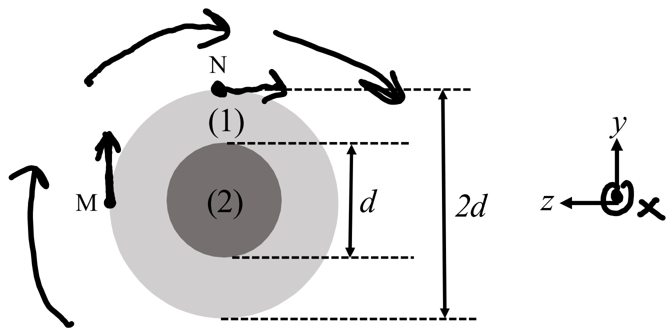
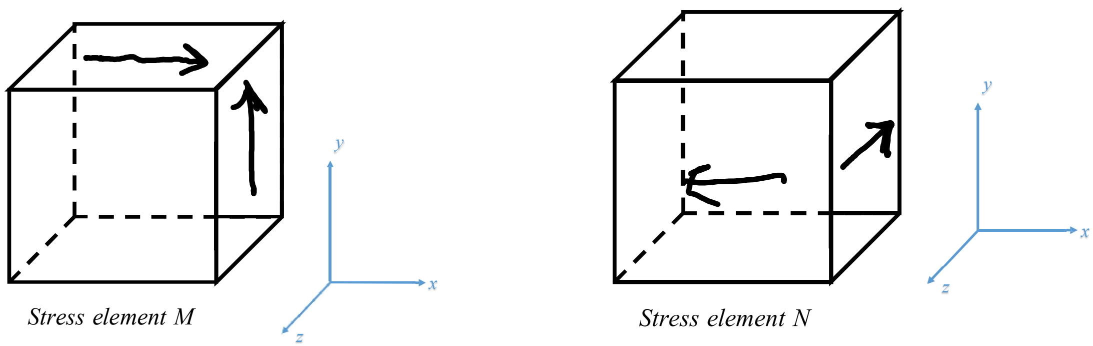


Fig. 3(c)

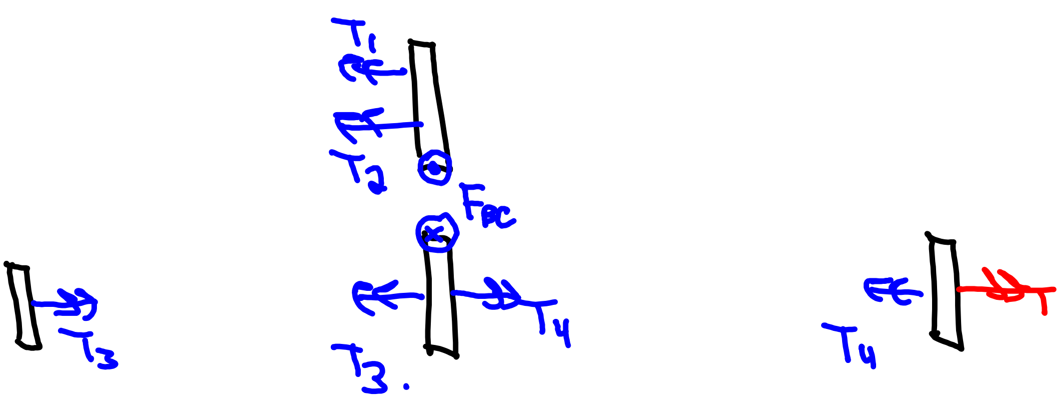


Stress element M

Stress element N

Fig. 3(d)

1.)



$$\left.\begin{aligned}
(\Sigma M)_A &= T - T_4 = 0 \Rightarrow T_4 = T \\
(\Sigma M)_B &= T_4 - F_{BC} r_B = 0 \Rightarrow F_{BC} = \frac{T}{r_B} \\
(\Sigma M)_C &= -T_1 - T_2 - F_{BC} r_C = 0 \Rightarrow \underbrace{T_1 + T_2}_{(1)} = -T\left(\frac{r_C}{r_B}\right)
\end{aligned}\right\} \quad \begin{aligned}&4 \text{ unknowns} \\ &3 \text{ eqns.}\end{aligned}$$

2.) $\Delta\phi_1 = \dfrac{T_1 L_1}{G_1 I_{p1}}$ $\Delta\phi_2 = \dfrac{T_2 L_2}{G_2 I_{p2}}$.

$I_{p1} = \frac{\pi}{2}\left[\left(\frac{2d}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right]$ $I_{p2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4$

$\frac{\pi}{2}\left[\left(\frac{2d}{2}\right) - \left(\frac{d}{2}\right)\right]^4$

3.) Compatibility

$\Delta\phi_1 = \Delta\phi_2$.

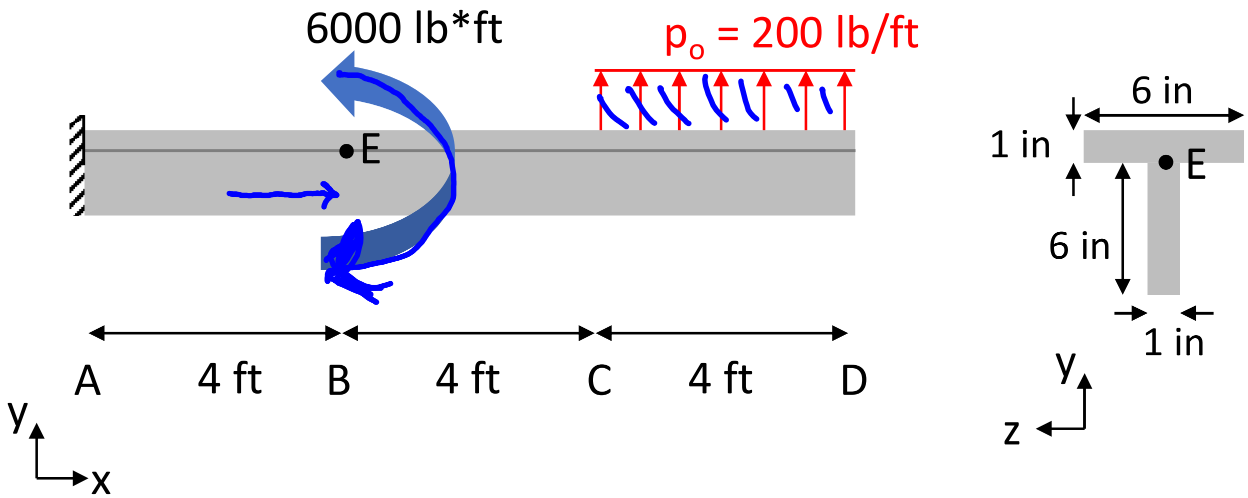
$$\frac{T_1 L}{2G\left(\frac{15}{32}\pi d^4\right)} = \frac{T_2 L}{G\left(\frac{\pi}{32}d^4\right)} \quad (2)$$

4.) Solve.

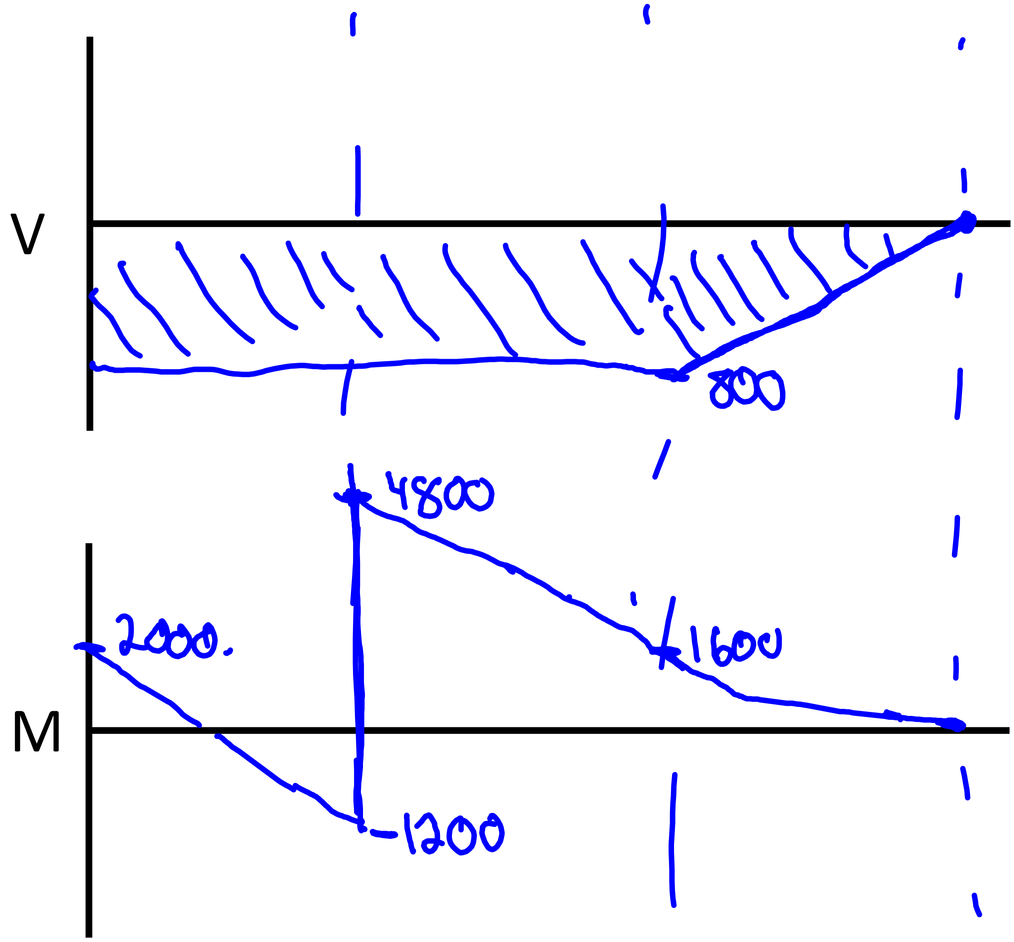
(1) + (2) $T_1 = -\frac{60}{31}T$ -

5) 0

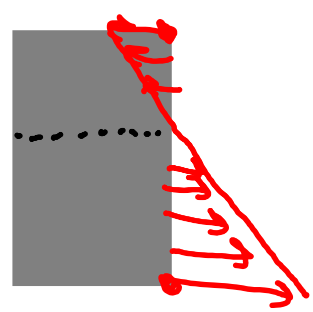
Exam 1 Example Question

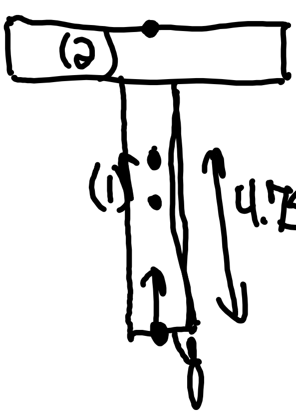


a) Draw the shear force and bending moment diagrams.
b) For the cross-section with the largest normal stress, draw the profile of the normal stress on the cross-section. Include the numerical values of the maximum and minimum normal stresses.
c) What is the shear stress at point E^-? What is the shear stress at point E^+?
d) Draw the profile of the shear stress on the cross-section. Include numerical values for the key points.



b) $\sigma = -\dfrac{My}{I}$





$$A\bar{y} = A_1\bar{y}_1 + A_2\bar{y}_2$$

$$12\bar{y} = (6)(1)(3) + (6)(1)(6.5)$$

$$\bar{y} = 4.75 \text{ in.}$$

$$I = (I_1)_0 + (I_2)_0$$

$$I = \frac{(1)(6)^3}{12} + (6)(1.75)^2 + \frac{(6)(1)^3}{12} + (6)(1.25 + 0.5 \text{ in})^2$$

$$I = 55.25 \text{ in}^4$$

Fall 2019 Exam 1

ME 323 Examination # 1 **Name** ______________________
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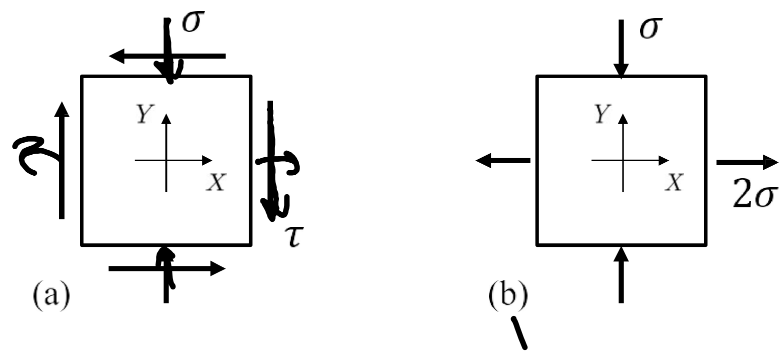
PROBLEM #4 (25 Points):
PART A – 6 points

For each state of plane stress shown below, i.e., for configurations (a) and (b), indicate whether each component of the state of strain is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

The material is linear elastic with Poisson's ratio ν ($0 < \nu < 0.5$), and the deformations are small.



	(a)	(b)
ϵ_x	>0	
ϵ_y	<0	
ϵ_z	>0	
γ_{xy}	<0	
γ_{xz}	$=0$	
γ_{yz}	$=0$	

Fill in with '= 0', '> 0', or '< 0'.

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

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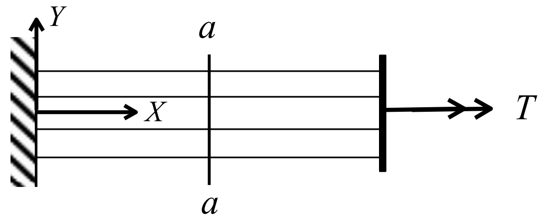
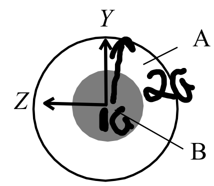
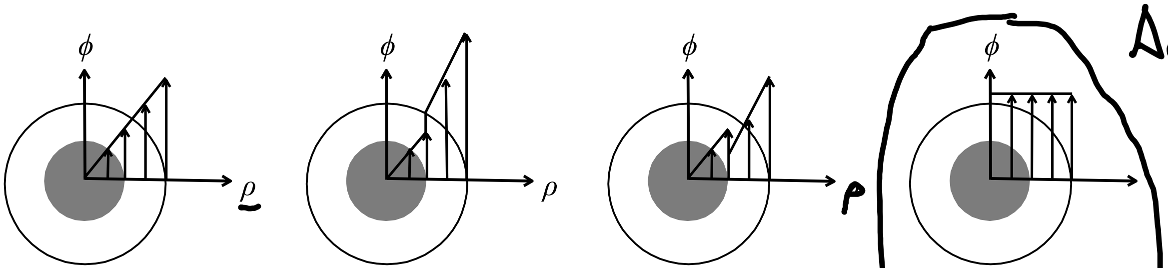
4.2. (6 Points)

A bimetallic bar with circular cross section consists of a shell A and a core B. The bimetallic bar is subjected to a torque T. The shear moduli of the core and shell are known to be $G_A = 2G_B$, and polar moment of inertia $I_{PA} = 0.5\, I_{PB}$.





Which figure shows the correct distribution of the twist angle in the cross section aa?

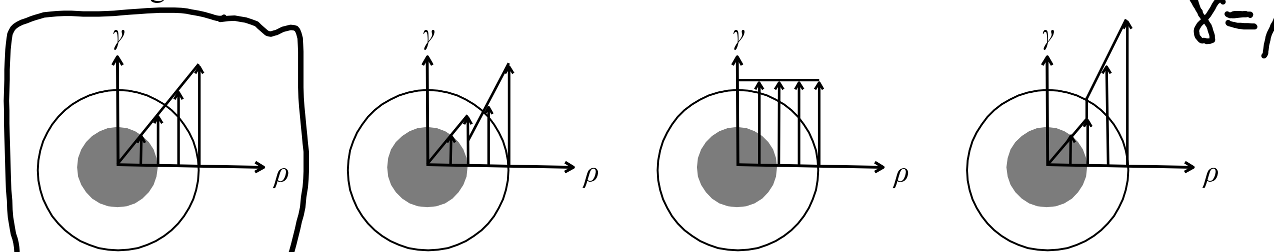


(a) (b) (c) (d)

$\Delta\phi = \frac{TL}{GI_p}$

$I_p = \frac{\pi}{2}R^4$

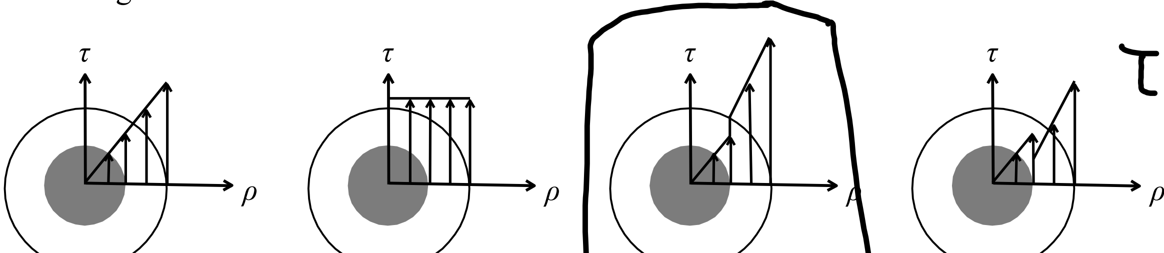
Which figure shows the correct distribution of the shear strain in the cross section aa?



(a) (b) (c) (d)

$\gamma = \rho \frac{d\phi}{dx}$.

Which figure shows the correct distribution of the shear stress in the cross section aa?



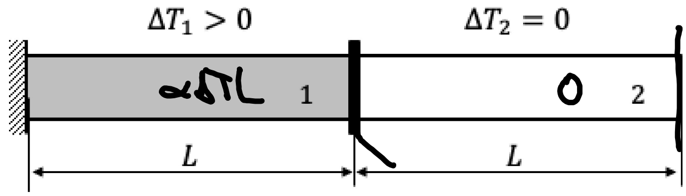
(a) (b) (c) (d)

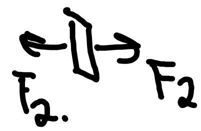
$\tau = \rho G \frac{d\phi}{dx}$.

PROBLEM #4 (cont.):

PART B – 4 points

A rod is made up of elastic elements 1 and 2, each having a length L and cross-sectional area A. Elements 1 and 2 have the same Young's modulus $E_1 = E_2$ and coefficient of thermal expansion $\alpha_1 = \alpha_2$. Let F_1 and F_2 represent the axial load carried by elements 1 and 2, respectively, when the temperature of element 1 is increased by $\Delta T_1 > 0$ — while the temperature of element 2 is kept constant $\Delta T_2 = 0$.





Circle the correct answer:

a) $|e_1| > |e_2|$

b) $|e_1| = |e_2|$

c) $|e_1| < |e_2|$

Circle the correct answer:

a) $|F_1| > |F_2|$

b) $|F_1| = |F_2| = 0$

c) $|F_1| < |F_2|$

Circle the correct answer:

a) Elastic element 1 is under tension

b) Elastic element 1 is under compression

c) Elastic element 1 is stress-free

Circle the correct answer:

a) Elastic element 2 is under tension

b) Elastic element 2 is under compression

c) Elastic element 2 is stress-free