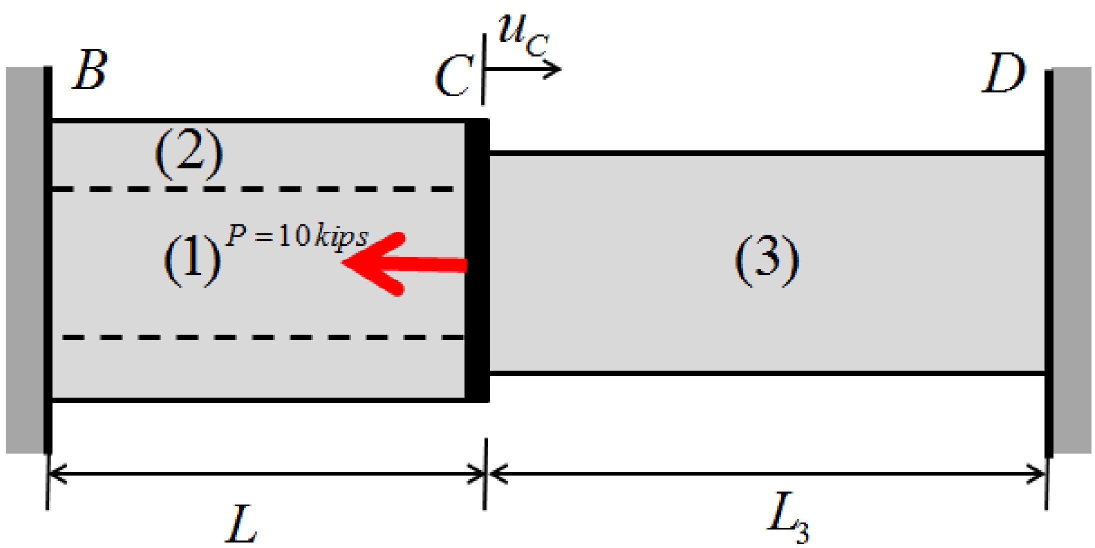
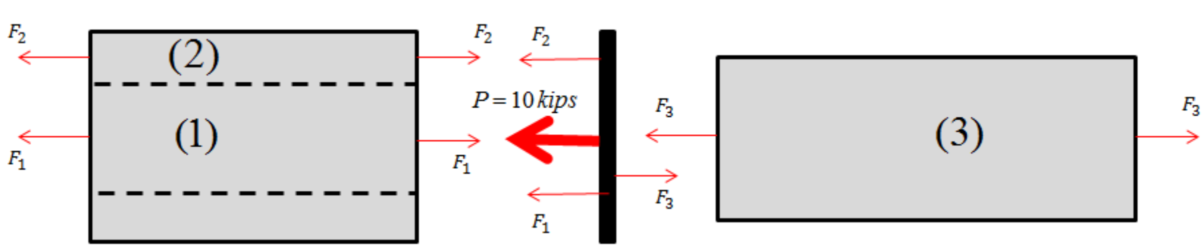
A magnesium-alloy rod ($E_1 = E_m = 6.5 \times 10^3 ksi$) of diameter $d_1 = 1.5\ in.$ is encased and securely bonded to a brass sleeve ($E_2 = E_b = 15 \times 10^3 ksi$) with outer diameter $d_2 = 2\ in;$ the length $L_1 = L_2 = L = 20\ in.$ The two rods are connected to an aluminum rod ($E_3 = E_a = 10 \times 10^3 ksi$) with diameter $d_3 = 1.75\ in.$ and length $L_3 = 40\ in.$ as shown in the Figure. A load $P = 10\ kips$ is applied at point C.

a) Determine the axial stresses $\sigma_1, \sigma_2, and\ \sigma_3$, in the three rods, and

b) Determine the displacement U_C of point C.



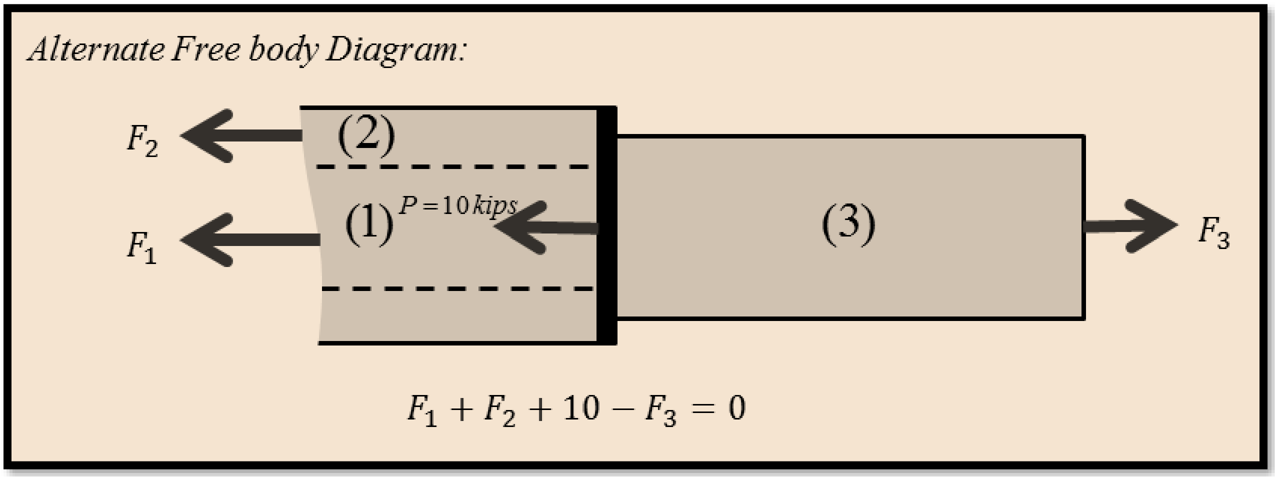
FBD:



Force Balance Equation:

$$P + F_2 + F_1 - F_3 = 0$$

$$=> F_1 + F_2 - F_3 = -10\ kip$$



Geometric Compatibility:

$$\delta_1 = \delta_2 = -\delta_3$$

Force Deformation:

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1}; \delta_2 = \frac{F_2 L_2}{A_2 E_2}; \delta_3 = \frac{F_3 L_3}{A_3 E_3}$$

$$\frac{F_3 - F_2 - 10}{A_1 E_1} = \frac{F_2}{A_2 E_2}$$

$1.795\ F_3 - 2.795\ F_2 = 17.95$ ....(1)

$$\frac{F_2}{F_3} = -\left(\frac{L_3}{L_2}\right)\left(\frac{A_2}{A_3}\right)\left(\frac{E_2}{E_3}\right) = \left(\frac{40}{20}\right)\left(\frac{1.75}{1.75^2}\right)\left(\frac{15}{10}\right) = -1.7143$$

$F_2 = -1.7143 F_3$..(2)

From (1) and (2),

$F_3 = 2.725\ kips$

$F_2 = -4.672\ kips$

$F_1 = -10 - F_2 + F_3 = -2.608\ kips$

$\delta_c = \frac{F_1 L_1}{A_1 E_1} = \frac{-2.608*20}{\frac{\pi}{4}*1.5^2*6.5x10^3} = -4.54x10^{-3}\ in$

Displacement of point C

$u_c = -4.54x10^{-3} in$

Axial stress:

$$\sigma_1 = \frac{F_1}{A_1} = -\frac{2.608}{\frac{\pi}{4}*1.5^2} = -1.476\ ksi$$

$$\sigma_2 = -\frac{4.672}{\frac{\pi}{4}*1.75} = -3.39\ ksi$$

$$\sigma_3 = \frac{2.725}{\frac{\pi}{4}*1.75^2} = 1.133\ ksi$$