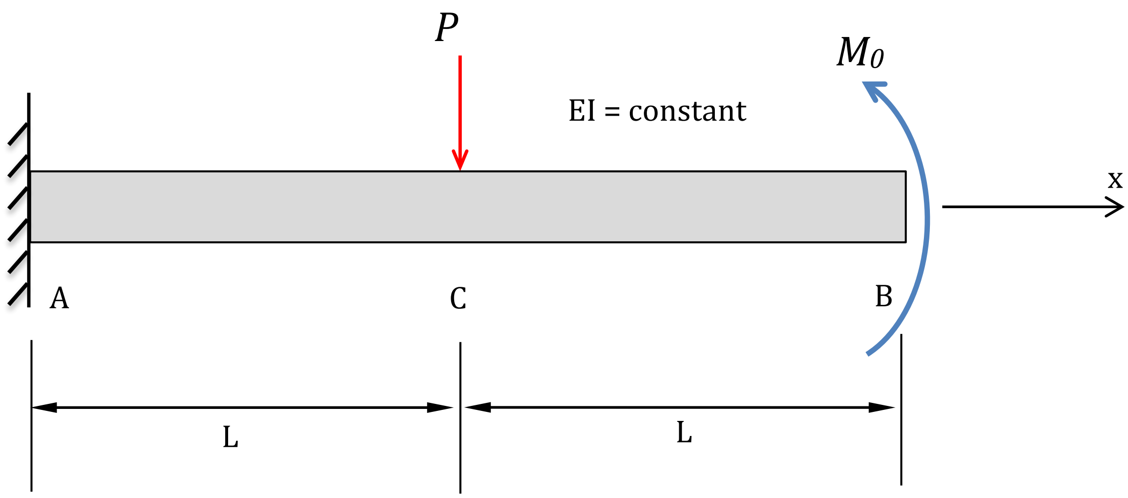
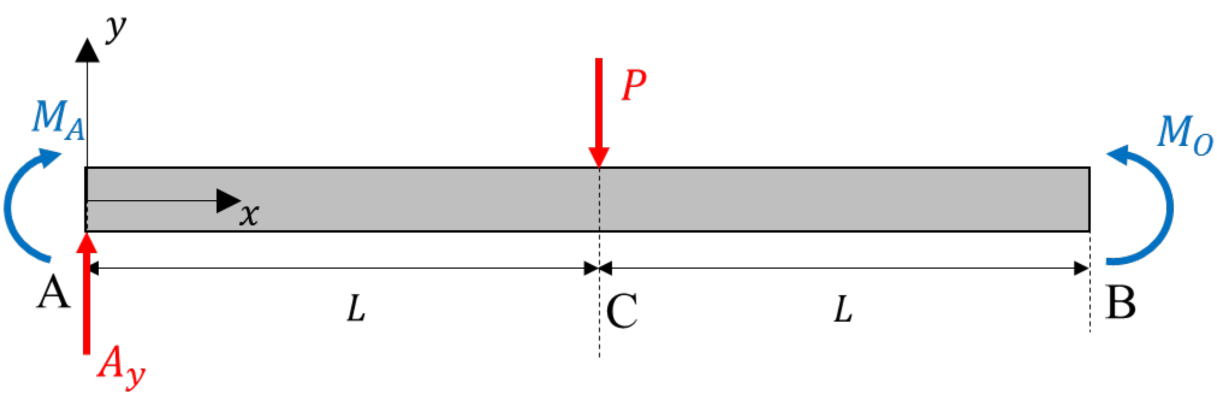
Q1 (10 Points): Beam AB is fixed at the rigid walls A, and is subjected to the concentrated force P at C and moment M_0 at B as shown below. Use the integration method to determine the deflection function of the beam.



Solution: (1). From the FBD of beam AB,



$$\Sigma F_y: A_y = P \quad (1)$$

$$\Sigma M_A: -M_A - PL + M_0 = 0 \Rightarrow M_A = M_0 - PL \quad (2)$$

Geometric BCs: At fixed end A, $\theta_A = 0$, $v_A = 0$.

Bending moment:

$$M(x) = M_A + A_y x = M_0 - PL + Px \qquad [0 \le x \le L] \quad (3)$$

$$M(x) = M_A + A_y x - P(x - L) = M_0 \qquad [L \le x \le 2L] \quad (4)$$

Slope function:

$$\theta(x) = \theta_A + \frac{1}{EI}\int_0^x M(x)dx = 0 + \frac{1}{EI}\int_0^x (M_0 - PL + Px)dx \qquad [0 \le x \le L]$$

$$\Rightarrow \theta(x) = \frac{1}{EI}\left\{(M_0 - PL)x + \frac{Px^2}{2}\right\} \qquad (5)$$

$$\Rightarrow \theta_C = \theta(L) = \frac{1}{EI}\left(M_0 L - \frac{PL^2}{2}\right)$$

$$\theta(x) = \theta_C + \frac{1}{EI}\int_L^x M(x)dx = \frac{1}{EI}\left(M_0 L - \frac{PL^2}{2}\right) + \frac{1}{EI}\int_L^x M_0 dx \qquad [L \le x \le 2L]$$

$$\Rightarrow \theta(x) = \frac{1}{EI}\left(M_0 L - \frac{PL^2}{2}\right) + \frac{1}{EI}[M_0(x - L)] = \frac{1}{EI}\left(M_0 x - \frac{PL^2}{2}\right) \qquad (6)$$

Deflection function:

$$v(x) = v_A + \int_0^x \theta(x)dx \qquad [0 \le x \le L]$$

$$\Rightarrow v(x) = 0 + \int_0^x \frac{1}{EI}\left\{(M_0 - PL)x + \frac{Px^2}{2}\right\}dx = \frac{1}{EI}\left[\frac{(M_0 - PL)x^2}{2} + \frac{Px^3}{6}\right] \qquad (7)$$

$$\Rightarrow v_C = v(L) = \frac{1}{EI}\left[\frac{(M_0 - PL)L^2}{2} + \frac{PL^3}{6}\right] = \frac{1}{EI}\left[\frac{M_0 L^2}{2} - \frac{PL^3}{3}\right]$$

$$v(x) = v_C + \int_L^x \theta(x)dx \qquad [L \le x \le 2L]$$

$$\Rightarrow v(x) = \frac{1}{EI}\left[\frac{M_0 L^2}{2} - \frac{PL^3}{3}\right] + \int_L^x \frac{1}{EI}\left(M_0 x - \frac{PL^2}{2}\right)dx$$

$$\Rightarrow v(x) = \frac{1}{EI}\left[\frac{M_0 L^2}{2} - \frac{PL^3}{3}\right] + \frac{1}{EI}\left[\frac{M_0(x^2 - L^2)}{2} - \frac{PL^2}{2}(x - L)\right]$$

$$\Rightarrow v(x) = \frac{1}{EI}\left[\frac{M_0 x^2}{2} - \frac{PL^2 x}{2} + \frac{PL^3}{6}\right] \qquad (8)$$