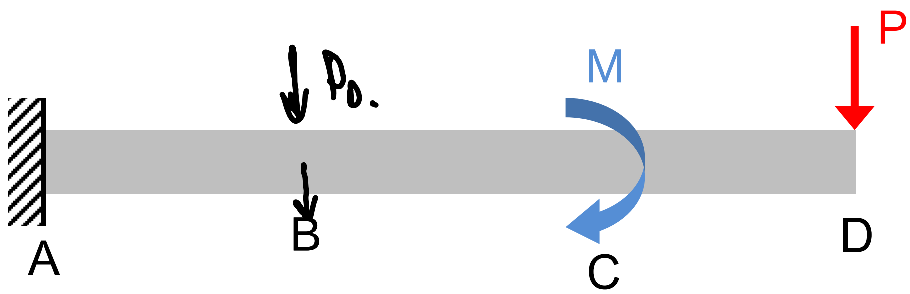
Castigliano’s Review

Question 1:

In the beam, the displacement at B is found by:

- The partial derivative of U with respect to P.
- The partial derivative of U with respect to M.
- The partial derivative of U with respect to a dummy load.
- The partial derivative of U with respect to a dummy moment.



$$\frac{\partial U}{\partial P_i} = \Delta_i$$

Example 16.8

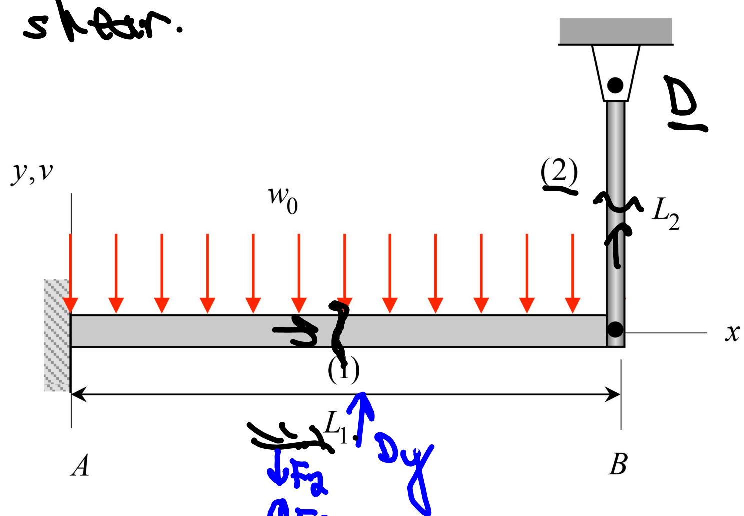
For the following examples, ***set up*** the problem for determining the requested deflections using Castigliano's method:

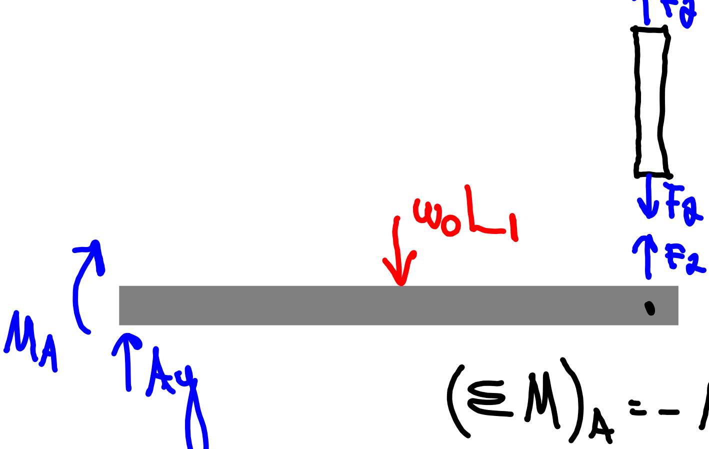
- draw appropriate FBDs;
- determine internal results for each section;
- set up the integrals for calculating the required deflections;
- explain how Castigliano's method is used to solve. Discuss the application of dummy forces (when needed) and how to handle redundant forces for indeterminate structures.

Problem A

Find the load carried by member (2) of the structure below. Let E and A be the Young's modulus and cross-sectional area, respectively, of member (2), whereas E and I are the Young's modulus and second area moment of the cross section of (1), respectively.

Don't neglect shear.



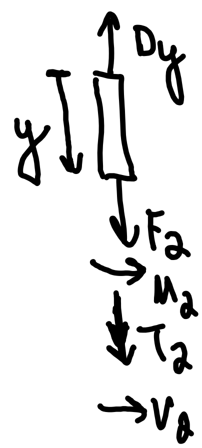


$$(\Sigma M)_A = -M_A - w_0 L\left(\frac{L}{2}\right) + F_2 L = 0$$

$$(\Sigma F_y) = A_y - w_0 L + F_2 = 0$$

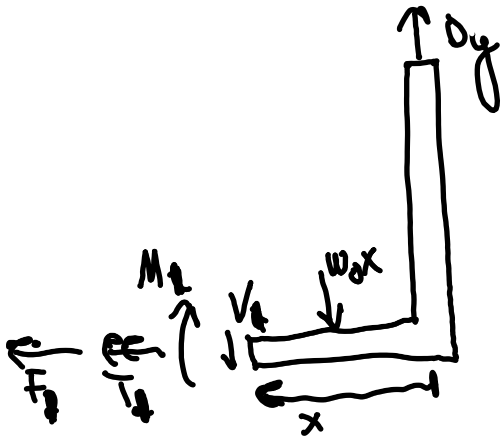
3 unknowns
2 eqns
1

$$(\Sigma F_y)_D : F_2 = D_y$$



$$\Sigma F_y = D_y - F_2 = 0$$

$$F_2(y) = D_y$$



$$\Sigma F_y = D_y - V_1 - w_0 x = 0$$

$$V_1(x) = D_y - w_0 x.$$

$$(\Sigma M)_2 = D_y x - w_0 x \left(\frac{x}{2}\right) - M_1 = 0$$

$$M_1(x) = D_y x - w_0 \frac{x^2}{2}.$$

$$U = U_{M1} + U_{V_1} + U_{F_2}$$

$$U = \frac{1}{2EI}\int_0^{L_1} M_1^2\,dx + \frac{f_s}{2GA}\int_0^{L_1} V_1^2\,dx + \frac{1}{2EA}\int_0^{L_2} F_1^2\,dy$$

$$\frac{\partial U}{\partial D_y} = 0 \qquad \frac{\partial F_2}{\partial D_y} = 1 \qquad \frac{\partial M_1}{\partial D_y} = x \qquad \frac{\partial V_1}{\partial D_y} = 1.$$

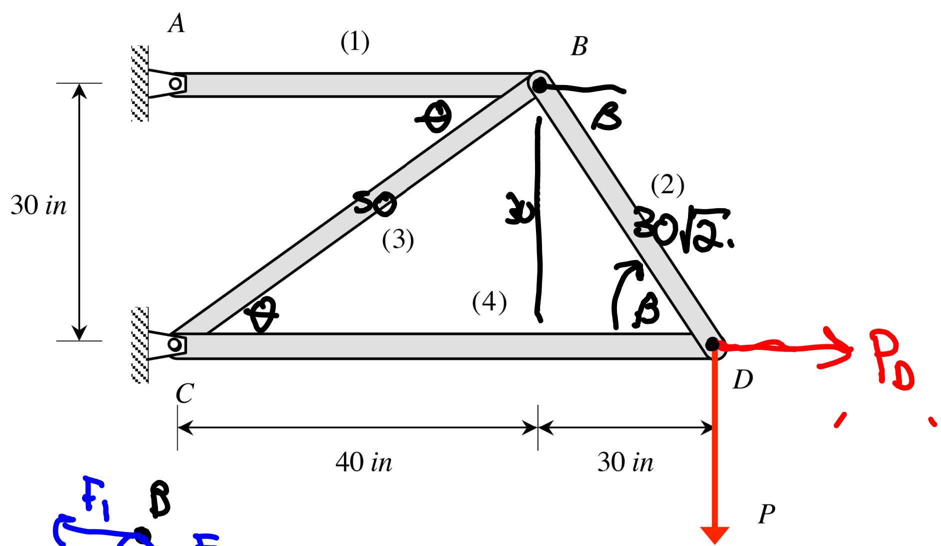
$$\frac{\partial U}{\partial D_y} = 0 = \frac{1}{EI}\int_0^{L_1}\left(D_y x - w_0 \frac{x^2}{2}\right)x\,dx + \frac{f_s}{GA}\int_0^{L_1}(D_y - w_0 x)\,dx + \frac{1}{EA}\int_0^{L_2} D_y\,dy$$

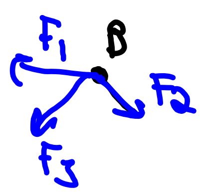
$$= \frac{D_y L_2}{EA} = e_2.$$

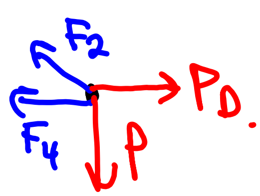
$$-e = \frac{1}{EI}\int_0^{L_1}\left(D_y x - w_0\frac{x^2}{2}\right)x\,dx + \frac{f_s}{GA}\int_0^{L_1}\left(D_y - w_0 x\right)dx.$$

Problem D

Determine the vertical and horizontal deflection of the truss at joint D. All members of the truss have a cross-sectional area of A and are made of a material with a Young's modulus of E.







$$(\Sigma F_x)_B = -F_1 - F_3\left(\tfrac{4}{5}\right) + F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0$$

$$(\Sigma F_y)_B = -F_3\left(\tfrac{3}{5}\right) - F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0$$

$$(\Sigma F_x)_D = P_D - F_4 - F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0$$

$$(\Sigma F_y)_D = -P + F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0.$$

4 unknowns

4 eqns.

$F_2 = \sqrt{2}\, P$

$F_4 = P_D - P \leftarrow$

$F_3 = -\left(\frac{5}{3}\right)P$

$F_1 = \frac{4}{3}P$

$$U = U_{F_1} + U_{F_2} + U_{F_3} + U_{F_4}$$

$$u = \left[\frac{\partial U}{\partial P_D}\right]_{P_D = 0}$$

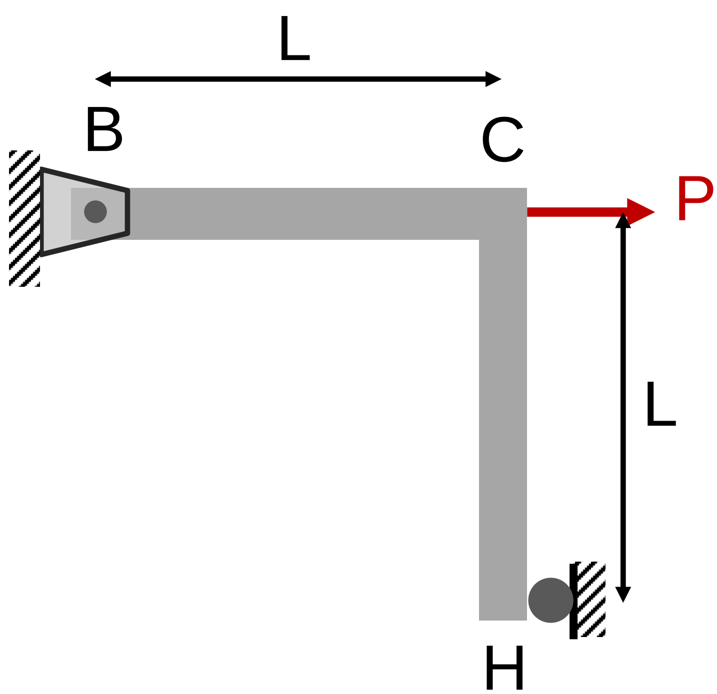
$$\underline{v} = -\left[\frac{\partial U}{\partial P}\right]_{P_D = 0}$$

$$\frac{\partial F_1}{\partial P_D} = \frac{\partial F_2}{\partial P_D} = \frac{\partial F_3}{\partial P_D} = 0 \qquad \frac{\partial F_4}{\partial P_D} = 1.$$

$$u = \frac{L_4}{EA}(-P)(1)$$

Lecture 24 Quiz

An L-shaped beam is subject to an applied force as shown in the diagram below. You are asked to use Castigliano's to solve for the angular change at H (θ_H). Structure BCH has a modulus of E and a second area moment of I.



(a) Draw the free body diagram.
(b) Solve for the angular change at H.

Ch16: 29.