12. Thin-walled pressure vessels

Objectives:
To study the combined axial and hoop stress state in the sidewalls of cylindrical vessels and in spherical pressure vessels.

Background:
Relationship between the resultant normal force F due to constant normal stress σ acting over an area A:

$$\sigma = \frac{F}{A}$$

Lecture topics:

a) Axial stress.
b) Hoop stress.
c) Combined state of stress.

Lecture Notes

We have so far understood the stresses, and deformations of thin rods/beams in (a) axial deformation, (b) torsion, and (c) in bending. In this class we will consider one more type of structure that is more "two-dimensional" compared the one-dimensional beam models.

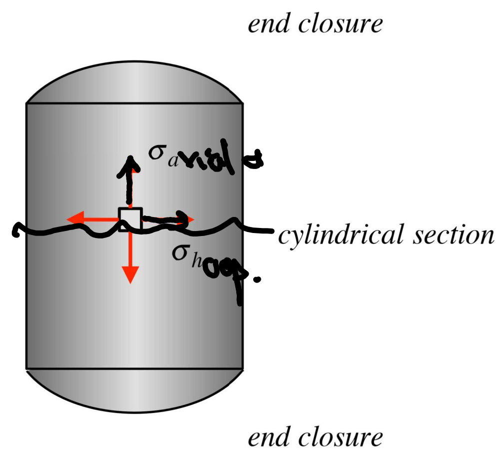
Thin-walled pressure vessels have a number of applications:

- Vacuum chambers
- Pressure vessels used for storing various kinds of fluids under high pressure
- Natural gas containers, hot air balloons, coke cans, gel and aerosol cans, chemical and nuclear reactors, oil refining containers, soap bubbles
- Liquid fuel containers in space vehicles
- Submarine hulls

To prevent the explosion or breakage of these pressure vessels it is important to design these to keep stresses within an acceptable level.

Cylindrical pressure vessels

Consider a thin-walled circular-cross section pressure vessel with an internal pressure of p, inner radius r and wall thickness t.



<u>*Circumferential cut through vessel*</u>

For equilibrium of the upper section of the vessel:

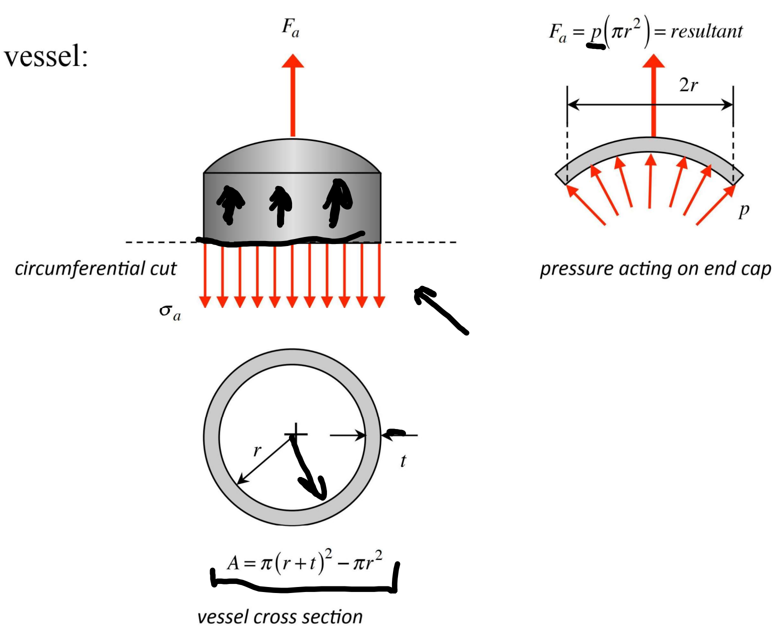
$$\sum F = F_a - \sigma_a A = 0 \quad \Rightarrow$$

$$\sigma_a = \frac{F_a}{A} = \frac{\pi r^2 p}{\pi\left(r+t\right)^2 - \pi r^2}$$

$$= \frac{\pi r^2 p}{\pi\left(r^2 + 2rt + t^2\right) - \pi r^2}$$

$$= \frac{r^2 p}{2rt + t^2}$$

$$= \frac{r^2 p}{2rt\left(1 + t/2r\right)} \approx \frac{pr}{2t} \quad ; \quad \frac{t}{r} << 1$$



σ_a is the *axial* component of normal stress in the vessel due to the internal pressure.

Longitudinal cut through vessel

For equilibrium of the left portion of a hoop section of the vessel (of height Δx):

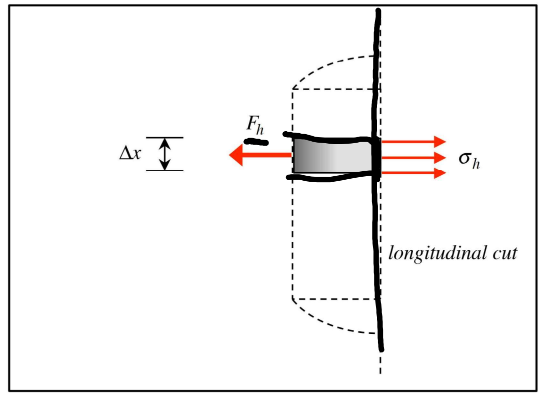
$$\sum F = F_h - \sigma_h A = 0 \quad \Rightarrow$$

$$\underline{\sigma_h} = \frac{F_h}{A} = \frac{2rp\Delta x}{2t\Delta x} = \frac{pr}{t}$$

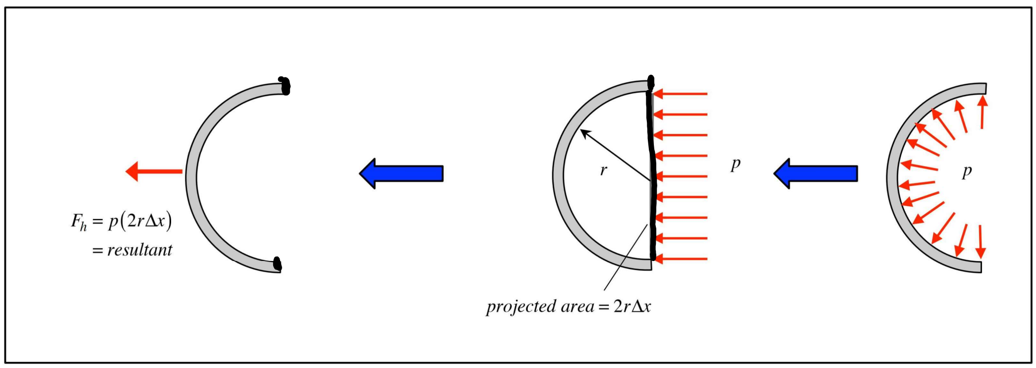
σ_h is the "*hoop*" component of normal stress in the vessel due to the internal pressure.

Note that the axial component of stress is exactly half of the hoop component of stress in a cylindrical pressure vessel.

SIDE view



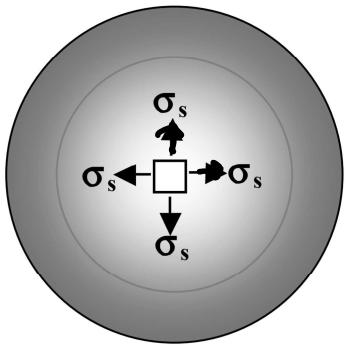
TOP view

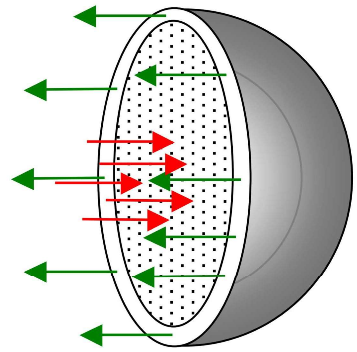


Spherical pressure vessels

Consider a thin-walled spherical pressure vessel with an internal pressure of p, inner radius r and wall thickness t. Using an equilibrium relationship on the hemispherical section of the tank gives a normal stress of:

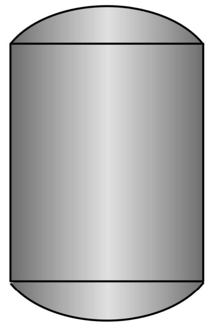
$$\sigma_s = \frac{pr}{2t}$$





Example 12.1

A steel propane tank for a barbecue grill has a 12-in inside diameter and a wall thickness of 1/8 in. The tank is pressurized to 200 psi. Determine the axial and hoop components of stress in the wall of the tank.



$$\sigma_a = \frac{pr}{2t}$$

$$\sigma_a = \frac{(200)(6)}{2(1/8)}$$

$$\sigma_a = 4800 \text{ psi}$$

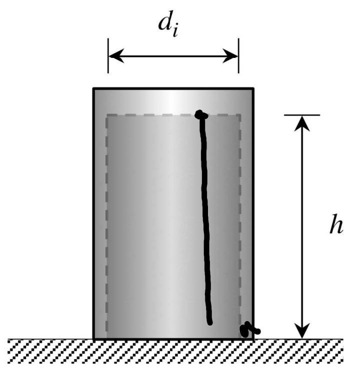
$$\sigma_h = \frac{pr}{t}$$

$$\sigma_h = \frac{200(6)}{(1/8)}$$

$$\sigma_h = 9600 \text{ psi}$$

Example 12.2

A vertical standpipe has an inside diameter of $d_i = 3m$ and is filled with water to depth of h = 5m. If the allowable hoop stress is 80MPa, what is the minimum wall thickness of the tank?



$\sigma_h = \frac{pr}{t}$ $t_{min} = \frac{pr}{\sigma_{allow}}$

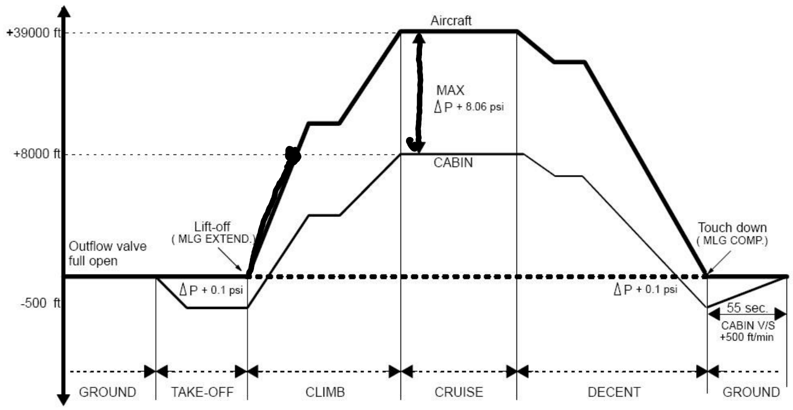
$p_{max} = \rho g h$

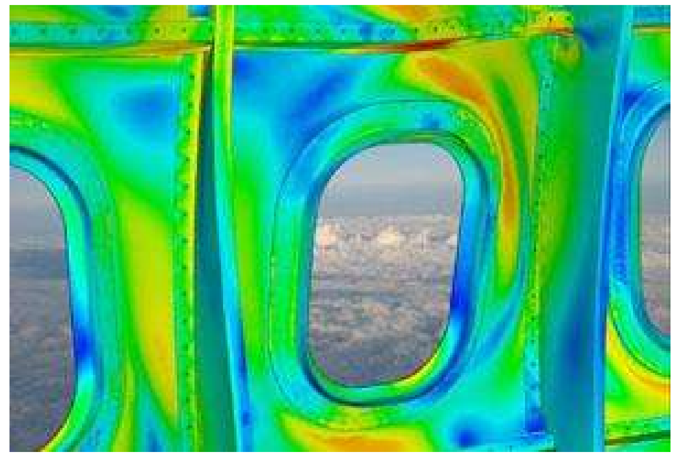
$t_{min} = \frac{(\rho g h) r}{\sigma_{allow}}$

$t_{min} = \frac{(1000)(9.8)(5)(1.5)}{80 \times 10^6}$

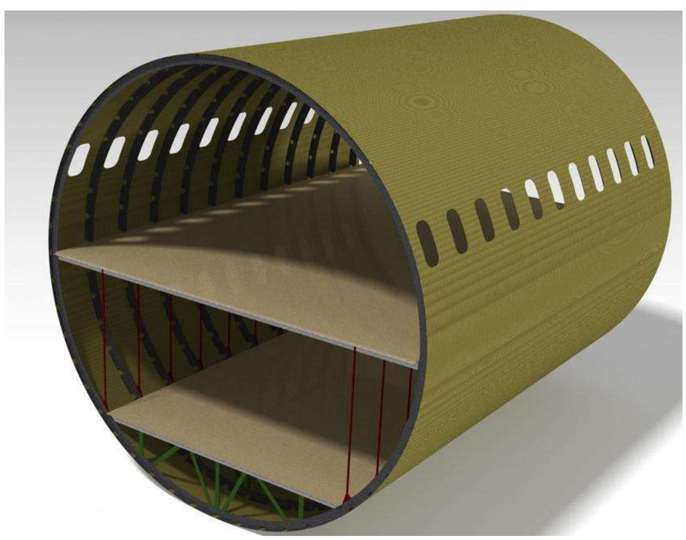
$t_{min} = 0.9$ mm.

Airplane as a Pressure Vessel





https://aviation.stackexchange.com/questions/19291/what-is-the-pressure-in-a-civil-aircraft-fuselage-at-flight-ceiling



An airplane exhibits a pressure difference of 56 kPa in the fuselage at 39 000 ft cruising altitude. The ~~radius~~ diameter of the fuselage is 4 m. The tensile yield strength of aircraft grade aluminum is 276 MPa.

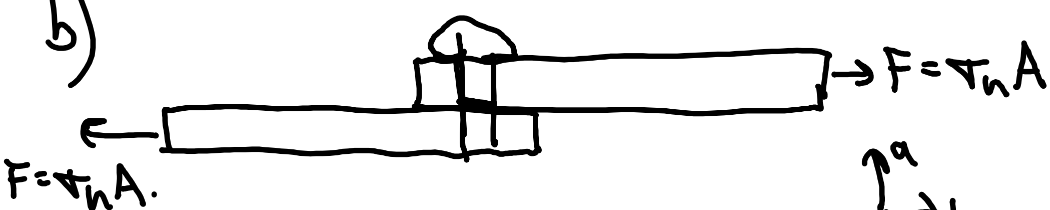
a) What thickness is required to achieve a factor of safety of 2.5?
b) Rivets hold the fuselage together. The rivets have a diameter of 3.175 mm and a shear strength of 95 MPa. What density of rivets are required to reach a factor of safety of 2.5?

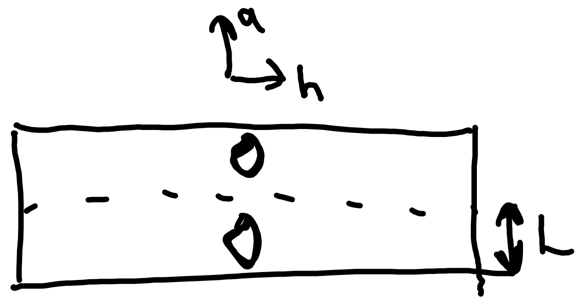
a) $\sigma_a = \frac{pr}{2t}$ $\qquad \sigma_h = \frac{pr}{t} = \frac{\sigma_Y}{FS} = \frac{\sigma_Y}{2.5}$

$$\frac{(56\times10^3)(2)}{t} = \frac{276\times10^6}{2.5}$$

$$t = 0.001\ m = 1\ mm$$ ←

b)





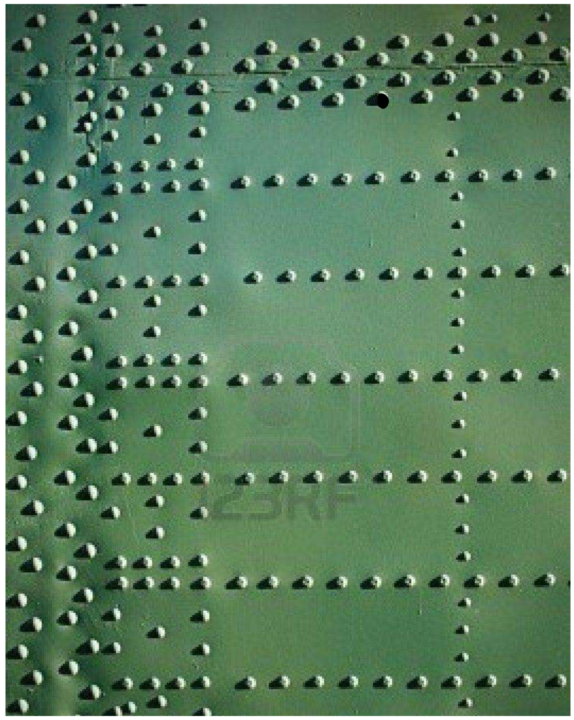
$\Sigma F_y = F = V$

$\tau_{rivet} A_{rivet} = \sigma_h A_h.$ $\qquad \sigma_h = 110.4\ MPa.$

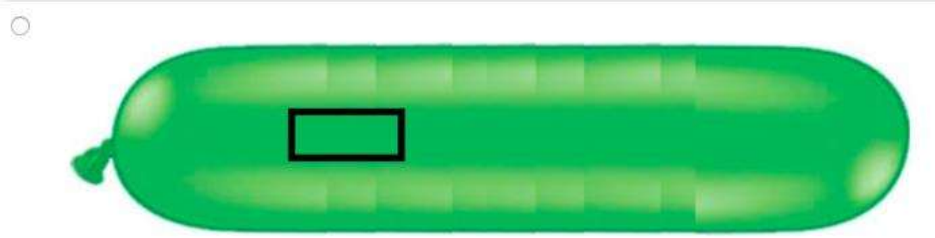
$$\left(\frac{\tau_s}{2.5}\right)\pi r^2 = \sigma_h (0.001)(L)$$

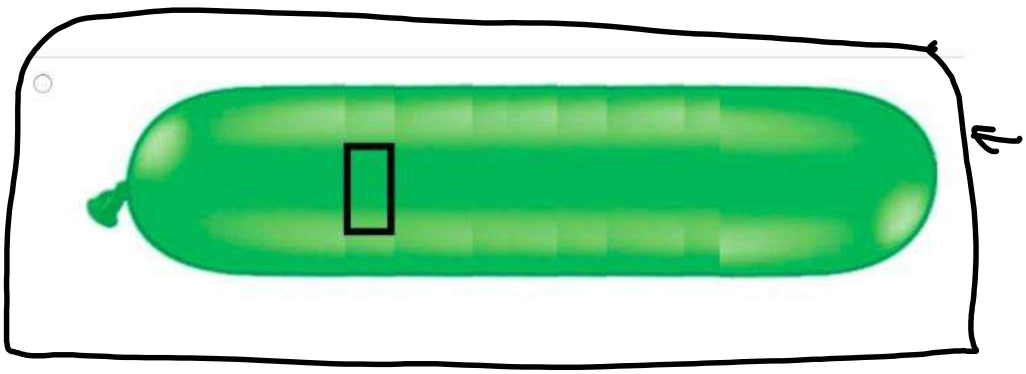
$$L = 0.0027\ m = 2.7\ mm.$$

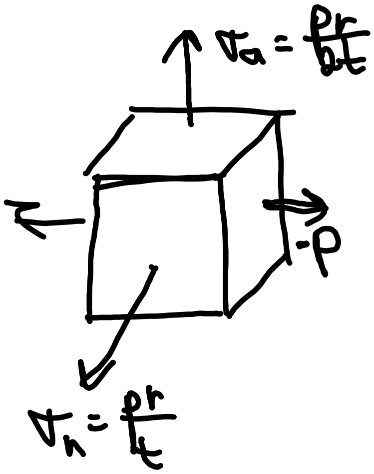
Example rivet pattern
for airplane fuselage.



A square was drawn on a cylindrical balloon before inflating it. The balloon was then inflated. Which one is the correct shape resulting from the original square after inflation?







$$\varepsilon_a = \frac{1}{E}\left(\sigma_a - \nu\left(\sigma_h + \sigma_t\right)\right)$$

$$\varepsilon_a = \frac{1}{E}\left(\frac{pr}{2t} - \nu\left(\frac{pr}{t} - p\right)\right) \qquad \nu = 0.5$$

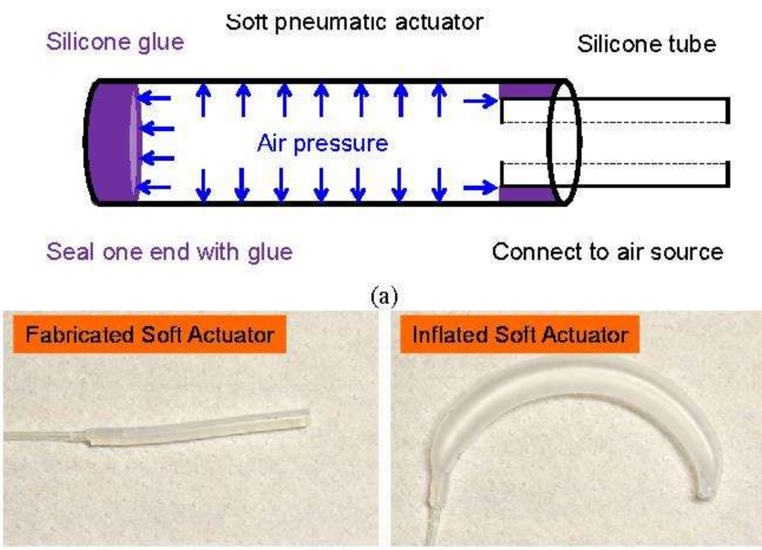
$$\varepsilon_a = \frac{1}{E}(\nu p) = \frac{p}{2E}.$$

$$\varepsilon_h = \frac{1}{E}\left(\frac{pr}{t} - \nu\left(\frac{pr}{2t} - p\right)\right)$$

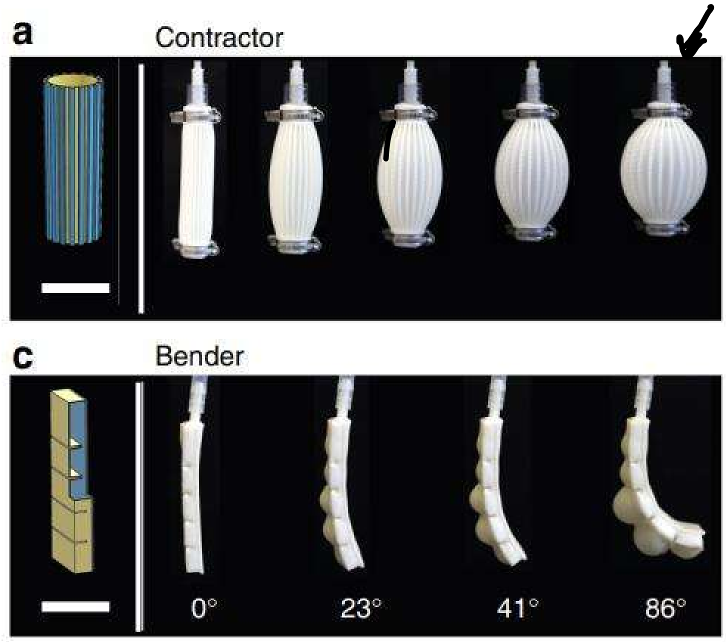
$$\varepsilon_h = \frac{1}{E}\left(\frac{pr}{t} - \frac{pr}{4t} + \nu \cdot p\right)$$

$$\varepsilon_h = \frac{1}{E}\left(\frac{3pr}{4t}\right) = \left(\frac{p}{2E}\right)\left(\frac{3}{2}\right)\left(\frac{r}{t}\right). \qquad r \gg t$$

Pneumatic Actuators



Jin Guo et al, IEEE ICMA, 2017



Schaffner et al, Nat Comm, 9:878, 2018.

1. Starting from the equations for stresses in chambers and the generalized strain equation, derive the relationships for the strain in the axial and hoop directions in a pressure vessel.

2. For an elastomer with a Poisson's ratio of 0.5, what is the strain in the axial and radial directions as a function of pressure?

3. How could you modify the materials properties or device structure ot improve the actuation strain and/or actuation stress in the axial direction?