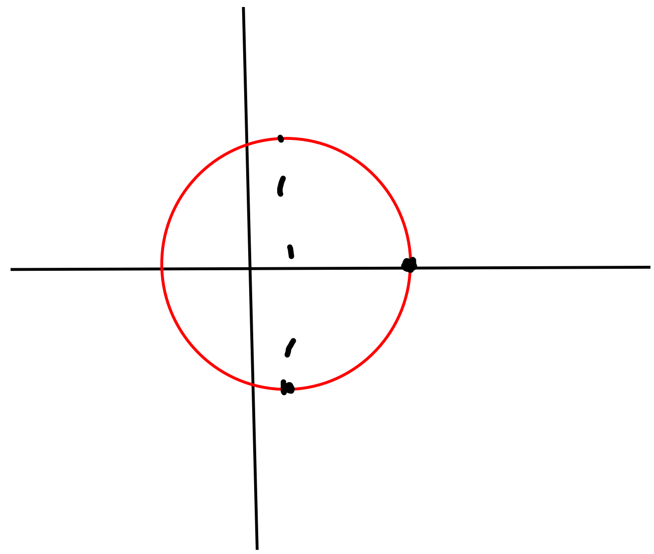
Stress Transformations

At the max normal stress, the shear stress is ______.
At the max shear stress, the normal stress is ______.

- $zero, zero$
- $\boxed{zero, \sigma_{avg}}$
- $zero, \sigma_{P1}$
- $\tau_{max}, zero$
- τ_{max}, σ_{avg}
- τ_{max}, σ_{P1}



$\sigma_{P2} = ?$

- τ_{max}
- R
- $\sigma_{avg} + R$
- $\boxed{\sigma_{avg} - R}$

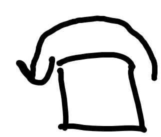
Lecture 30 Summary

$$\underline{\sigma_n} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

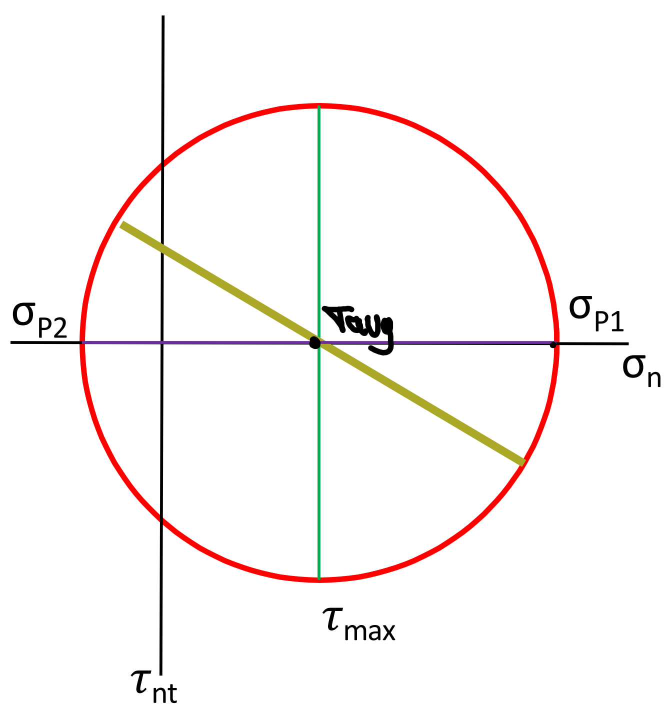
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\sigma_{P1} = \sigma_{avg} + R$$

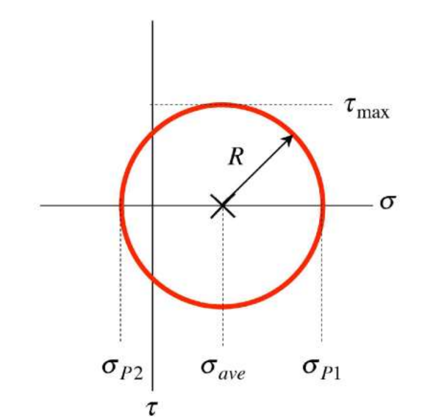
$$\sigma_{P2} = \sigma_{avg} - R$$

$$\tau_{max} = R$$



__Using Mohr's circle to locate planes of principal stresses and in-plane maximum shear stress__

Up to this point we have seen that Mohr's circle in the $\sigma-\tau$ plane provides us with information on the description of the state of plane stress: the stress states lie on a circle of radius $R=\sqrt{\tau_{xy}^2+\left(\sigma_x-\sigma_y\right)^2/4}$ and centered on $(\sigma,\tau)=(\sigma_{ave},0)$, where $\sigma_{ave}=\left(\sigma_x+\sigma_y\right)/2$, and where σ_x, σ_y and τ_{xy} are the two normal components of stress and shear component of stress, respectively, corresponding to a set of *x-y* coordinate axes.



$$\sigma_{ave}=\frac{\sigma_x+\sigma_y}{2} \qquad R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

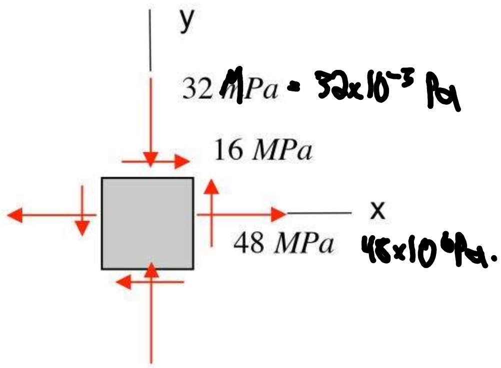
What we have *not* done at this point is discussed how to relate a transformed stress state through a rotation angle of θ to its location on the Mohr's circle in the $\sigma-\tau$ plane.

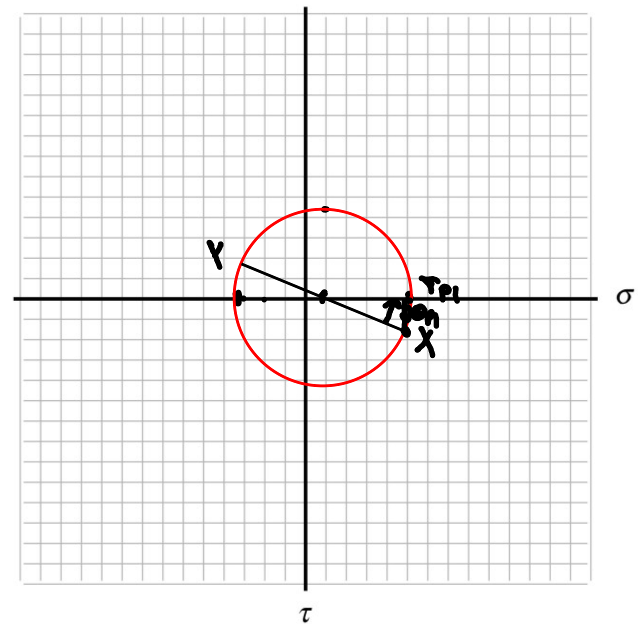
Before attempting this, let's review a couple points related to what we already know about stress states and Mohr's circle.

- Direction of positive shear stress in Mohr's circle. We have defined a positive shear stress on the x-face of the stress cube as being in the positive y-direction. Once we rotate this stress cube, this notation is equivalently stated as being positive in the n-face pointing in the t-direction. We will continue that here. However, here we will point the positive τ direction DOWNWARD in the $\sigma-\tau$ plane when constructing our Mohr's circle diagram. The reasoning behind this somewhat odd choice of positive direction is to maintain an equivalence in the direction of rotation of the element in the physical space with the direction of rotation (e.g., to insure that a CCW rotation in the physical space corresponds to a CCW rotation in Mohr's circle plane).

Example 13.4

Draw the in-plane Mohr's circle for the plane stress state shown below. What are the principal stresses and the maximum in-plane shear stress?





1 unit = 10 MPa.

$\sigma_{avg} = \frac{48-32}{2} = 8$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$R = 43$ MPa.

$X = (48, 16)$

$Y = (-32, -16)$

$\sigma_{p1} = \sigma_{avg} + R = 51$ MPa

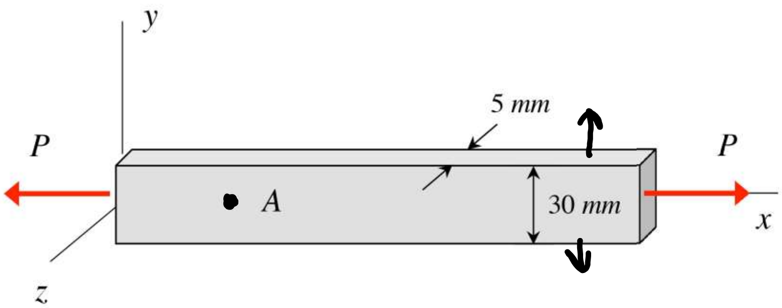
$\sigma_{p2} = \sigma_{avg} - R = -35$ MPa.

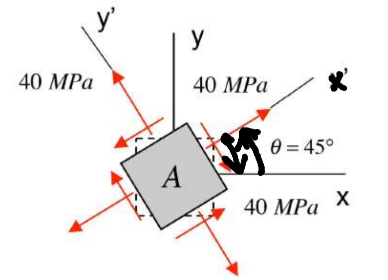
$2\theta_{p1} = \tan^{-1}\left(\frac{16}{48-8}\right)$

$\theta_{p1} = 10.9°$ CCW

Example 13.5

Draw the in-plane Mohr's circle for the plane stress state shown below. Determine the axial load P acting on the bar.



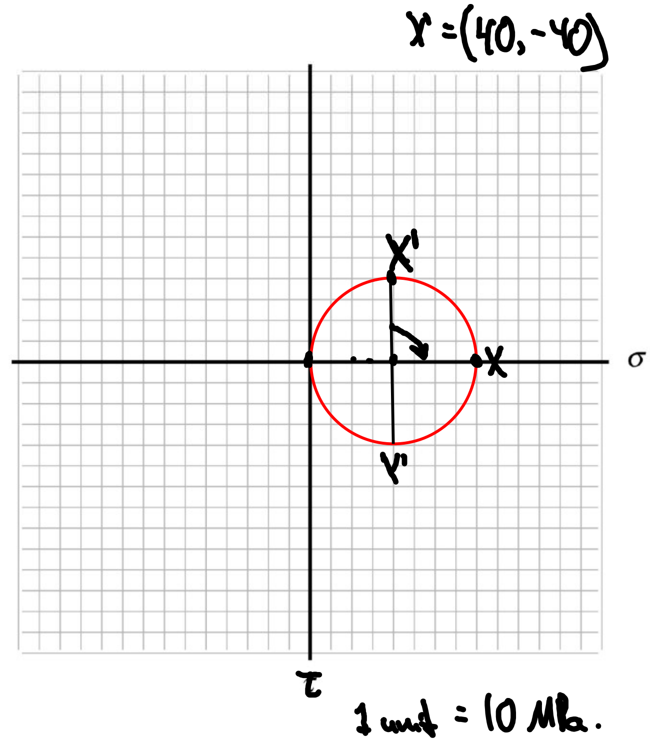


$$\sigma_{avg} = \frac{40+40}{2} = 40 \text{ MPa.}$$

$$R = \sqrt{\left(\frac{40-40}{2}\right)^2 + 40^2} = 40 \text{ MPa.}$$

$$\boxed{\sigma_{p1} = 80 \text{ MPa}} = \sigma_x$$

$$\sigma_{p2} = 0 \text{ MPa.}$$



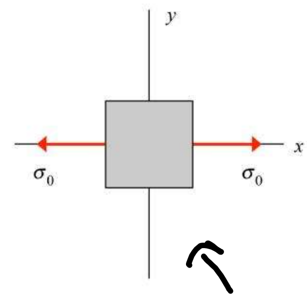
$$\sigma_x = \frac{P}{A}$$

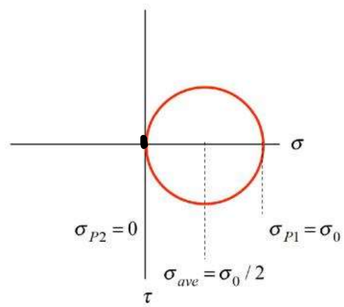
$$P = \sigma_x A = 80 \text{ MPa}(0.005 \text{ m})(0.03 \text{ m}) = 12 \text{ kN.}$$

Mohr's circle examples – some special stress states

Uniaxial stress *(e.g., uniaxial load on member)*

$$\left(\sigma_x, \sigma_y, \tau_{xy}\right) = \left(\sigma_0, 0, 0\right)$$





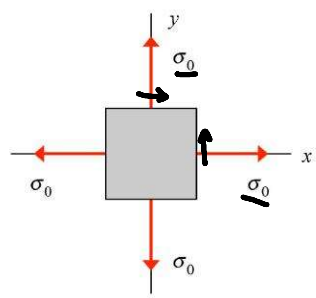
stress element **Mohr's circle**

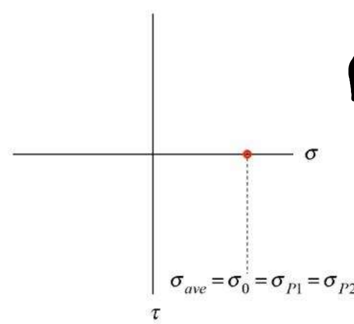
Hydrostatic stress *(e.g., thin-walled spherical pressure vessel)*

$$\left(\sigma_x, \sigma_y, \tau_{xy}\right) = \left(\sigma_0, \sigma_0, 0\right)$$

$\tau_{avg} = \tau_0$

$R = \sqrt{\left(\frac{\tau_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ 0





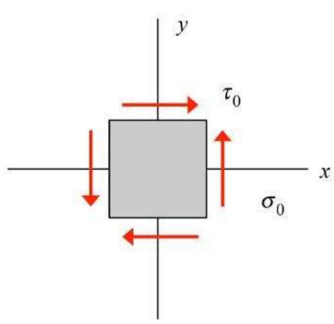
stress element **Mohr's circle**

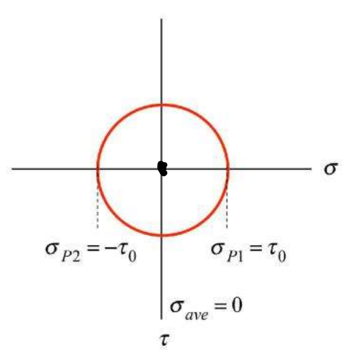
Pure shear stress *(e.g., axial torque on member)*

$$\left(\sigma_x, \sigma_y, \tau_{xy}\right) = \left(0, 0, \tau_0\right)$$

$\tau_{avg} = 0$

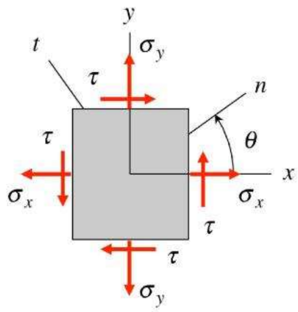
$R = \tau_0.$

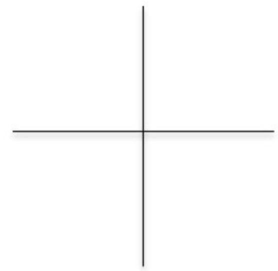




stress element **Mohr's circle**

Reflection: Stress transformations and Mohr's circle (for a state of plane stress)





a) *What* is a *stress element*?

b) *Why* are we interested in *stress transformations*?

Max σ and max τ $\Rightarrow$ when things will fail.

c) There are the *two important parameters* that we need to represent a state of stress – what are they?

d) What are *principal stresses*? How are these related to the two parameters mentioned in c) above?

e) What is the *maximum in-plane shear stress*? How is this related to the two parameters mentioned in c) above?

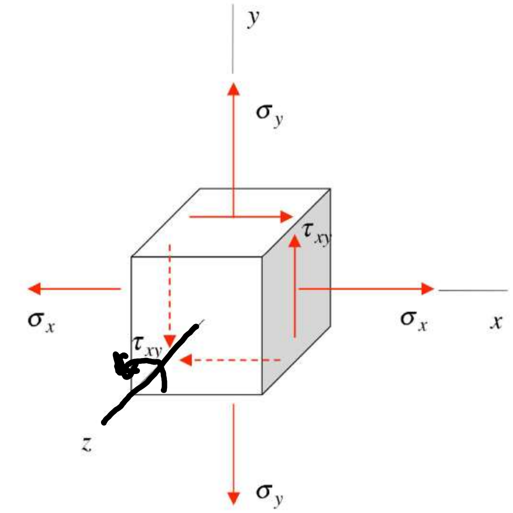
f) Where is the *center* of Mohr's circle? What is the *radius* of Mohr's circle?

g) *Why* do we choose the "*positive*" direction of τ as *downward*?

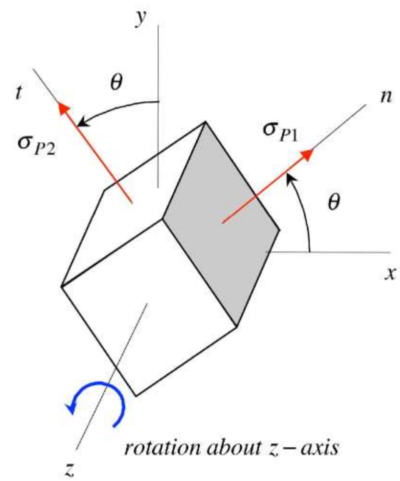
h) *How* do we know that a rotation of θ in the physical world correspond to a rotation of 2θ in Mohr's circle?

i) *How* can we use Mohr's circle to find the rotations of the stress element that correspond to the principal components of stress and the maximum in-plane shear stress?

e) Absolute maximum shear stress for plane stress



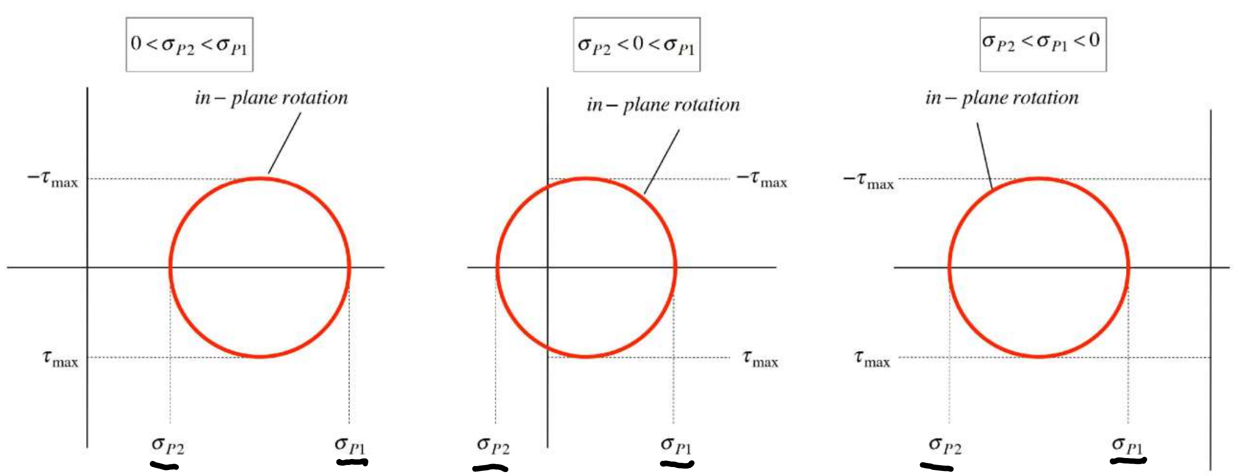
stress element in plane stress

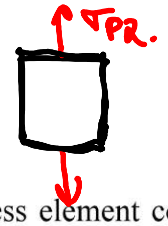


stress element in rotated to orientation for principal stresses

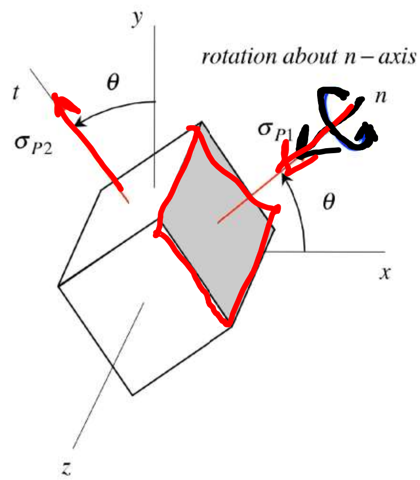
Recall for a state of plane stress, the stress element shows zero normal and shear stresses on the *z*-face. As we rotate the stress element about the *z*-axis, we observe the normal and shear stresses on the other four faces change according to our stress transformation equations. When rotated to the orientation showing the principal stresses, only the normal stresses appear on these four faces, as shown in the figure above right.

The Mohr's circle for the stress element rotation described above is shown in the following figures, considering the three possibilities on the signs of the principal stresses σ_{P1} and σ_{P2} .

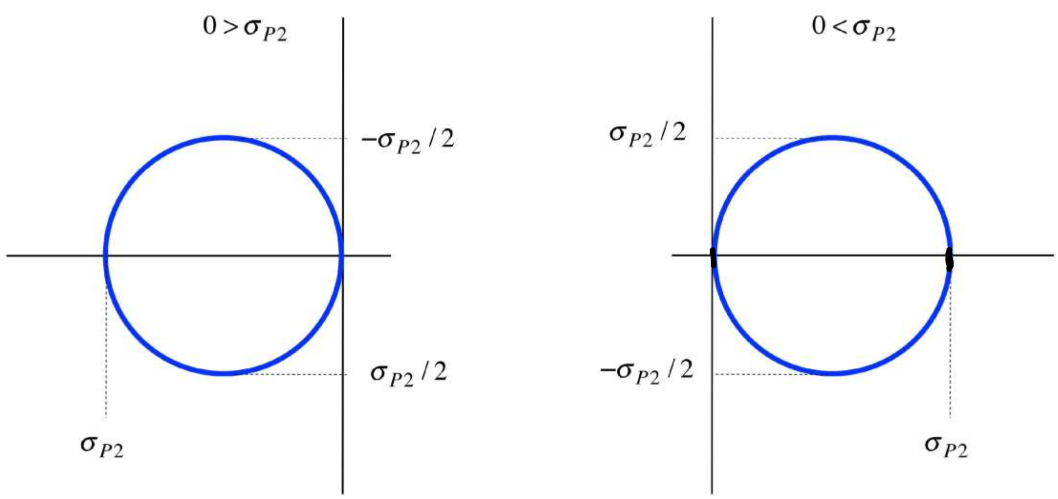




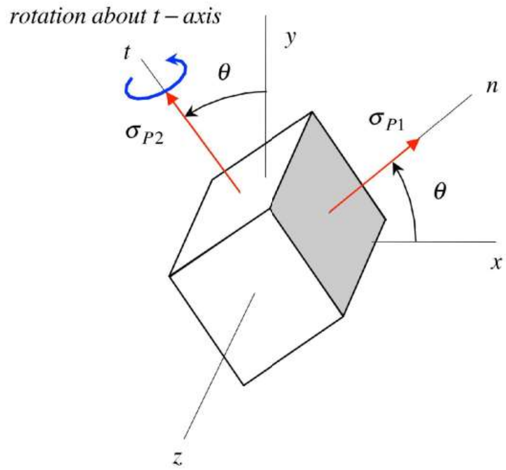
Now, starting with the orientation of the stress element corresponding to the principal stresses, say we rotate the stress element about the "*n*" axis, as shown below.

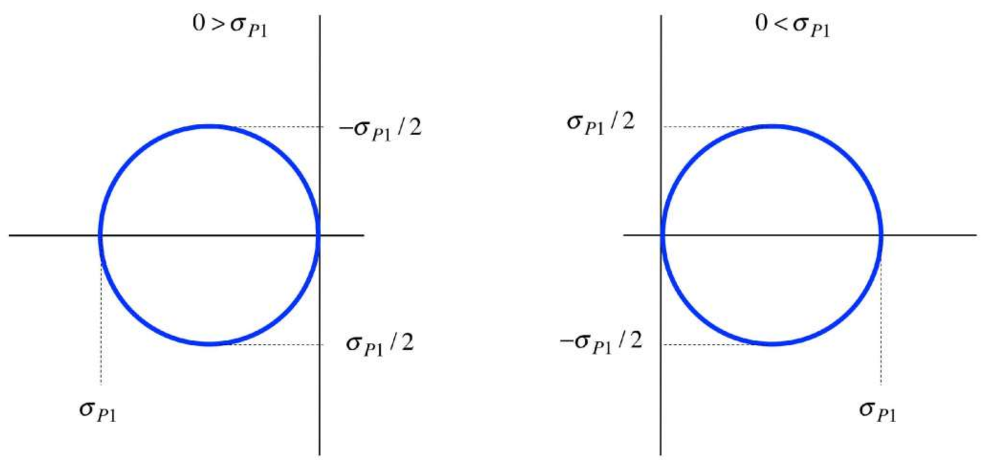


Since the *z*-face of the stress element is stress free, a 90° rotation produces zero normal and zero shear stress on the t-face. The Mohr's circle for this rotation is shown below, considering the two possibilities on the sign for σ_{P2}. Note that $\sigma = 0$ is an out-of-plane principal stress.

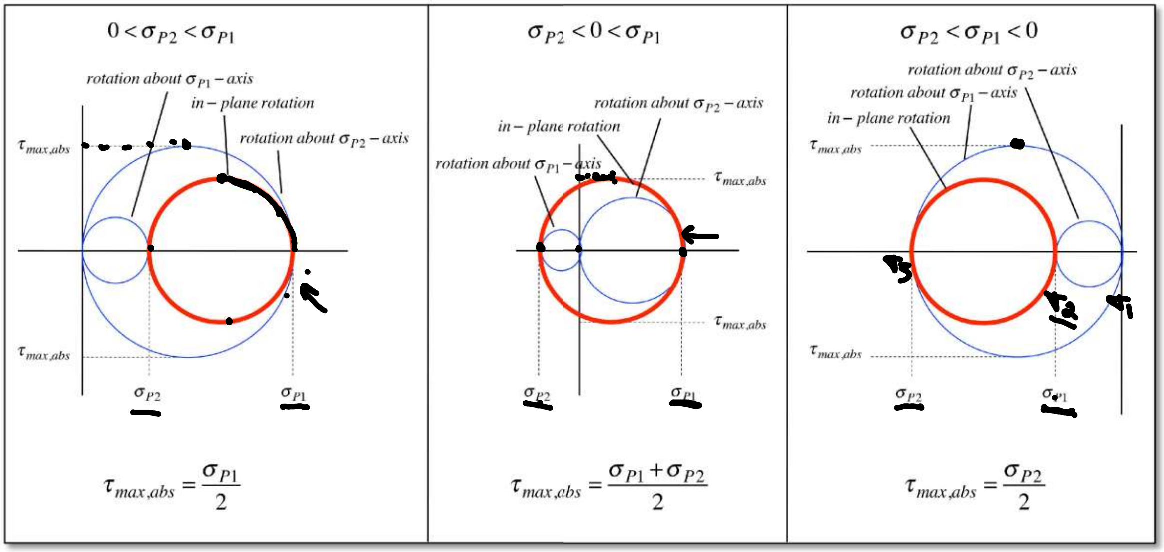


Similarly, a rotation of the stress element about the "t" axis, starting with the principal stresses orientation, produces the Mohr's circle shown below, depending on the sign of σ_1. Note that $\sigma = 0$ an the out-of-plane principal stress.





Suppose that we superimpose the Mohr's circle for these three stress element rotations. In the end, we will have three sets of Mohr's circles, depending on the signs of the principal stresses σ_{P1} and σ_{P2}, as shown in the following.



In conclusion, we see that the "absolute maximum shear stress", $\tau_{max,abs}$, (the largest shear stress observed for both in-plane and out-of-plane rotations) depends on the signs of the in-plane principal stresses, σ_{P1} and σ_{P2}: if they are of the same sign, then $\tau_{max,abs}$ is half of the larger of the two, and if they are of opposite signs, then $\tau_{max,abs}$ is the average value of the two in-plane principal stresses.

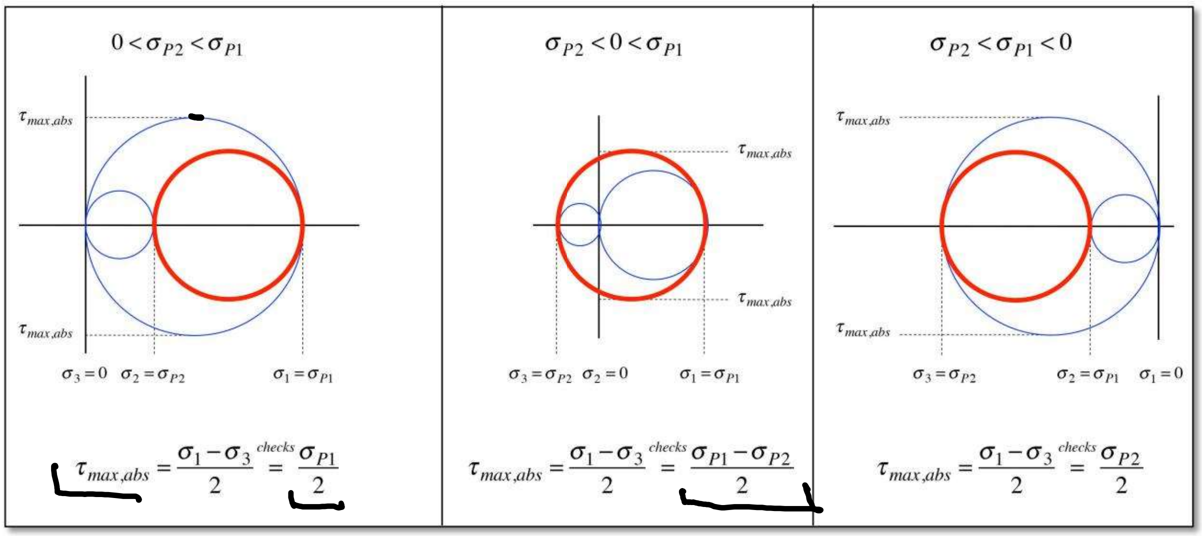
An alternate way (and easier to remember way) to express this result is to first introduce the following notation for principal stresses: consider the two in-plane principal stresses σ_{P1} and σ_{P2}, and the out-of-plane principal stress (which is zero) and rename these in the following way:

- σ_1 is the *largest* of the three → most positive.
- σ_3 is the *smallest* of the three → most negative.
- σ_2 is the *intermediate* of the three

That is, $\sigma_3 \le \sigma_2 \le \sigma_1$. With this notation, we can simply write:

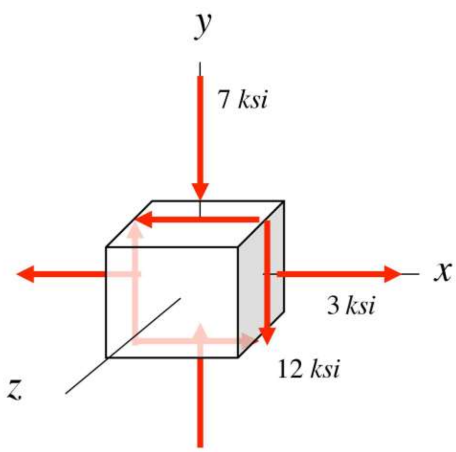
$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2}$$

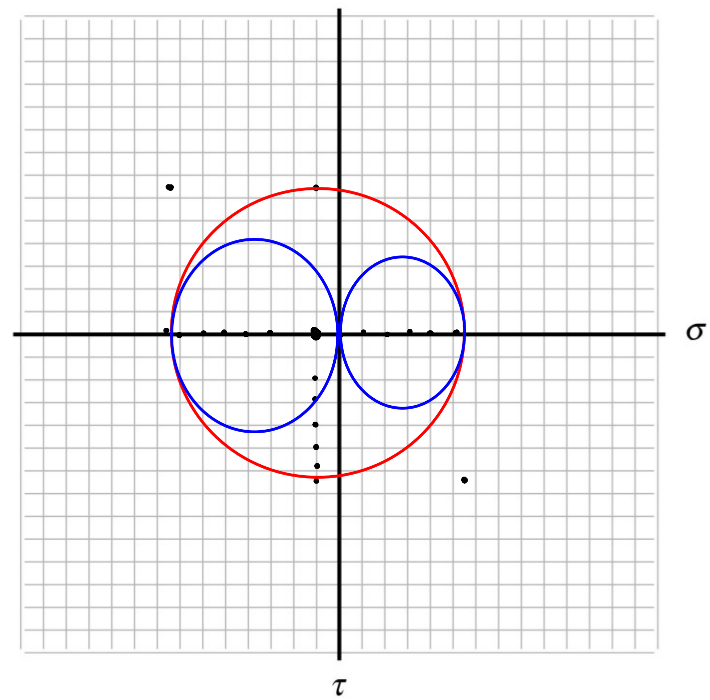
as summarized in the following.


$0 < \sigma_{P2} < \sigma_{P1}$
$\tau_{max,abs}$
$\tau_{max,abs}$
$\sigma_3 = 0$ $\sigma_2 = \sigma_{P2}$ $\sigma_1 = \sigma_{P1}$
$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \stackrel{checks}{=} \frac{\sigma_{P1}}{2}$
$\sigma_{P2} < 0 < \sigma_{P1}$
$\tau_{max,abs}$
$\tau_{max,abs}$
$\sigma_3 = \sigma_{P2}$ $\sigma_2 = 0$ $\sigma_1 = \sigma_{P1}$
$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \stackrel{checks}{=} \frac{\sigma_{P1} - \sigma_{P2}}{2}$
$\sigma_{P2} < \sigma_{P1} < 0$
$\tau_{max,abs}$
$\tau_{max,abs}$
$\sigma_3 = \sigma_{P2}$ $\sigma_2 = \sigma_{P1}$ $\sigma_1 = 0$
$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \stackrel{checks}{=} \frac{\sigma_{P2}}{2}$

Example 13.6

Determine the principal stresses and the absolute maximum shear stress for the plane stress element shown.





1 unit = 2ksi

$\sigma_{avg} = \frac{3-7}{2} = -2 \text{ ksi}$

$R = \sqrt{\left(\frac{3+7}{2}\right)^2 + 12^2} = 13 \text{ ksi}$

$\tau_{max,abs} = R$

Example 13.7

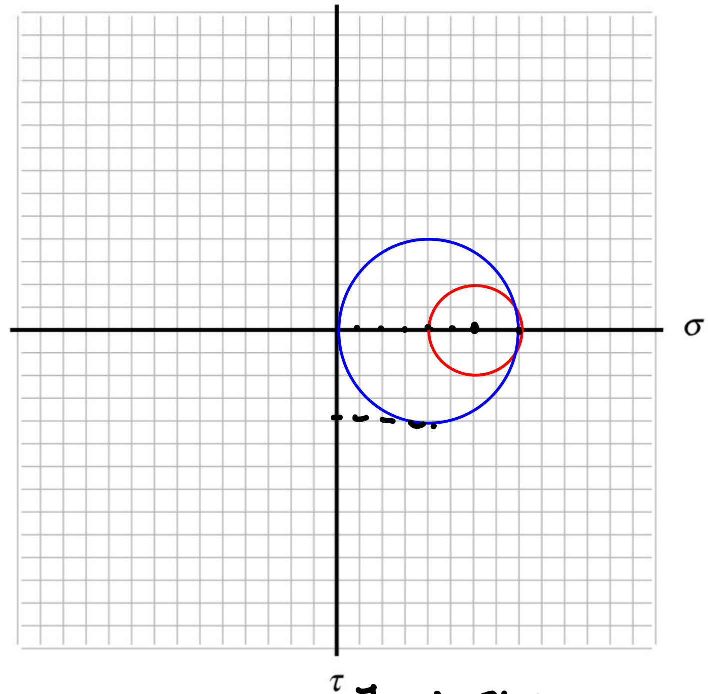
A state of plane stress is given by: $\sigma_x = 12\ ksi$, $\sigma_y = 12\ ksi$ and $\tau_{xy} = 4\ ksi$. Determine the principal stresses and the absolute maximum shear stress for this state of stress.

$\sigma_{avg} = \frac{12+12}{2} = 12\,ksi$

$R = \sqrt{\left(\frac{12-12}{2}\right)^2 + \tau_{xy}} = \tau_{xy}$

$R = 4\,ksi$

$\tau_{max,abs} = \frac{\sigma_{p1}}{2} = \frac{12+4}{2} = 8\,ksi$



1 unit = 2ksi