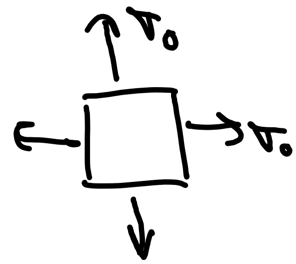
Lecture 33 Hotseat

For a state of hydrostatic stress, what is the relationship between the max in-plane shear stress and the absolute maximum shear stress?

X $\tau_{max,in-plane} > \tau_{max,abs}$

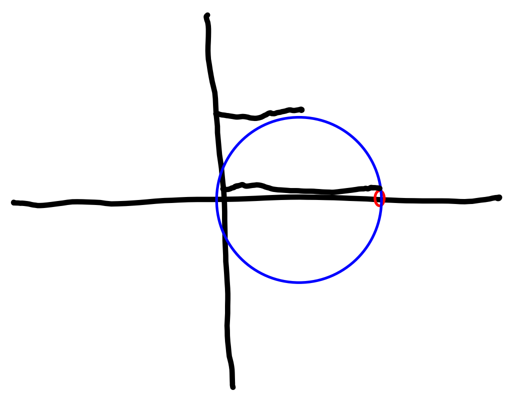
→ $\tau_{max,in-plane} = \tau_{max,abs}$

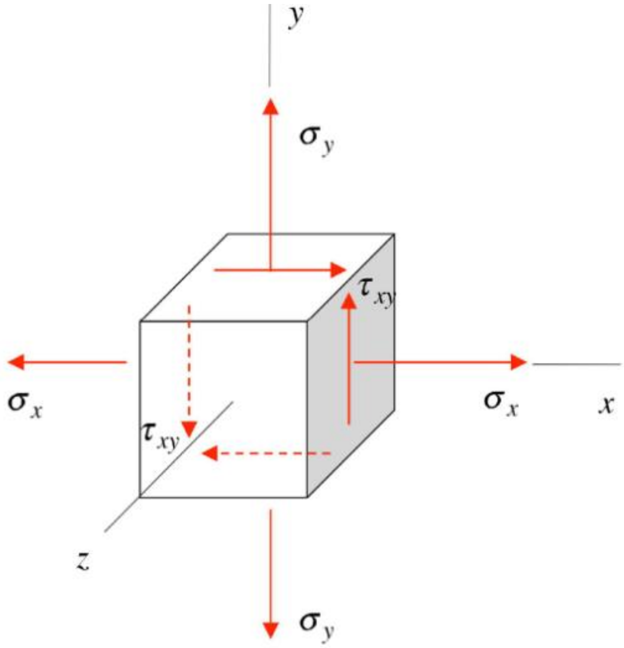
→ $\tau_{max,in-plane} < \tau_{max,abs}$



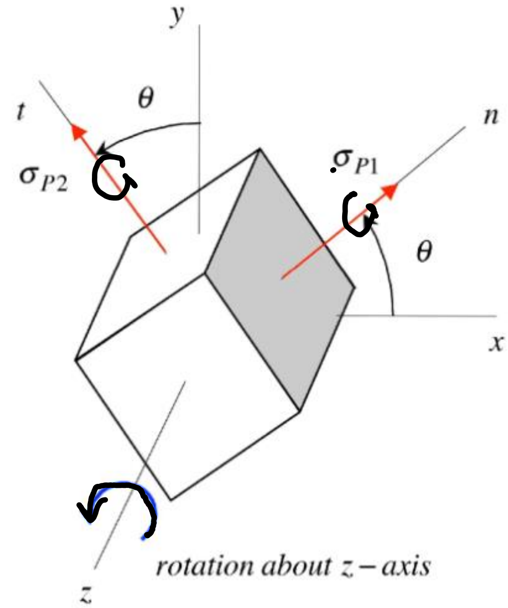
$\sigma_{avg} = \sigma_0$

$R = 0$



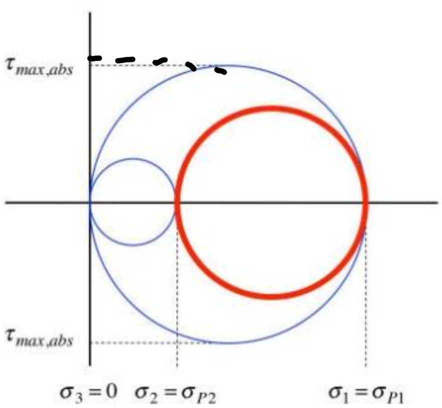


stress element in plane stress



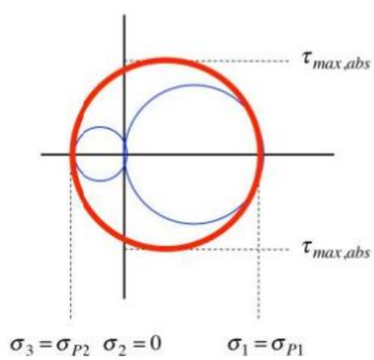
stress element in rotated to orientation for principal stresses

$0 < \sigma_{P2} < \sigma_{P1}$



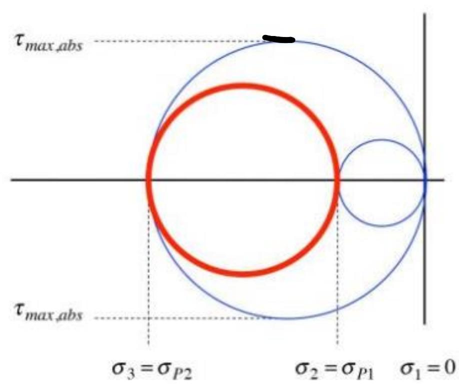
$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \overset{checks}{=} \frac{\sigma_{P1}}{2}$$

$\sigma_{P2} < 0 < \sigma_{P1}$



$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \overset{checks}{=} \frac{\sigma_{P1} - \sigma_{P2}}{2}$$

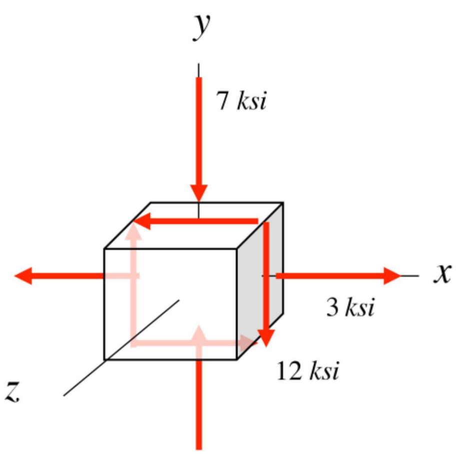
$\sigma_{P2} < \sigma_{P1} < 0$

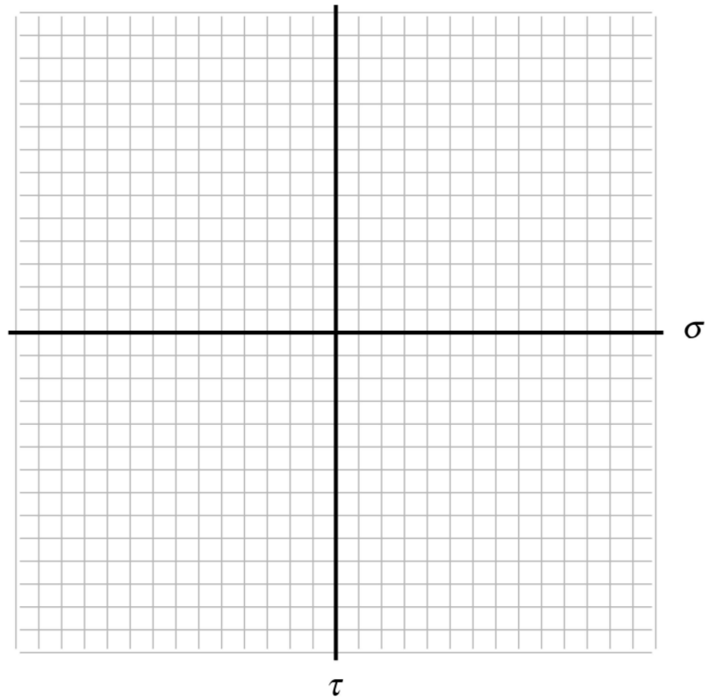


$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} \overset{checks}{=} \frac{\sigma_{P2}}{2}$$

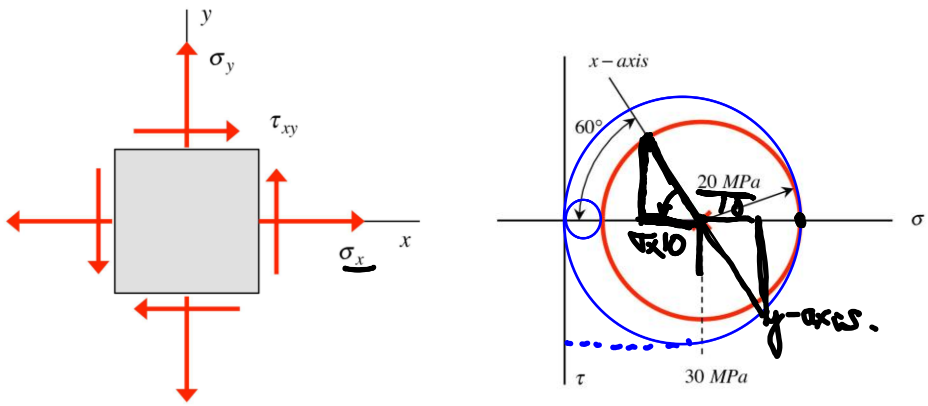
Example 13.6 ✓

Determine the principal stresses and the absolute maximum shear stress for the plane stress element shown.





Example 13.8



The Mohr's circle for a stress state is presented above.

a) Show the locations of the y-axis in the Mohr's circle above.

b) Determine the principal stresses and the absolute maximum shear stress for this state.

c) Determine the values for σ_x, σ_y and τ_{xy} of this stress state.

b) $\sigma_{P1} = \sigma_{avg} + R = 30 + 20 = 50 \text{ MPa}$

$\sigma_{P2} = \sigma_{avg} - R = 30 - 20 = 10 \text{ MPa}.$

$\tau_{max,abs} = \frac{\sigma_{P1}}{2} = 25 \text{ MPa}.$

c) $\cos(2\theta) = \frac{(\sigma_{avg} - \sigma_x)}{R}$

$\sigma_x = \sigma_{avg} - R\cos(60°) = 20 \text{ MPa}$

$\sigma_y = 2\sigma_{avg} - \sigma_x = 40 \text{ MPa}.$]

$\sigma_y = \sigma_{avg} + R\cos(60°)$

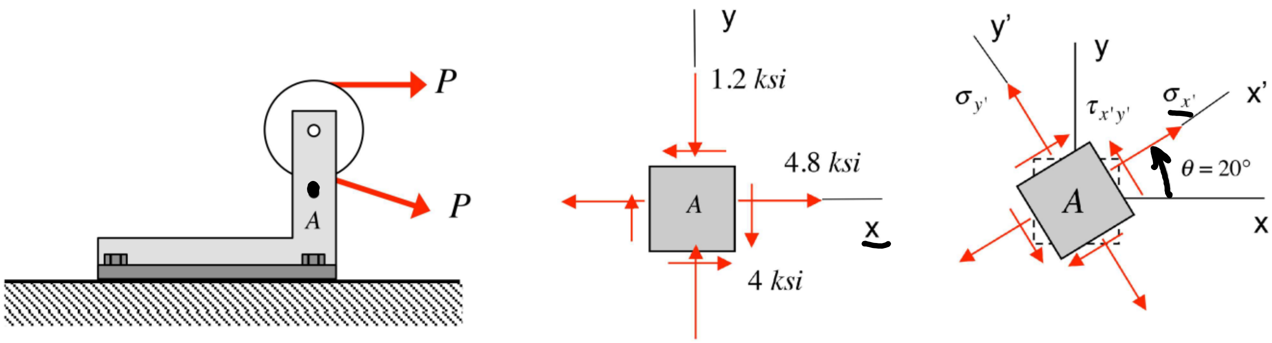
$\tau_{xy} = -R\sin(60°) = -20\left(\frac{\sqrt{3}}{2}\right)$

Example 13.9

Consider the loaded pulley bracket shown below.

a) Determine $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$ corresponding to $\theta = 20°$.

b) Determine the principal stresses and maximum in-plane shear stress, along with the corresponding rotation angles.



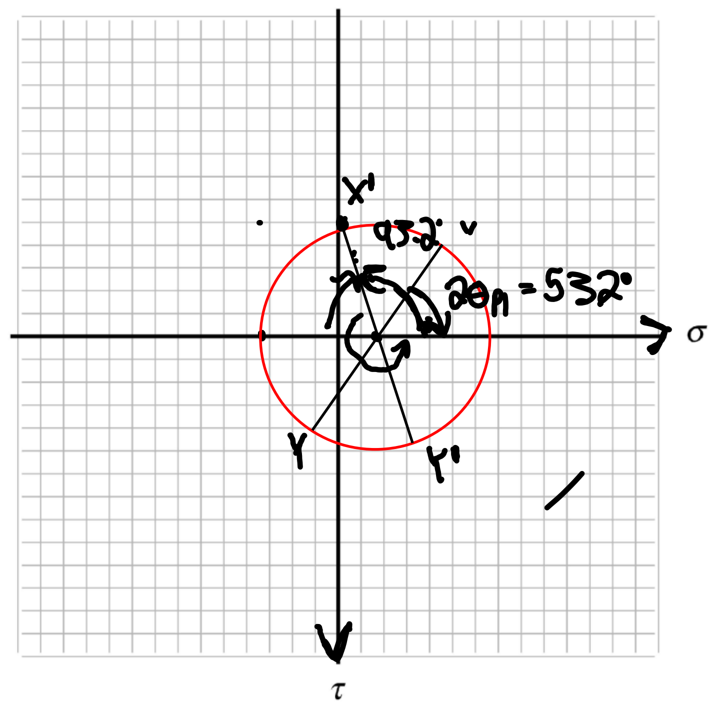
b) $\sigma_{avg} = \frac{4.8-1.2}{2} = 1.8 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$R = \sqrt{\left(\frac{4.8+1.2}{2}\right)^2 + 4^2}$

$R = 5$

$X = (4.8, -4)$



1 unit = 1 ksi

$2\theta_s = \tan^{-1}\left(\frac{4.8-1.8}{4}\right)$

$\theta_s = 18.4°$

$2\theta_{p1} = 2\theta_s + 270°$

$\Rightarrow \theta_{p1} = 153.4° \text{ CCW}$

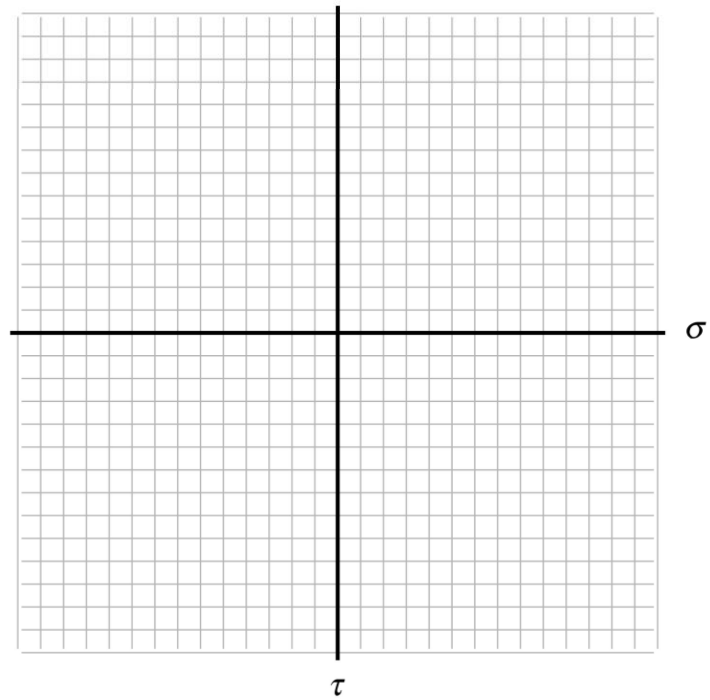
$\theta_{p1} = 26.6° \text{ CW}$

a) $\sigma_{x'} = \sigma_{avg} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta)$

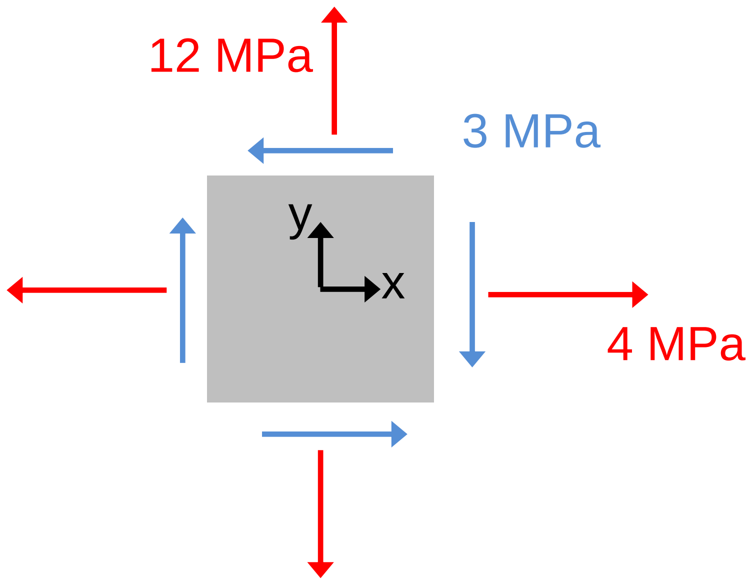
$\sigma_{x'} = 1.8 + 3\cos(40) - 4\sin(40) = 1.53 \text{ ksi}$

Example 13.7

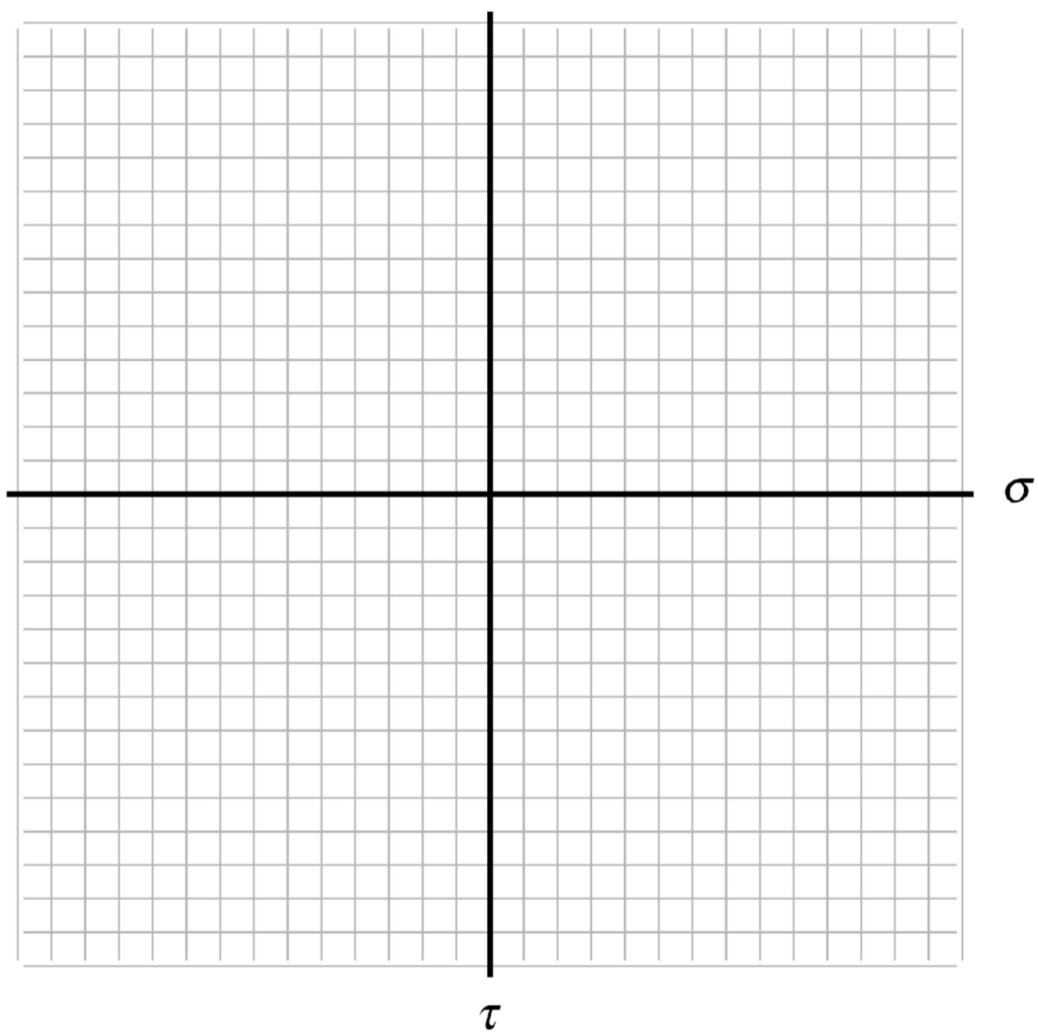
A state of plane stress is given by: $\sigma_x = 12\ ksi$, $\sigma_y = 12\ ksi$ and $\tau_{xy} = 4\ ksi$. Determine the principal stresses and the absolute maximum shear stress for this state of stress.



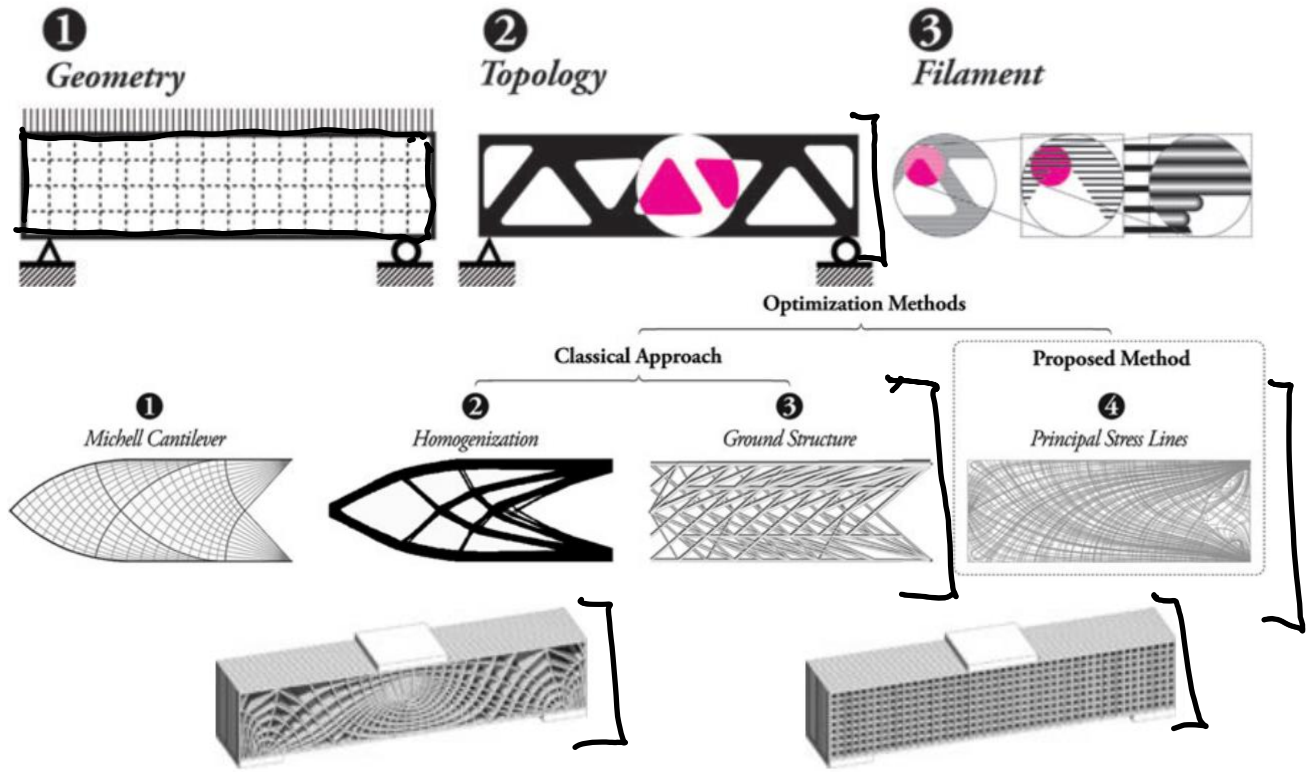
Lecture 33 Quiz: Practice with Mohr's Circles



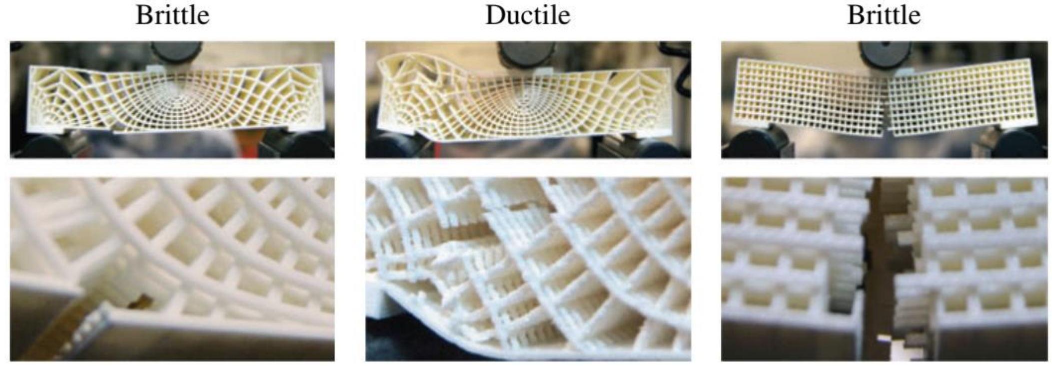
a) Draw the Mohr's circle.
b) Determine the principal stresses.
c) Determine the in-plane maximum shear stress.
d) Determine the absolute maximum shear stress.
e) Relative to the defined x-y axis, at what angle is the first principal stress?



Printing Along Principal Stress Lines

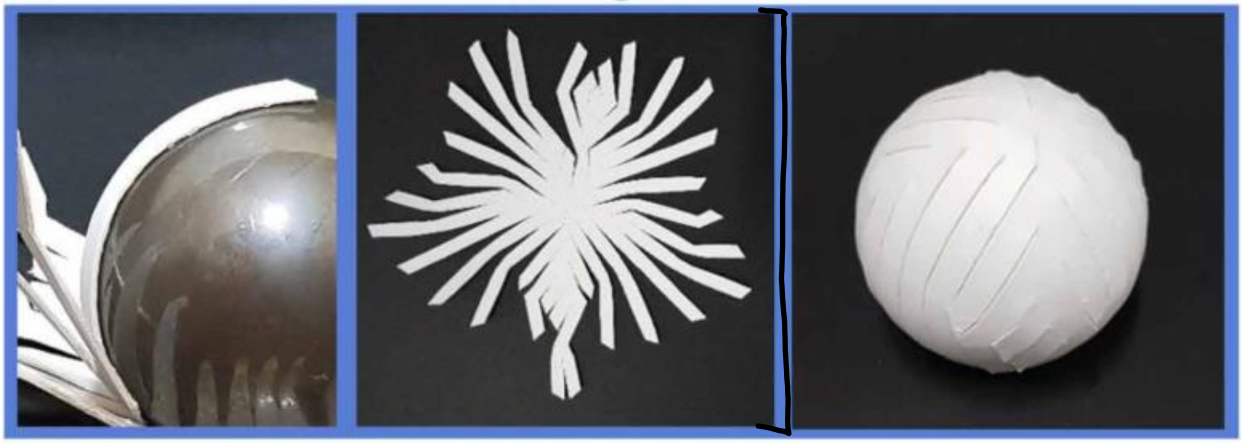


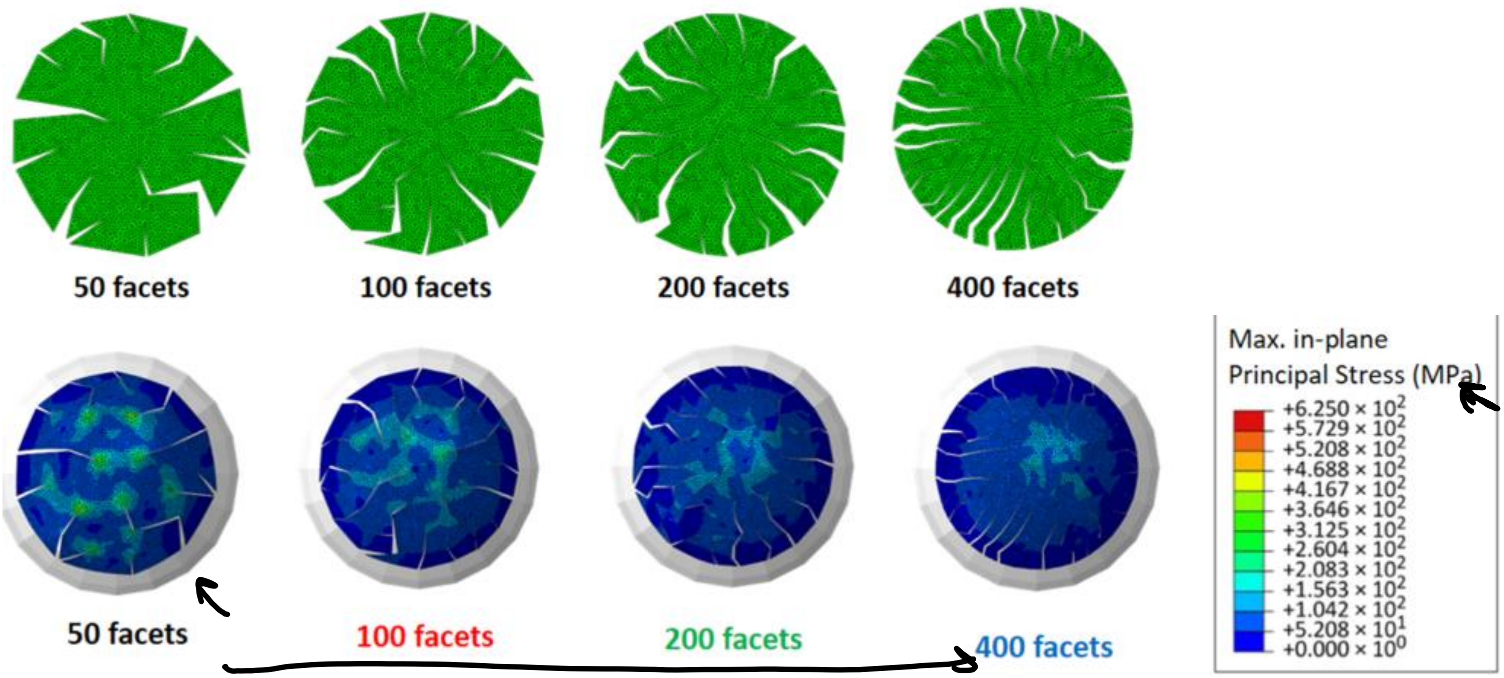
Case	*SLAM-XY*	*SLAM-XZ*	*Grid-XY*
Predicted normalized yield load and displacement	2495.1 N/N at 0.79 mm		2250.1 N/N at 5.12 mm
Average normalized yield load and displacement	2537.7 N/N at 2.26 mm	541.6 N/N at 0.64 mm	1783.0 N/N at 2.93 mm
Average normalized ultimate load and displacement	2847.8 N/N at 2.92 mm	810.9 N/N at 2.46 mm	2799.9 N/N at 7.54 mm
Average elastic stiffness normalized by specimen mass	1111.8 (N/N)/mm	836.4 (N/N)/mm	601.6 (N/N)/mm
Failure mode description	Simultaneous gross section failures (tension) at concentrated location	Delamination between layers (tension) along Y axis, progressive failure at multiple locations	Gross section (tension), multiple locations
Failure type	Brittle	Ductile	Brittle
Failure profiles Global geometry Member view			

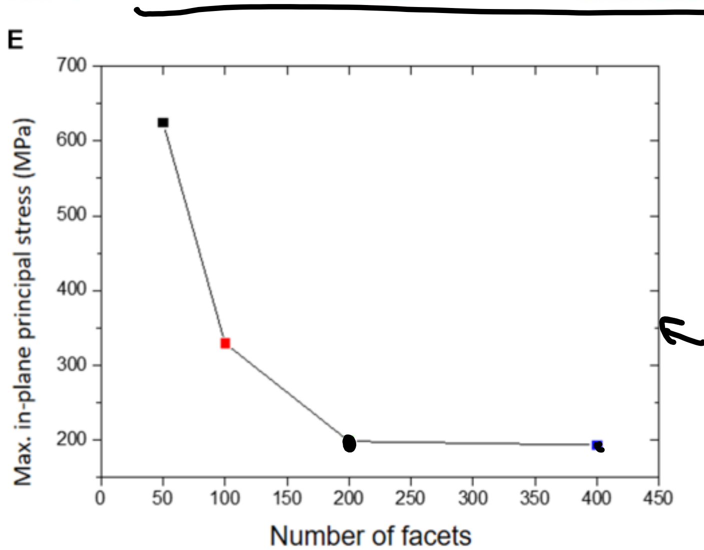


Tam and Mueller, 3D Printing and Additive Manufacturing, "Additive Manufacturing Along Principal Stress Lines", 2017.

Computational Wrapping







Lee et al, Sci. Adv., "Computational wrapping: A universal method to wrap 3D-curved surfaces with nonstretchable materials for conformal devices", 6:eaax6212, 2020.

Course Roadmap

Ch 13: Mohr's Circles
- Given the loading conditions at a point, what are the stress states at different angles?
- At what angle does the max normal stress and max shear stress occur?

Ch 14: Combined Loading
- What are the normal and shear stresses at points on a cross section due to combined axial, torsion, and bending loading?
- Determine the principal stresses and max shear stress at these points – use Mohr's circles.

Ch 15: Failure Analysis
- Given the stress states at a point, under what condition will a 3D structure fail?

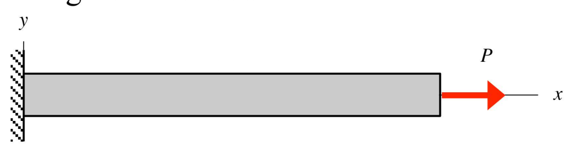
14. Stresses due to combined loadings

Objectives:

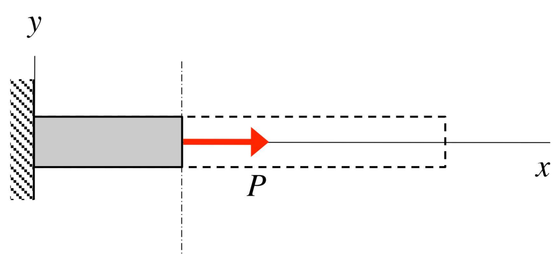
To study the combined effects of axial, torsion and bending loads on the principal and maximum shear components of stress at a point.

Background:
For each of the following three loading situations, consider the i) internal loading; ii) stress distribution; and , iii) the corresponding stress element.

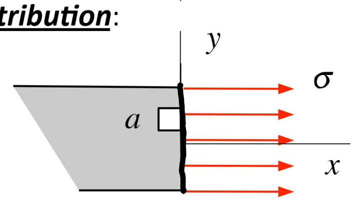
AXIAL LOADING



Internal loading:



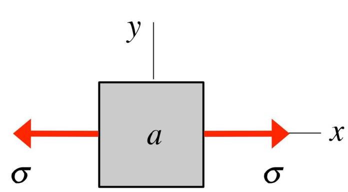
Stress distribution:



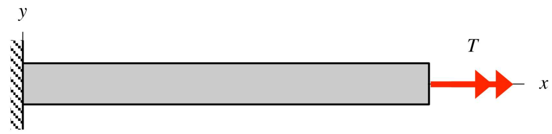
$$sign\ convention : \sigma\ \underline{positive}\ OUTWARD\ on\ face\ (tension)$$

$$\sigma = \frac{P}{A} = constant\ in\ y \quad ; \quad A = cross\text{-}sectional\ area$$

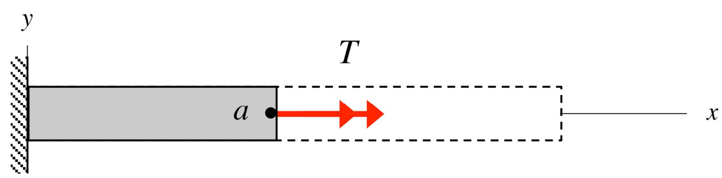
Stress element:



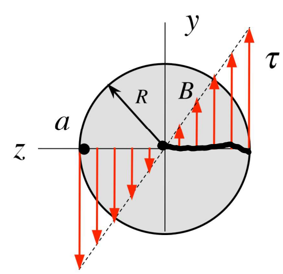
TORSIONAL LOADING



Internal loading:



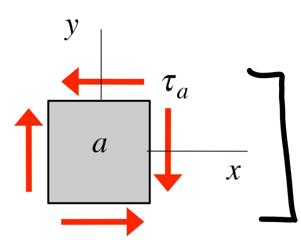
Stress distribution:



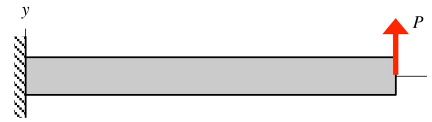
$sign\ convention: T\ \underline{positive}\ OUTWARD\ on\ face\ (by\ right-hand\ rule)$

$$\tau_a = \frac{TR}{I_P} = linear\ in\ radial\ position \quad ; \quad I_P = polar\ area\ moment$$

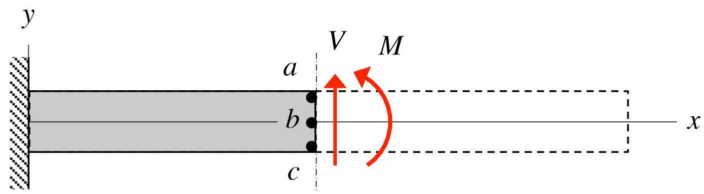
Stress element:



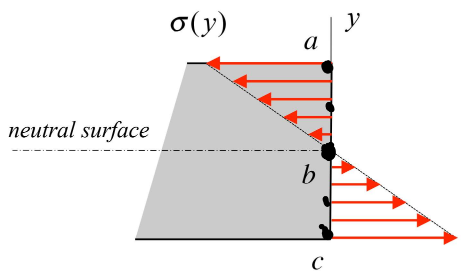
TRANSVERSE LOADING (e.g., rectangular cross section)



Internal loading:

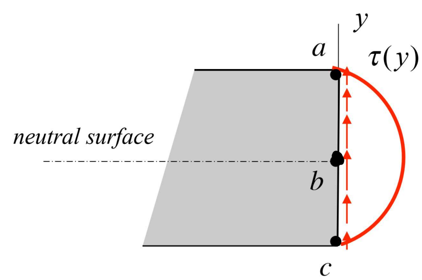


Normal stress distribution:



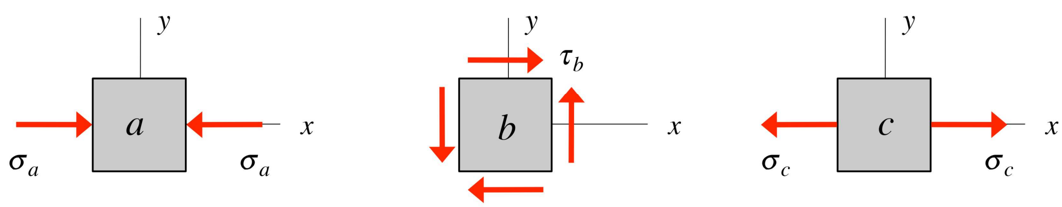
$$|\sigma_a| = \frac{M|y_a|}{I} \qquad \sigma_b = 0 \qquad |\sigma_c| = \frac{M|y_c|}{I}$$

Shear stress distribution:

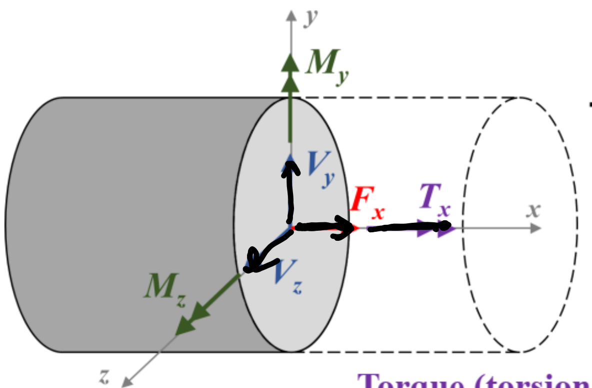


$$\tau_a = 0 \qquad |\tau_b| = \frac{3}{2}\frac{|V|}{A} \qquad \tau_c = 0$$

Stress element:



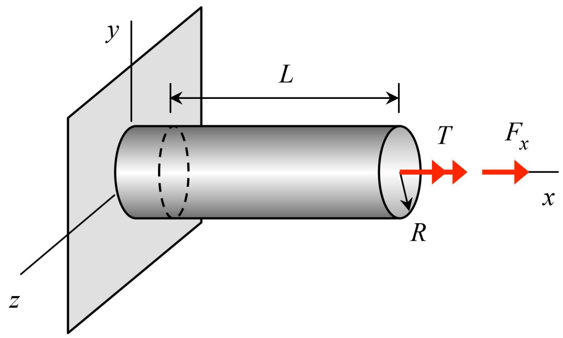
Combined Loading

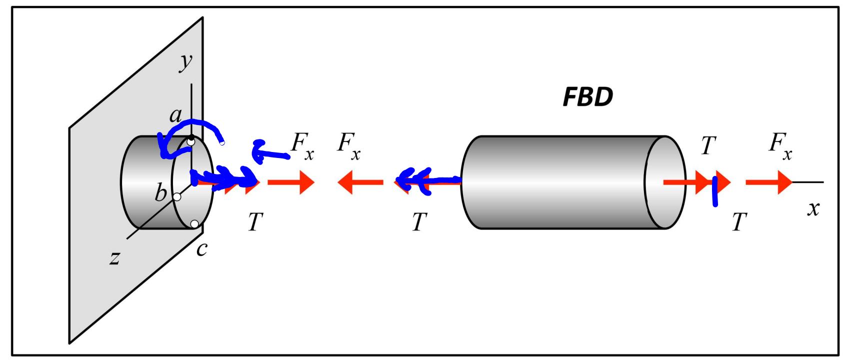


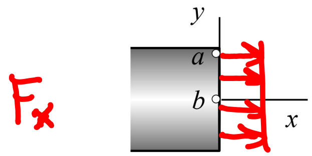
Load	Type of stress	Stress distribution	Lecture book ch.
Axial force F_x	**Normal**	$\sigma_x = F_x / A$	**Ch. 6**
Shear force V_y	**Shear**	$\tau_{xy} = \frac{V_y Q}{I_{zz} t}$	**Ch. 10**
Shear force V_z	**Shear**	$\tau_{xz} = \frac{V_z Q}{I_{yy} t}$	**Ch. 10**
Torque (torsional moment) T_x	**Shear**	$\tau = T\rho / I_p$	**Ch. 8**
Bending moment M_y	**Normal**	$\sigma_x = M_y z / I_{yy}$	**Ch. 10**
Bending moment M_z	**Normal**	$\sigma_x = -M_z y / I_{zz}$	**Ch. 10**

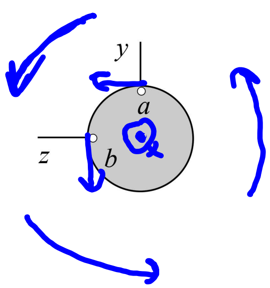
For combined loads, use superposition (possible because of the assumption of linearity).

Example I



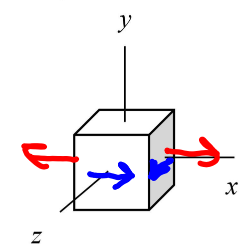




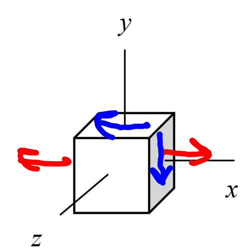


loading	stress comp. @ "a"	stress comp. @ "b"
F_x	$\sigma_x = \frac{F_x}{A}$	$\sigma_x = \frac{F_x}{A}$
T_x	$\tau_{xz} = \frac{TR}{I_p}$	$\tau_{xy} = -\frac{TR}{I_p}$

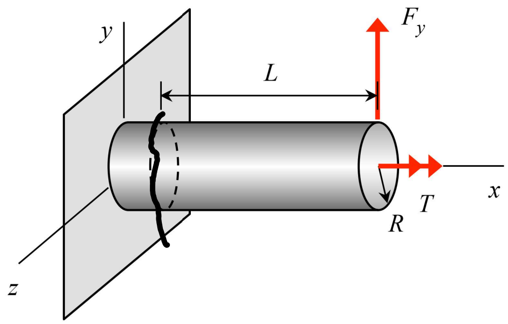
point "a"

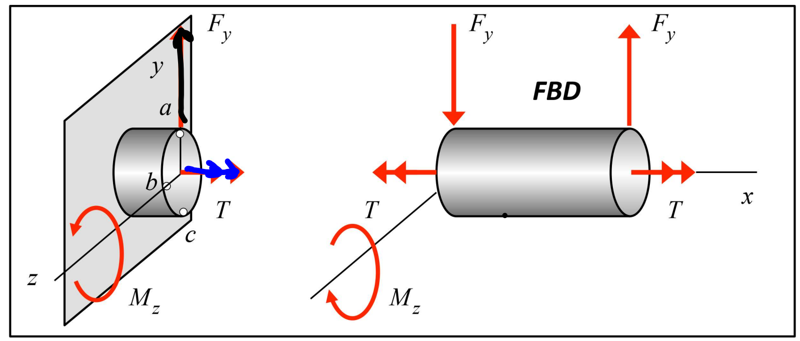


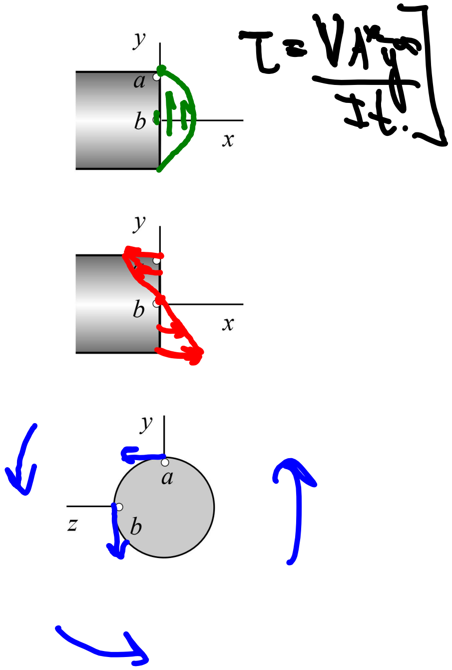
point "b"



Example II

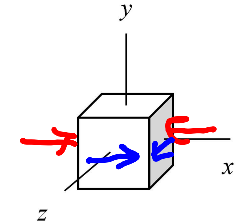






loading	*stress comp. @ "a"*	*stress comp. @ "b"*
V_y	0	$\tau_{xy} = \frac{4}{3}\frac{V}{A}$
M_z	$\sigma_x = -\frac{\lvert M_z \rvert R}{I}$	0
T_x	$\tau_{xz} = \frac{TR}{I_p}$	$\tau_{xy} = \frac{-TR}{I_p}$

point "a"



point "b"