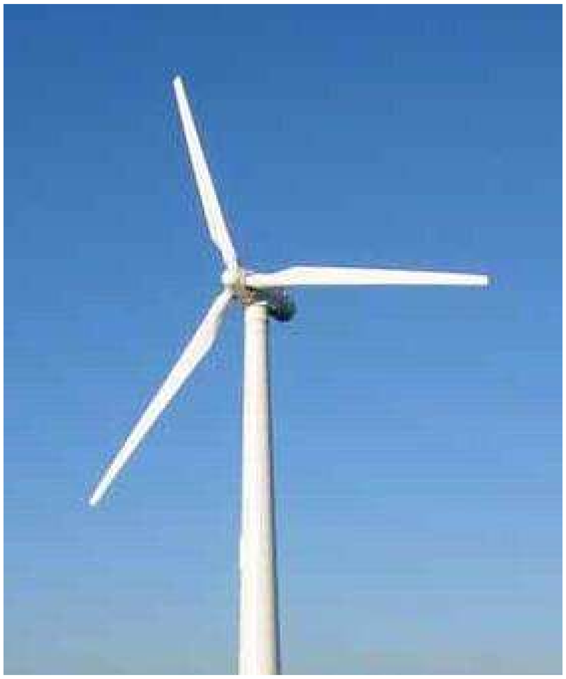
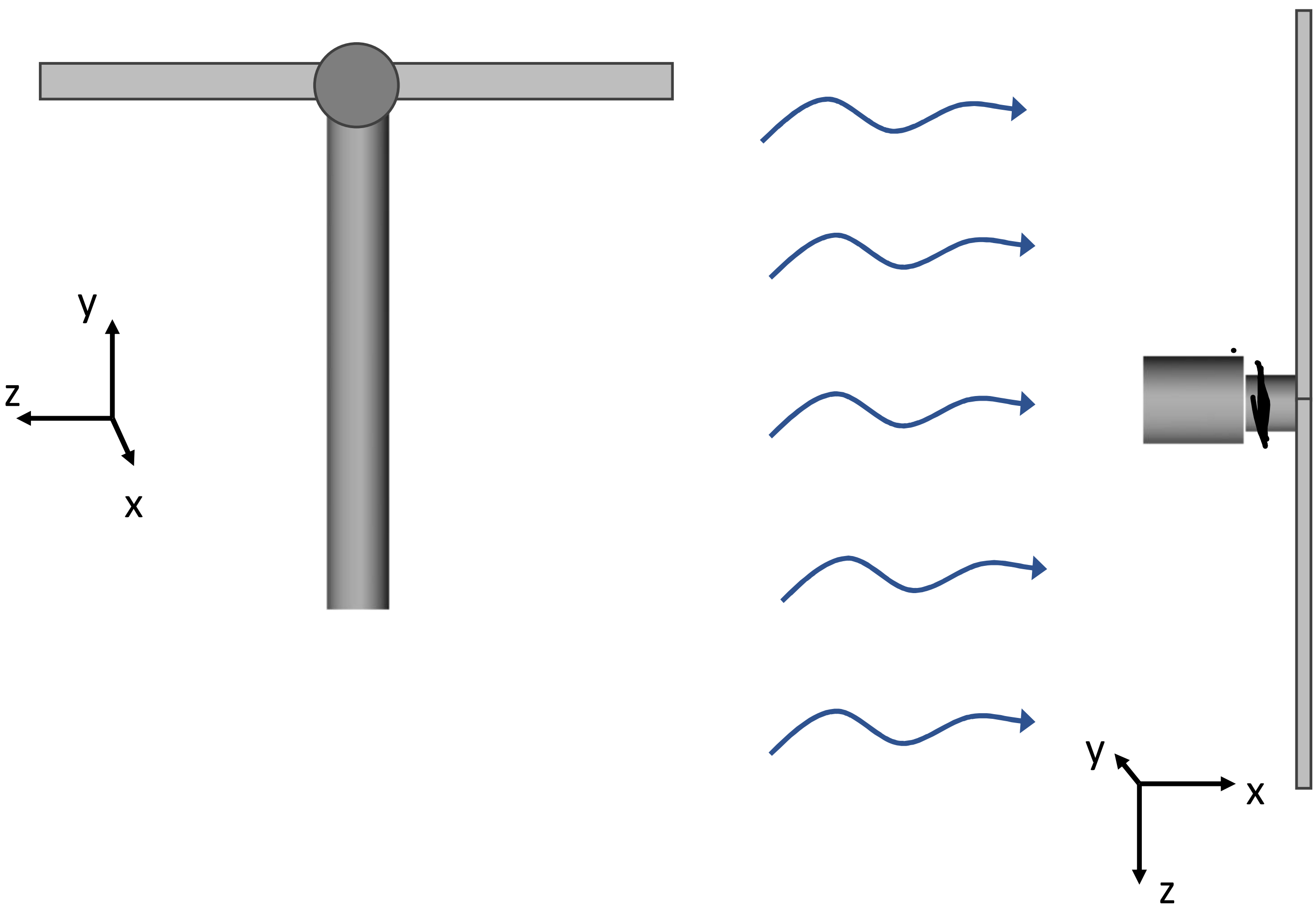
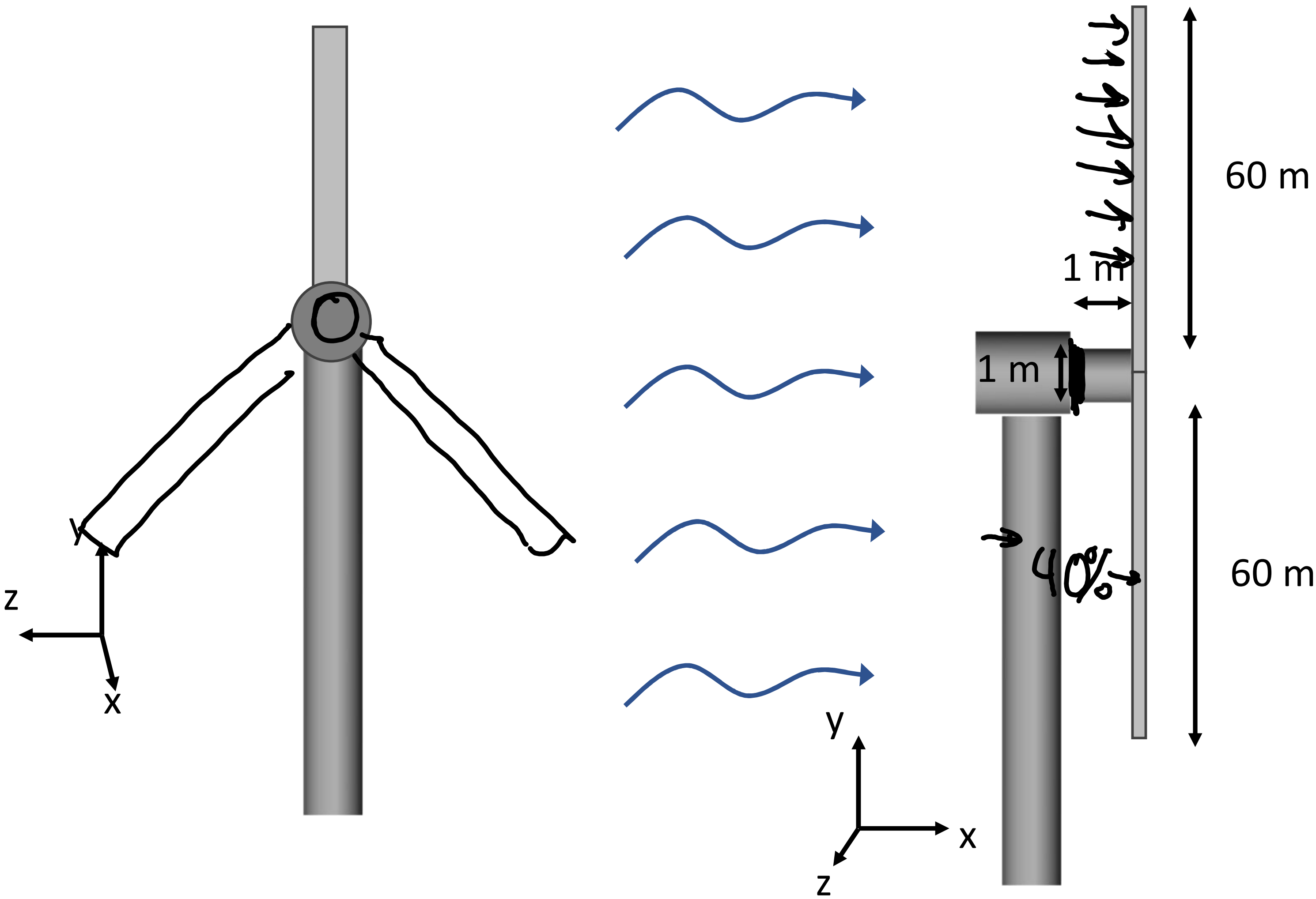
Combined Loading in Windmills



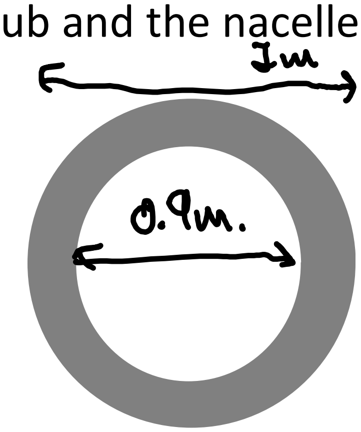
Why does a windmill have 3 blades?

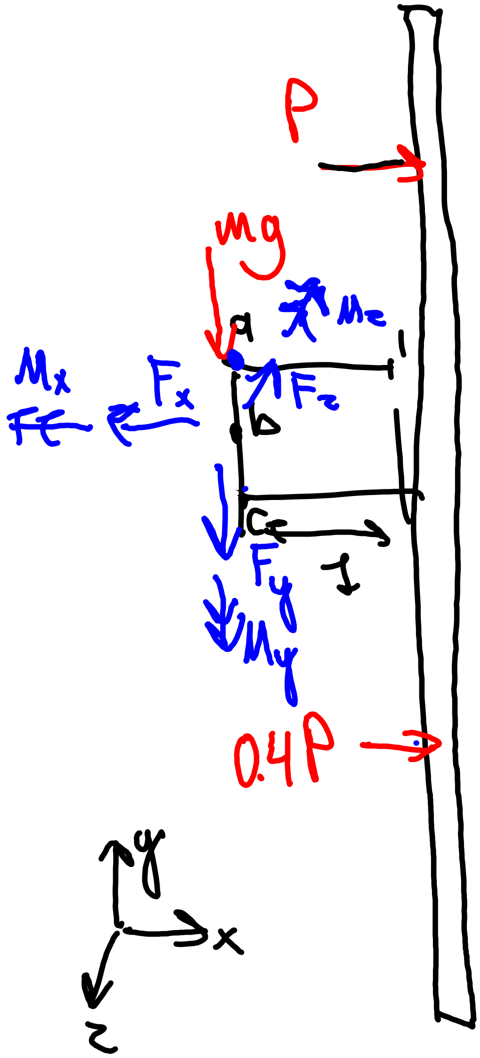


Combined Loading in Windmills



The rotor assembly weighs ~8 000 kg.
A distributed force of 300 N/m acts over the blades, which are each 60 m long.
Find the stresses at the interface between the hub and the nacelle.





$$\Sigma F = -F_x\hat{i} - F_y\hat{j} - F_z\hat{k} + P\hat{i} + 0.4P\hat{i} - mg\hat{j} = 0$$

$$F_x = 1.4P$$

$$F_y = -mg$$

$$F_z = 0$$

$$\Sigma M = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = 0$$

$$\vec{r}_1 = (1, 30, 0) \qquad \vec{F}_1 = (P, 0, 0)$$

$$\vec{r}_2 = (I, -30, 0) \qquad \vec{F}_2 = (0.4P, 0, 0).$$

$$\vec{r}_1 \times \vec{F}_1 = (0, 0, -30P)$$

$$\vec{r}_2 \times \vec{F}_2 = (0, 0, 12P)$$

$$0 = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} + 12P\hat{k} - 30P\hat{k}$$

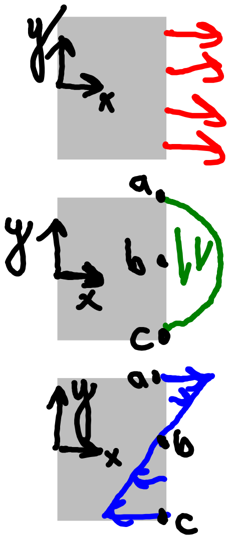
$$M_x = 0 \qquad M_y = 0$$

$$M_z = -18P$$

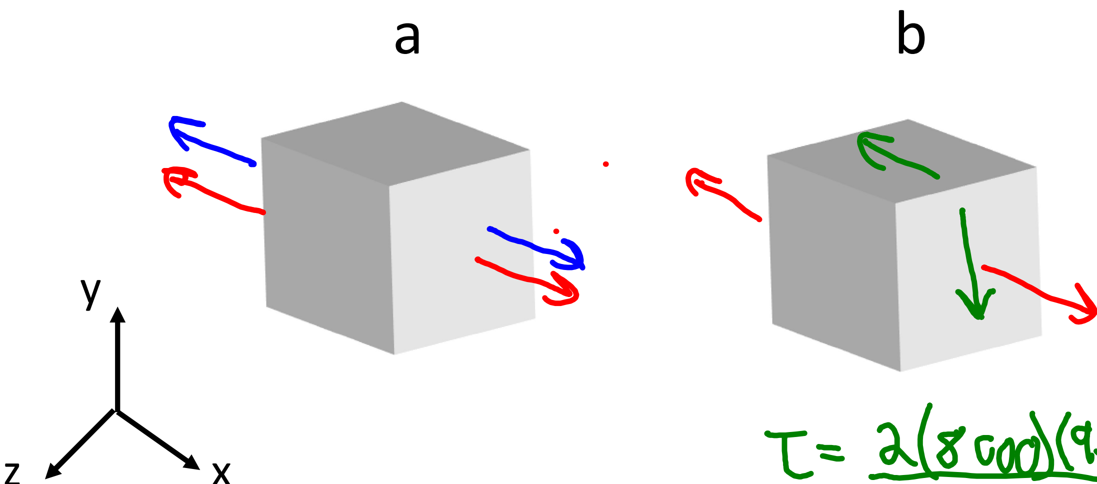
$$P = (300\ N/m)(60\ m) = 18\,000\ N$$

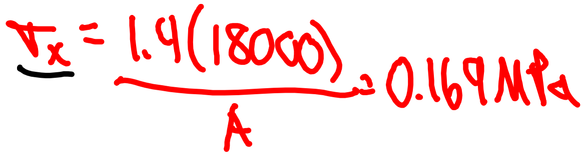
$$M_z = -18(18\,000) = -324\,000\ N\cdot m$$

Windmills



Force	a	b	c
F_x	$\sigma_x = \frac{F_x}{A}$	$\sigma_x = \frac{F_x}{A}$	$\sigma_x = \frac{F_x}{A}$
F_y	0	$\tau_{xy} = \frac{2mg}{A}$	0
M_z	$\sigma_x = \frac{My}{I}$	0	$\sigma_x = -\frac{My}{I}$





$$\sigma_x = \frac{1.4(18000)}{A} = 0.169\,MPa$$

$$\tau = \frac{2(8000)(9.81)}{0.15\,m^2} = 1.045\,MPa.$$

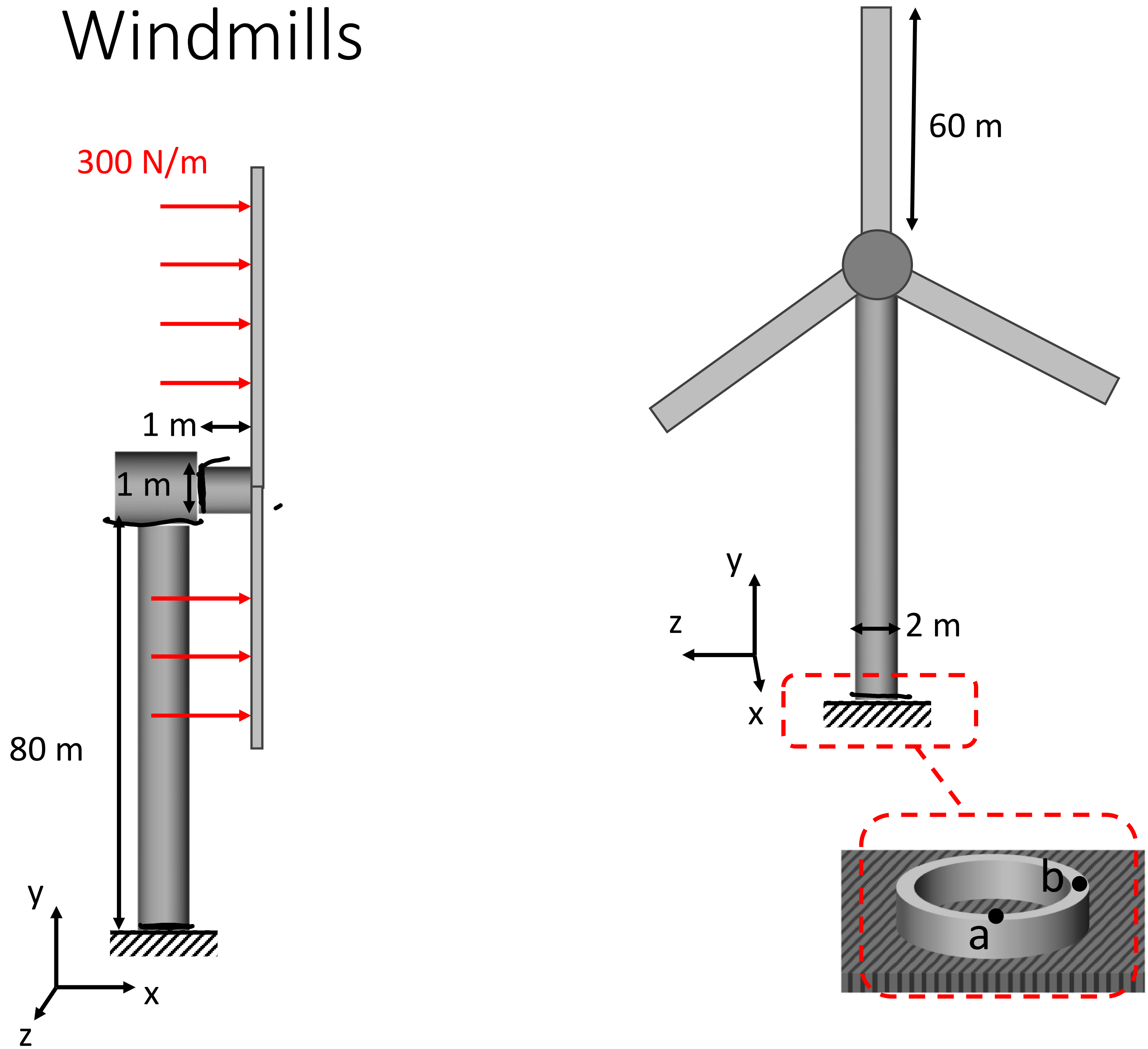
$$A = \pi(r_o^2 - r_i^2) = 0.15\,m^2$$

$\frac{My}{I} = 9.586$ MPa.

a: 9.75 MPa.

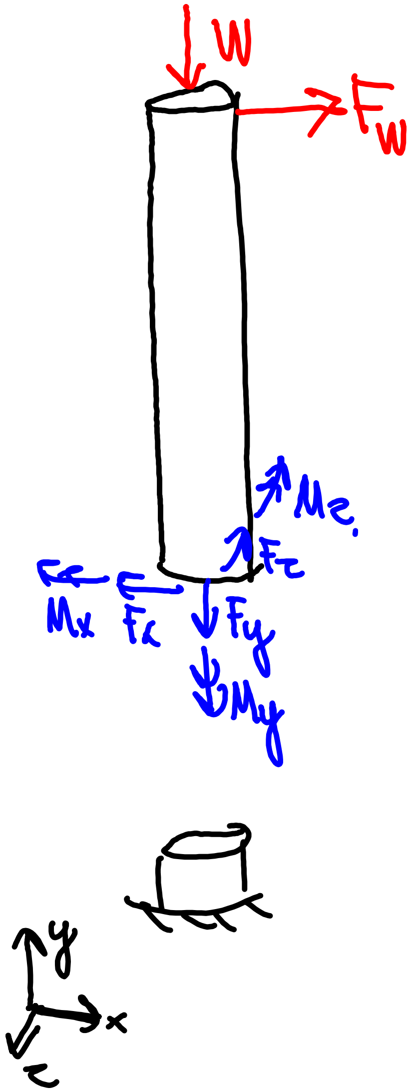
c: -9.4 MPa.

Windmills



The wind applies a distributed load of 300 N/m on each of three blades that are 60 m long. The rotor assembly and nacelle together weigh ~74 000 kg. The shaft of the windmill is 2 m in diameter at the base and is made of steel that has a thickness of 0.1 m. The weight of the shaft can be neglected compared to the weight of the rotor assembly and nacelle.

Find the state of stress at point **a** and point **b** at the bottom of the shaft. Draw the 3D stress element for the loading conditions.



$F_W = 3(300\ N/m)(60\ m) = 54\ 000\ N$

$W = (74\ 000\ kg)(10) = 740\ 000\ N.$

$\Sigma F = -F_x\hat{i} - F_y\hat{j} - F_z\hat{k} + F_W\hat{i} - W\hat{j} = 0$

$F_x = F_W$

$F_y = -W$

$F_z = 0$

$\Sigma M = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} + \vec{r} \times \vec{F} = 0$

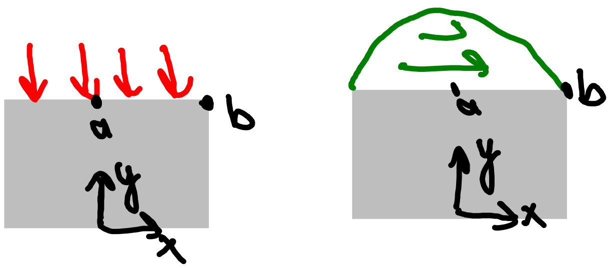
$\vec{r} = (0, 80, 0)$ $\quad \vec{F} = (F_W, -W, 0)$

$\vec{r} \times \vec{F} = (0, 0, -80P)$

$0 = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} - 80P\hat{k}$

$M_z = -80P$

Windmills



Force	a	b
F_x	$\tau_{xy} = \frac{2V}{A}$	0
F_y	$\sigma_y = -\frac{F_y}{A}$	$\sigma_y = -\frac{F_y}{A}$
M_z