

## Exam 2

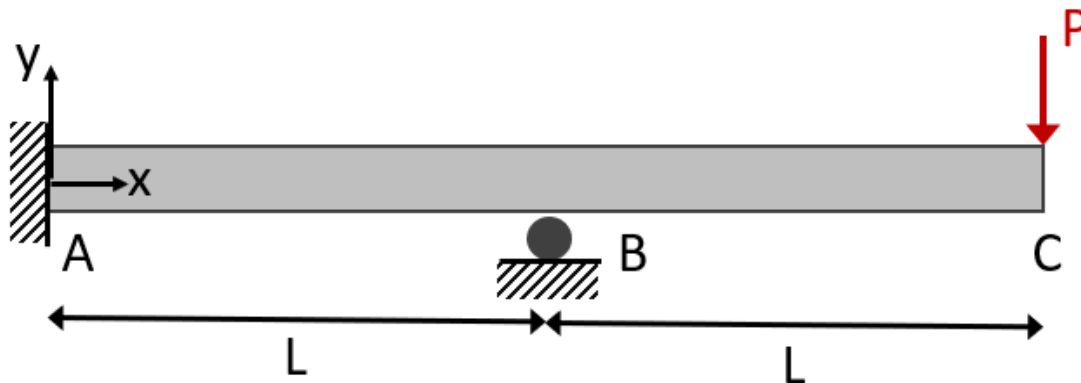
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**PROBLEM #1 (25 points)**

Beam ABC is fixed at end A and is supported by a roller support at B. A concentrated force  $P$  acts at C.  $E$  and  $I$  are constant along the beam. Use the **second order integration method** to calculate the following:

- Draw a free body diagram and write the equilibrium equations.
- Find the reactions on the beam at A and B in terms of  $P$ .
- Find the equation for the vertical displacement,  $v(x)$  using the  $x$ -direction shown in the figure, throughout the beam in terms of  $P$ ,  $L$ ,  $E$ , and  $I$ .
- Find the slope ( $\theta$ ) at point B in terms of  $P$ ,  $L$ ,  $E$ , and  $I$ .



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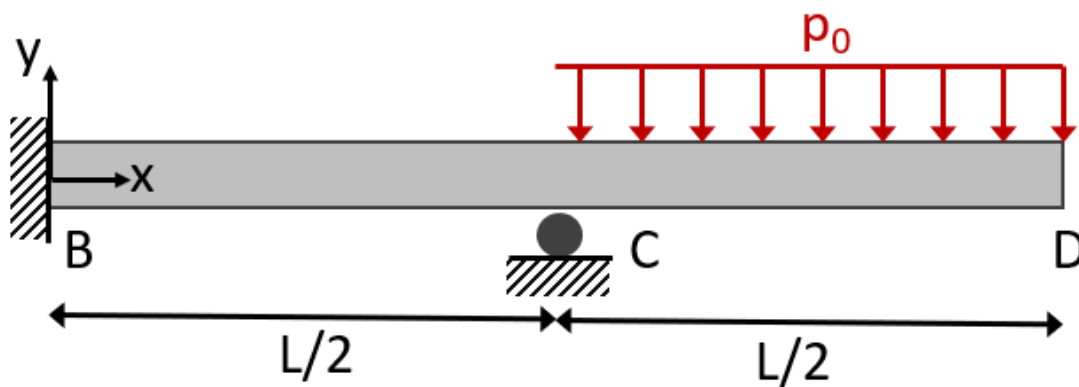
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**PROBLEM #2 (25 points)**

A cantilever beam BCD has a distributed load  $p_0$  acting between C and D.

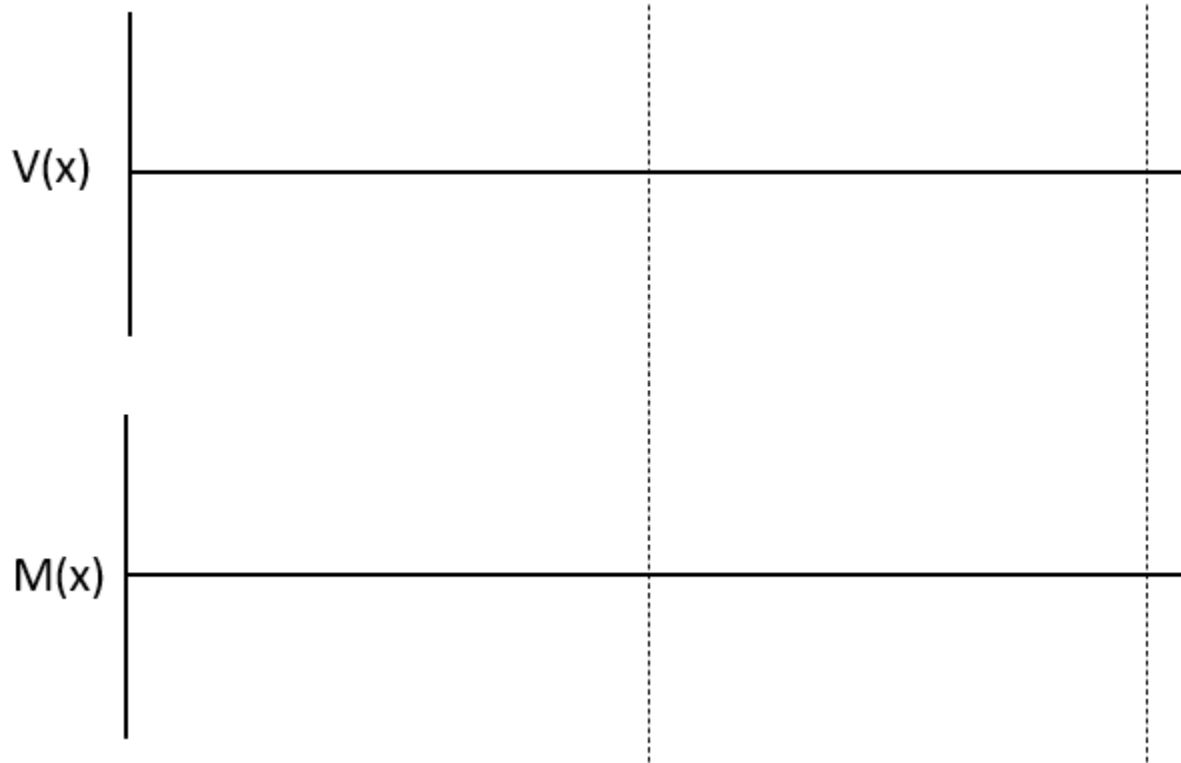
- Draw a free body diagram and write the equilibrium equations.
- Use the superposition principle** and the superposition tables provided to calculate the values of the reactions at B and C. Leave your answers in terms of  $p_0$  and  $L$ .
- Draw the internal moment  $M(x)$  and shear force  $V(x)$  along the beam on the axes on the next page. Label the values of  $M(x)$  and  $V(x)$  at points B, C, and D.



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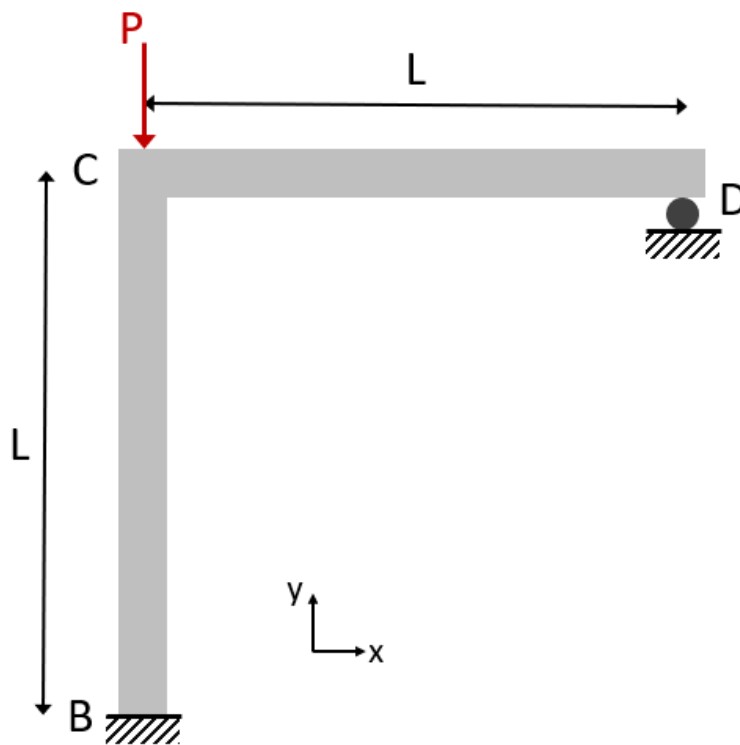
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**PROBLEM #3 (25 points)**

An L-shaped beam is rigidly fixed at B and supported by a roller at D. A point load is applied at C in the y-direction. You can assume that the value of P is small enough that BC does not buckle. You can neglect shear energy in your analysis.

- Make a free body diagram and write the equilibrium equations.
- Use **Castigliano's Second Theorem** to determine the reactions at B and D.
- Use **Castigliano's Second Theorem** to determine the displacement of point C in the y-direction.



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**PROBLEM #4 – PART A (6 points)**

Beam (i) and (ii) are identical cylindrical beams except that beam (i) is made of steel and beam (ii) is made of aluminum.  $E_{\text{steel}} > E_{\text{aluminum}}$ .



Beam (a) Steel



Beam (b) Aluminum

(a) Circle the correct relationship between the maximum shear stresses in the two beams (**1 point**).

$$|\tau_{max,a}| < |\tau_{max,b}|$$

$$|\tau_{max,a}| = |\tau_{max,b}| = 0$$

$$|\tau_{max,a}| = |\tau_{max,b}| \neq 0$$

$$|\tau_{max,a}| > |\tau_{max,b}|$$

(b) Circle the correct relationship between the maximum shear stresses in the two beams (**1 point**).

$$|\sigma_{max,a}| < |\sigma_{max,b}|$$

$$|\sigma_{max,a}| = |\sigma_{max,b}| = 0$$

$$|\sigma_{max,a}| = |\sigma_{max,b}| \neq 0$$

$$|\sigma_{max,a}| > |\sigma_{max,b}|$$

(c) Circle the correct relationship between the maximum deflection  $v(x)$  in the two beams (**1 point**).

$$|v_{max,a}| < |v_{max,b}|$$

$$|v_{max,a}| = |v_{max,b}| = 0$$

$$|v_{max,a}| = |v_{max,b}| \neq 0$$

$$|v_{max,a}| > |v_{max,b}|$$

(d) The diameter of the original beams is  $D$ . If the diameter is doubled to  $2D$ , how will the new deflection of the new beam ( $v_{max}^*$ ) with diameter of  $2D$  compare to the deflection of the original beam ( $v_{max}$ ) with diameter of  $D$  (**3 points**):

$$v_{max}^* = v_{max}$$

$$v_{max}^* = 2v_{max}$$

$$v_{max}^* = 4v_{max}$$

$$v_{max}^* = 8v_{max}$$

$$v_{max}^* = 16v_{max}$$

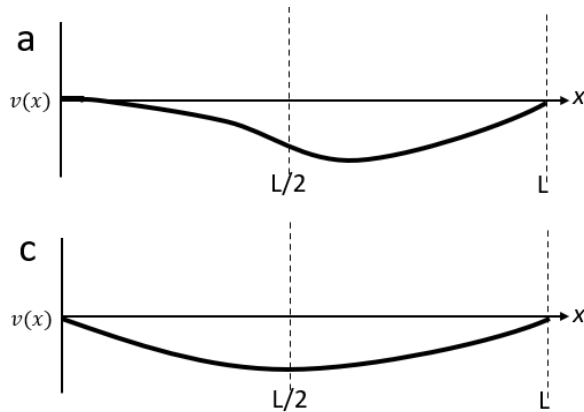
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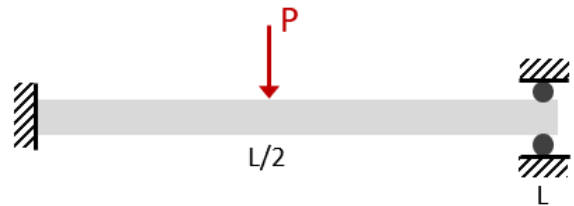
**PROBLEM 4 – PART B (6 points)**

Figures a-d indicate the deflection curve along four different beams.



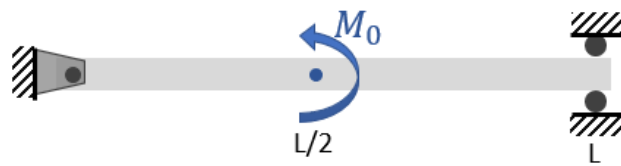
(i) Circle the deflection curve that corresponds to the given beam and loading conditions (2 points):

a                  b                  c                  d



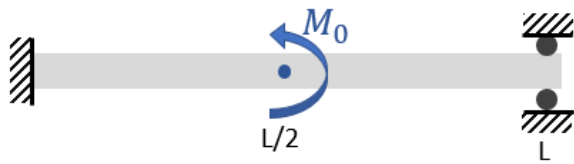
(ii) Circle the deflection curve that corresponds to the given beam and loading conditions (2 points):

a                  b                  c                  d



(iii) Circle the deflection curve that corresponds to the given beam and loading conditions (2 points):

a                  b                  c                  d



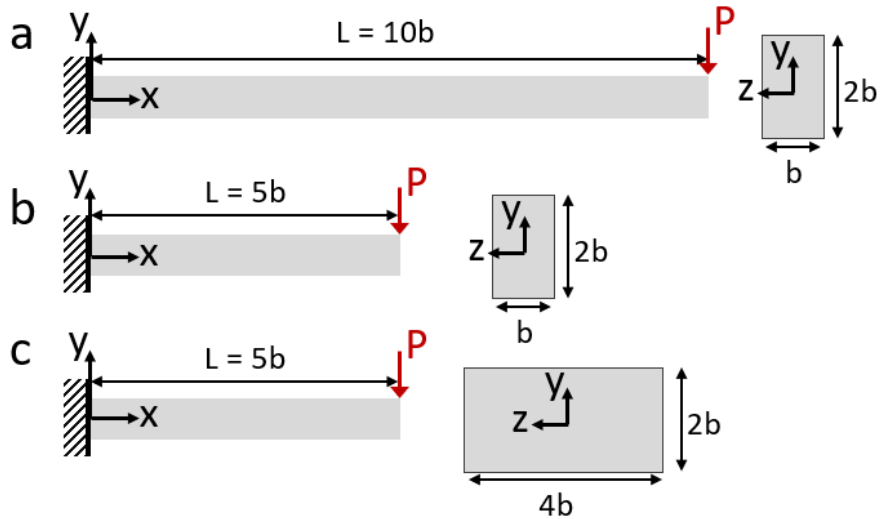
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**PROBLEM 4 – PART C (2 points)**

Based on the assumptions used when deriving the equation for shear stress on a beam cross-section ( $\tau = VQ/It$ ), choose the correct ranking for the accuracy of the shear stress predicted by this equation for the three beams shown below:



	Option 1	Option 2	Option 3	Option 4	Option 5
Most accurate	a	a	b	c	All have the same accuracy
	b	c	a	b	
Least accurate	c	b	c	a	

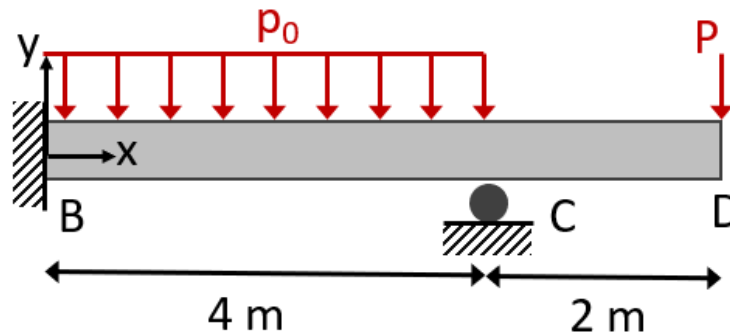
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**PROBLEM 4 – PART D (6 points)**

A beam is loaded with a distributed load from 0 to 4 m and a point load at 6 m.

Circle the value(s) that will be zero at  $x = 0\text{m}$  (2 points):

$V(0)$        $M(0)$        $\theta(0)$        $v(0)$

Circle the value(s) that will be zero at  $x = 4\text{m}$  (2 points):

$V(4)$        $M(4)$        $\theta(4)$        $v(4)$

Circle the value(s) that will be zero at  $x = 6\text{m}$  (2 points):

$V(6)$        $M(6)$        $\theta(6)$        $v(6)$

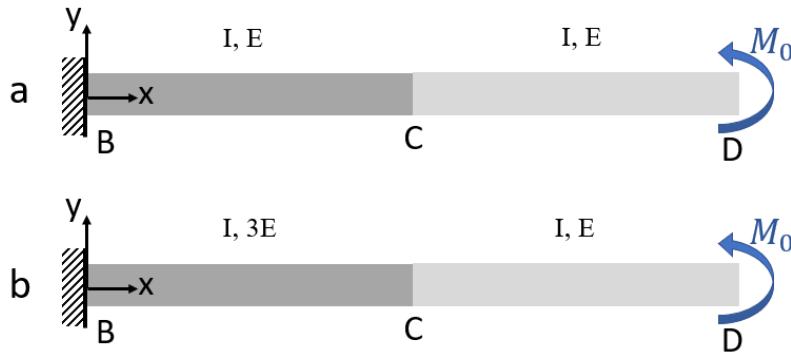
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**PROBLEM 4 – PART E (5 points)**

A simple cantilever is composed of two sections with an applied moment at the end.



(i) (3 points) In beam (a), the two sections both have the same Young's moduli of E. In beam (b), one of the sections has a Young's modulus of 3E, while one has a Young's modulus of E. How does the total strain energy of these two beams compare?:

$U_{total,a} > U_{total,b}$

$U_{total,a} = U_{total,b}$

$U_{total,a} < U_{total,b}$

(ii) (2 points) Circle the loading condition below (c to f) that would be used if we want to calculate the deflection at point C in the y-direction.

