Review for Midterm Exam #2

Zhao Section

2024.4.1

- Students must exhibit highest standard of honor. Any misconduct of academic integrity will be addressed.
- The exam is closed-book and closed-notes. There will be three full-length problems and one multiple-choice problem with multiple parts.
- Equation Sheet is posted on course blog and will be handed out in the exam.
- Calculator: please bring the allowed type of calculator as described in syllabus: TI-30X and TI-36X models, fx-115 and fx-991 models.
- **Exam Date & Time: April 3, 2024. Time: 8:00 – 10:00 PM**
- Exam will cover Lectures 17-28 and HW6-9.
- **Exam Room: PHYS114.**
- Please arrive to exam room at least 15 minutes prior to the start of exam.
- Exam Submission Window (30 Minutes): When you complete your exam, you may use your phone to scan your solution and upload to Gradescope. Specifically, your solutions will be scanned and submitted to Gradescope session "ME 323 - Spring 2024 - Exams". You are responsible for scanning your exam into a single PDF and uploading your exam into Gradescope immediately after completion of your exam. To accommodate the time needed to do this, the deadline to have your exam scanned and uploaded to Gradescope will be 10:30PM (EST), giving 30 minutes to complete this process. The time limit will be strictly enforced.
- Assigning Pages for Your Exam: As part of the submission process, you will need to identify the page numbers for Problem 1, 2, ... separately. If you need extra papers, please use your own but make sure to arrange the pages in the correct order in your submission. Do not submit the equation sheet.

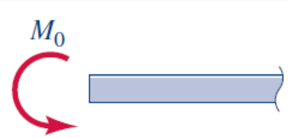
Coverage:

17 F	16-Feb	Beams deflections– statically determinate structures	Chap. 11	HW 5
18 M	19-Feb	Beam deflections - indeterminate structures	Chap. 11	
19 W	21-Feb	Beam deflections – superposition methods	Chap. 11	
20 F	23-Feb	Energy methods – Castigliano's theorems	Chap. 16	HW. 6
21 M	26-Feb	Review		
W	28-Feb	***Examination 1, 8-10pm: no lecture on Wednesday***		
22 F	1-Mar	Energy methods – Castigliano's theorems	Chap. 16	
23 M	4-Mar	Energy methods – Castigliano's theorems	Chap. 16	
24 W	6-Mar	Energy methods – Castigliano's theorems	Chap. 16	
25 F	8-Mar	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	HW 7
		Spring Break, March 11-15: no class		
26 M	18-Mar	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
27 W	20-Mar	Energy methods – introduction to finite element methods	Chap. 17	
28 F	22-Mar	Energy methods – introduction to finite element methods	Chap. 17	HW 8

Deflection of beams

2nd order integration method: $EIv'' = M(x)$

TABLE 7.1 Boundary Conditions

	Type	Symbol*	2nd Order
BC	Fixed end		$v = 0$ $v' = 0$
	Simple support		$v = 0$
	Free end		No BC
	Concentrated force		No BC
	Concentrated couple		No BC

*These boundary conditions also apply if the boundary under c
the other end of the beam (i.e., $x = L$).

Energy method

Axial energy $$U = \frac{1}{2}\int_0^L \frac{F^2 dx}{EA}$$

Torsional energy $$U = \frac{1}{2}\int_0^L \frac{T^2 dx}{GI_p}$$

Flexural energy $$U_\sigma = \frac{1}{2}\int_0^L \frac{M^2 dx}{EI}$$

Shear energy $$U_\tau = \frac{1}{2}\int_0^L \frac{f_s V^2 dx}{GA}$$

Castigliano's 2nd theorem

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \qquad \theta_{M_i} = \frac{\partial U}{\partial M_i} \qquad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

- Dummy load
- Indeterminate structures

Finite element method

- <u>*Defining the nodes and elements for the problem*</u>. Choose a set of N+1 *nodes* along the length of the rod at locations $x_1(=0), x_2, x_3, \ldots, x_N, x_{N+1}(=L)$. The subdomain of $x_i < x < x_{i+1}$ is known as the ith *element* of length $L_i = x_{i+1} - x_i$, for $i = 1, 2, \ldots, N$. The value of $k_i = (EA)_i / L_i$ is determined through the average value of EA over the ith element and the element length L_i.
- <u>*Constructing the global stiffness matrix*</u>. Construct the stiffness matrix [K]. The resulting matrix will be tri-diagonal and of size $(N+1)\times(N+1)$.
- <u>*Constructing the force vector*</u>
  Construct the force vector $\{F\}$ as being made up on the resultant *external* force acting on each node. The resulting vector will be of length *N+1*.
- <u>*Enforcing fixed-displacement boundary conditions*</u>. The fixed-displacement boundary conditions are enforced through the elimination of appropriate terms in the resulting stiffness matrix $[K]$ and forcing vector $\{F\}$. For example, if the ith node has a fixed (zero) displacement, we eliminate the ith row and ith column of $[K]$ and the ith row of $\{F\}$. If the problem has "n" fixed nodal displacements, then the stiffness matrix and force vector will be of sizes $(N-n+1)\times(N-n+1)$ and $N-n+1$, respectively[1].
- <u>*Solving*</u>. The nodal displacements u_k ; $k = 1, 2, \ldots, N-n+1$ are found from the solution of the algebraic equilibrium equations:
  $$[K]\{u\} = \{F\}$$
  through a linear equation solver in an application such as Matlab or Mathematica.
- <u>*Stress calculations*</u>
  The average stress across the ith element is found from:
  $$\sigma_i = \frac{E_i}{L_i}\left(u_{i+1} - u_i\right)$$