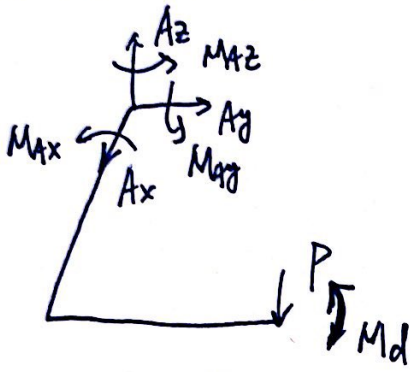
Quiz 8 solution:

To determine the rotation angle at C, a dummy moment M_d is applied at C

FBD:



$A_x = A_y = 0$

$A_z = P$

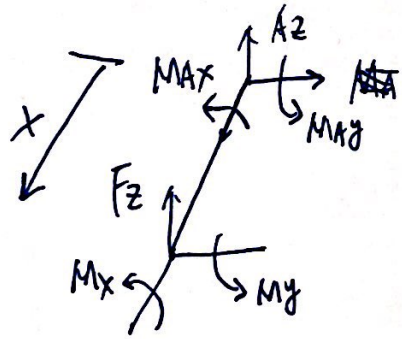
$$\Sigma M_A = M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k} + M_d\hat{i} + (L\hat{i} + L\hat{j}) \times (-P\hat{k})$$

$$= M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k} + M_d\hat{i} + PL\hat{j} - PL\hat{i}$$

$$= 0$$

$M_{Ax} = PL - M_d$, $M_{Ay} = -PL$

For AB section:



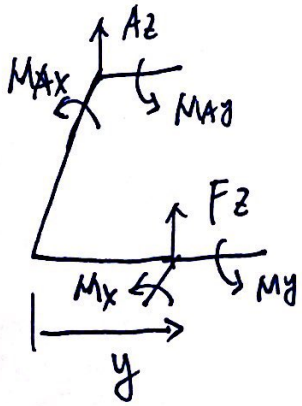
$F_z = -A_z = -P$

$$\Sigma M_x = (M_x + M_{Ax})\hat{i} + (M_y + M_{Ay})\hat{j} + [-x\hat{i}] \times P\hat{k}$$

$$= (M_x + M_{Ax})\hat{i} + (M_y + M_{Ay})\hat{j} + Px\hat{j} = 0$$

$M_x = -M_{Ax} = M_d - PL$, $M_y = -P(x-L)$

For BC:



$F_z = -A_z = -P$

$$\Sigma M_y = (M_x + M_{Ax})\hat{i} + (M_{Ay} + M_y)\hat{j} + (-L\hat{i} - y\hat{j}) \times P\hat{k}$$

$$= (M_x + M_{Ax})\hat{i} + (M_y + M_{Ay})\hat{j} + PL\hat{j} - Py\hat{i}$$

$$= 0$$

$$M_x = Py - M_{Ax} = P(y-L) + M_d, \quad M_y = 0$$

$$U_{total} = U_{AB} + U_{BC} = \frac{1}{2}\int_0^L \frac{(M_d - PL)^2}{GI_p}\,dx + \frac{1}{2}\int_0^L \frac{P^2(x-L)^2}{EI}\,dx$$

$$+ \frac{1}{2}\int_0^L \frac{[P(y-L)+M_d]^2}{EI}\,dy$$

$$\theta_C = \left.\frac{\partial U_{total}}{\partial M_d}\right|_{M_d=0} = \left[\int_0^L \frac{(M_d - PL)}{GI_p}\,dx + \int_0^L \frac{P(y-L)+M_d}{EI}\,dy\right]\Bigg|_{M_d=0}$$

$$= -\frac{PL^2}{GI_p} + \left.\frac{\frac{1}{2}P(y-L)^2}{EI}\right|_0^L$$

$$= -\frac{PL^2}{GI_p} + \frac{\frac{1}{2}PL^2}{EI}$$