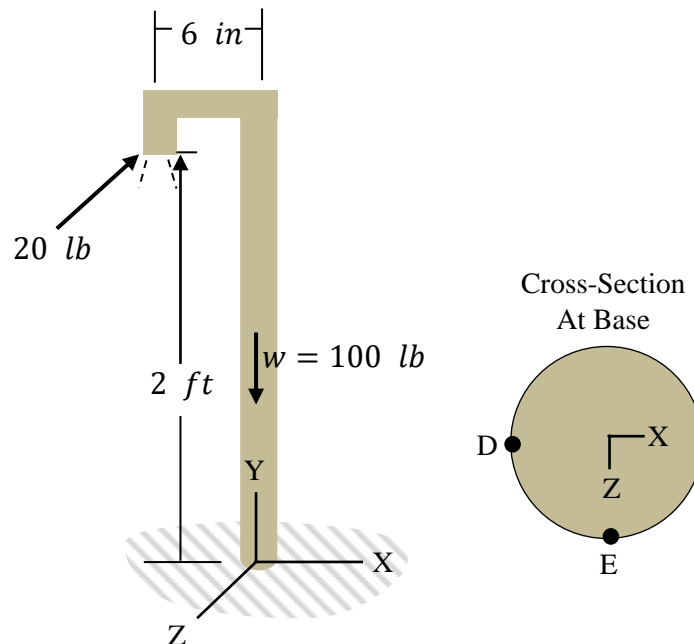Q1 (10 Points): A standing lamp is subject to a force 20 lb in the $-z$ direction shown below. The lamp is composed of a column of weight $= 100$ lb, and radius $r = 1$ in.

(a) Determine the state of stress at the location D, and draw the stress element.
(b) Determine the state of stress at the location E, and draw the stress element.

Cross-Section
At Base

Solution:

1. Bending stress due to the 20 lb load:
$$|M_x| = (20lb) \times (24in) = 480lb - in$$

$$\sigma_{y,E} = \frac{32|M_x|}{\pi d^3} = \frac{32 \times 480}{\pi(2)^3} = 611.15 \ psi \qquad \text{(tensile)}$$

$$\sigma_{y,D} = 0 \qquad \text{(on neutral plane)}$$

2. Torsional shear stress due to the 20 lb load:

$$|M_y| = (20lb) \times (6in) = 120lb - in$$

$$\tau_{xy,E} = \tau_{yz,D} = -\frac{16|M_y|}{\pi d^3} = -\frac{16 \times 120}{\pi(2)^3} = -76.39 \ psi$$

3. Shear stress due to the 20 lb load:

$$\tau_{xy,E} = 0 \qquad\qquad \text{(on neutral plane)}$$

$$\tau_{yz,D} = -\frac{4(V)}{3A} = -\frac{16(20)}{3\pi d^2} = -\frac{16(20)}{3\pi(2)^2} = -8.49 \ psi$$

4. Axial stress due to $100 \ lb$ weight:

$$\sigma_{y,E} = \sigma_{y,D} = -\frac{100}{(\pi d^2)/4} = -31.83 \ psi$$

At D,	At E,
$\sigma_{y,D} = -31.83 \ psi$	$\sigma_{y,D} = 611.15 - 31.83 \ psi = 579.32 \ psi$
$\tau_{yz,D} = -76.39 - 8.49 = -84.88 \ psi$	$\tau_{xy,E} = -76.39 psi$