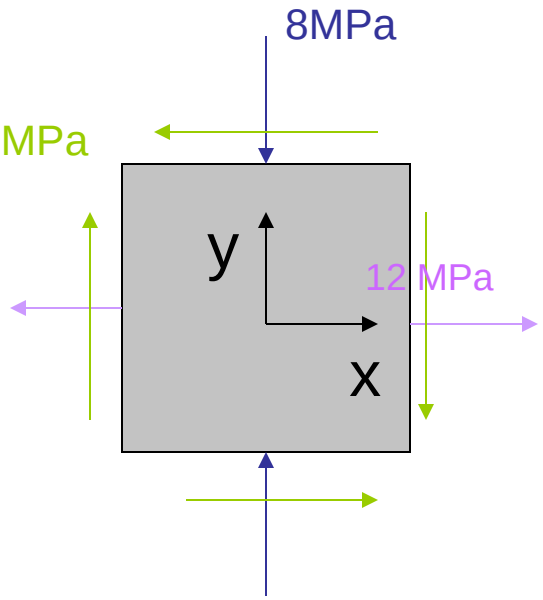
Example 2:

For the given stress state of plane stress, (a) sketch Mohr's circle of stress, using the Mohr's circle determine (b) the principal stresses (c) the maximum in plane shear stress and the normal stresses on the planes of maximum shear. Show these on a sketch of a **properly oriented stress element**.

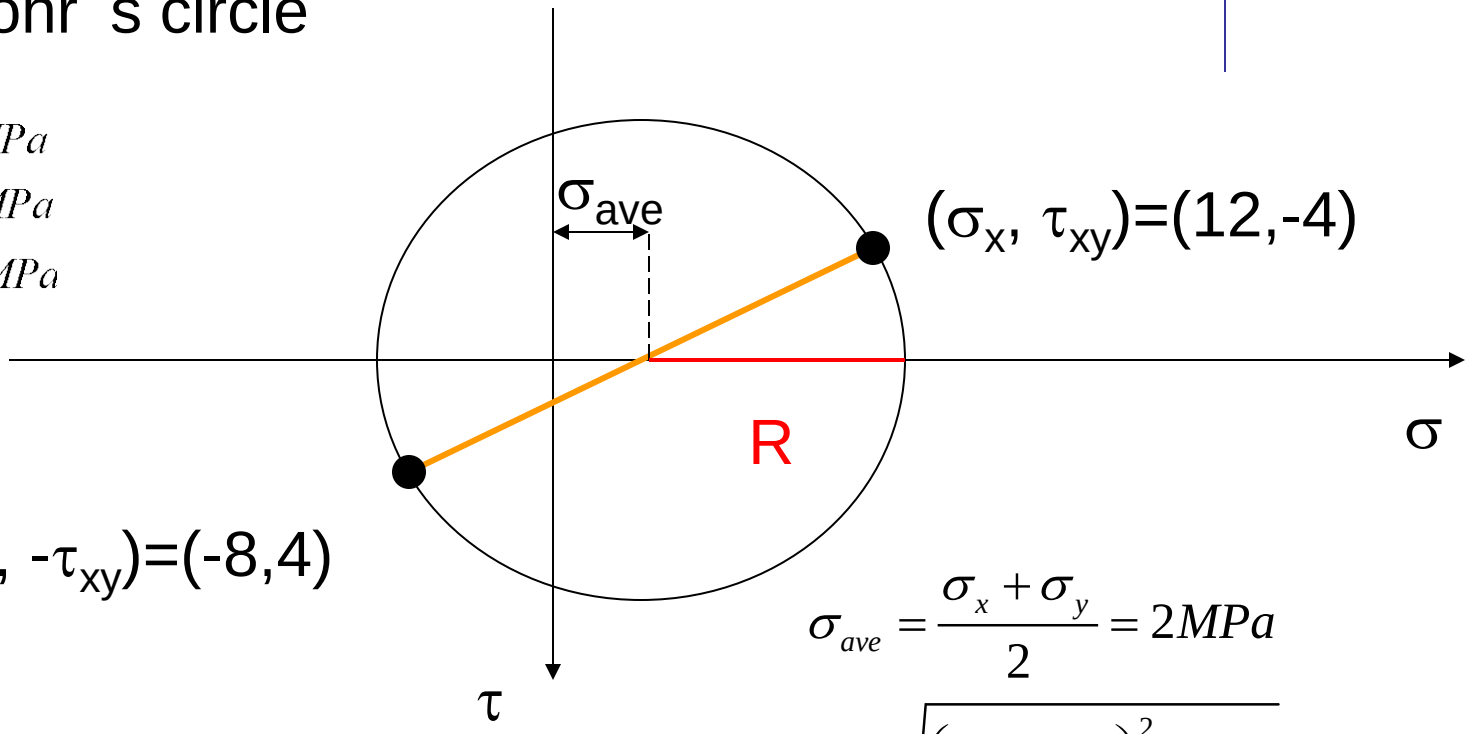


(a) Mohr's circle

$\sigma_x = 12MPa$

$\sigma_y = -8MPa$

$\tau_{xy} = -4MPa$



$(\sigma_x, \tau_{xy})=(12,-4)$

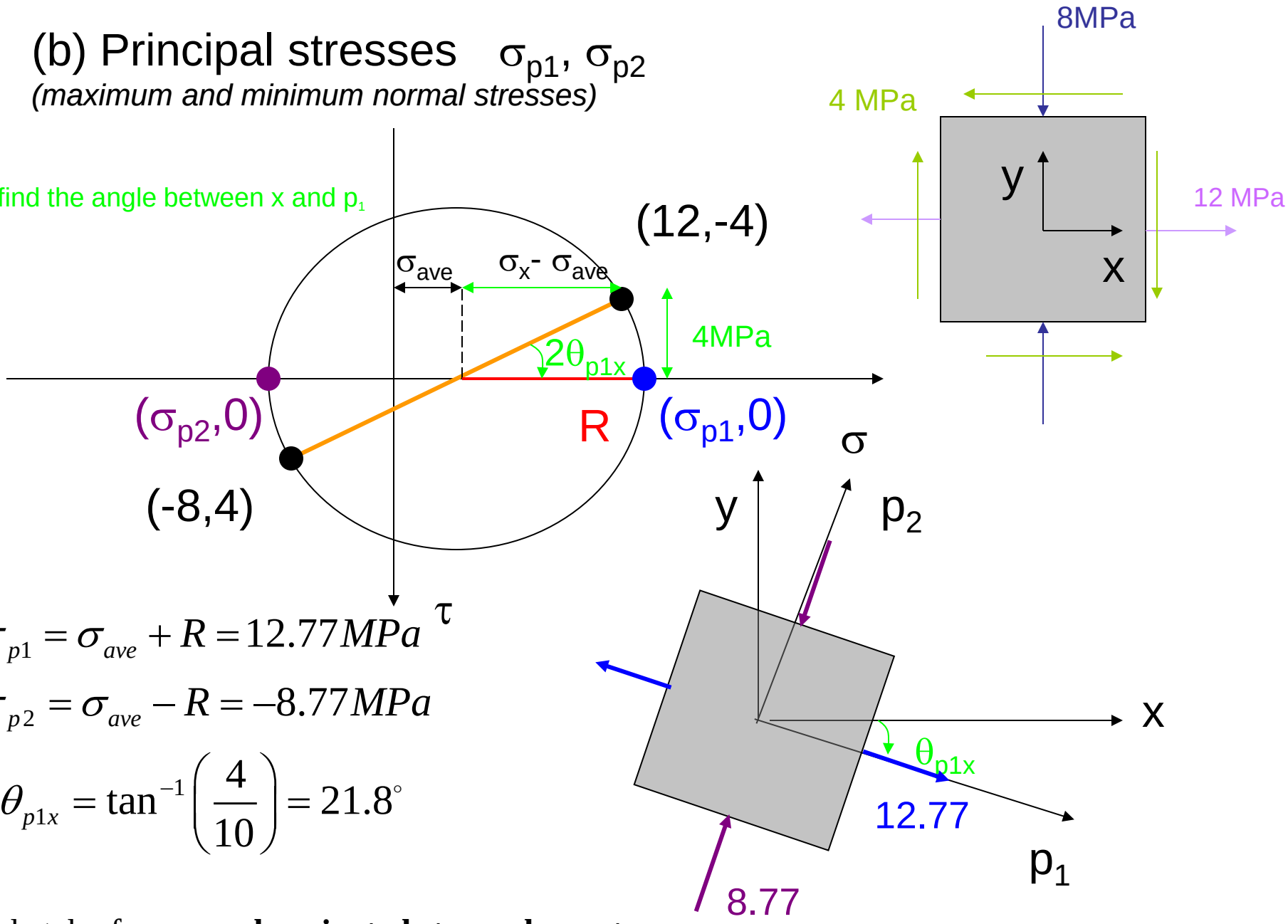
$(\sigma_y, -\tau_{xy})=(-8,4)$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 2MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = 10.77MPa$$

(b) Principal stresses σ_{p1}, σ_{p2}

(maximum and minimum normal stresses)



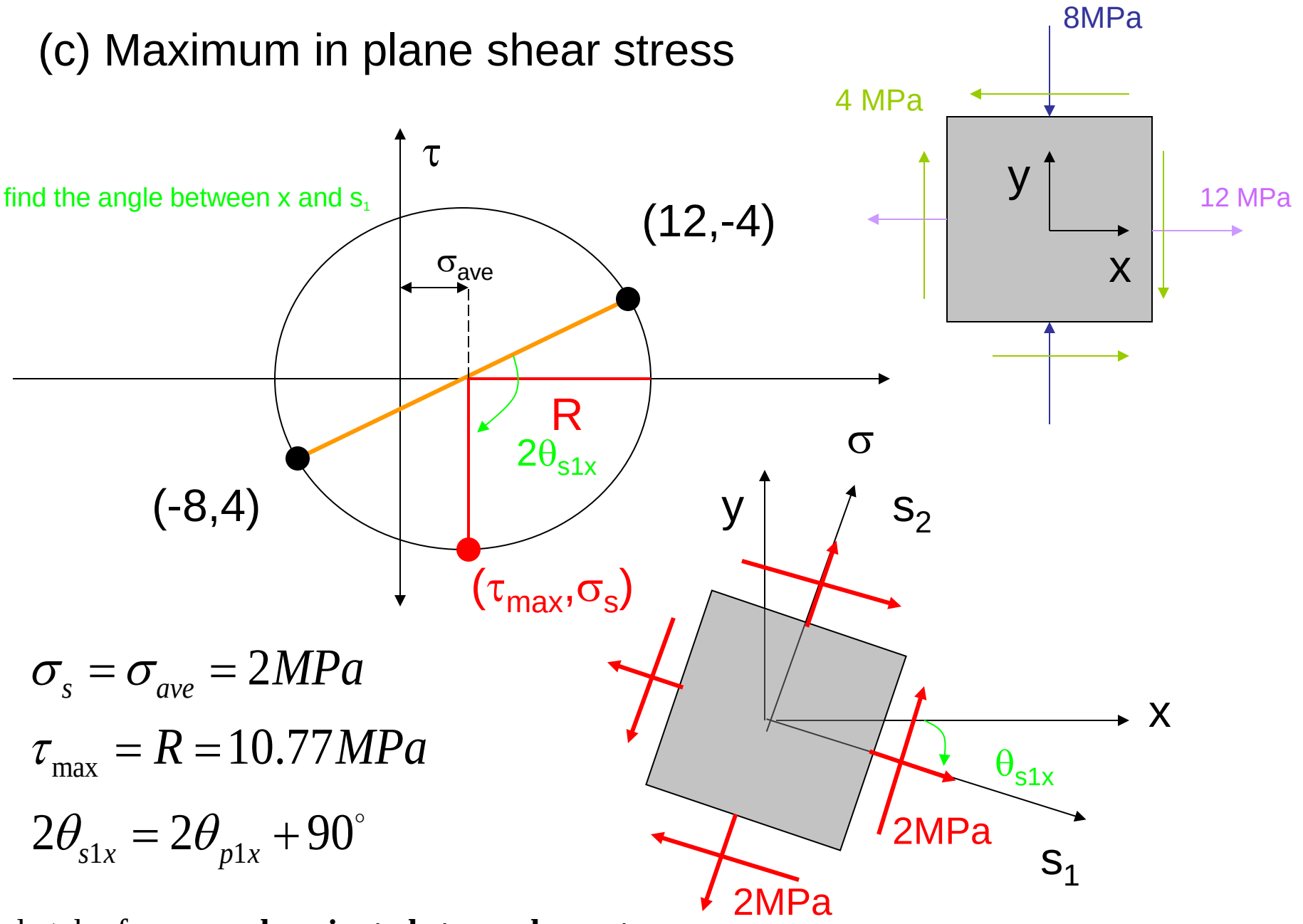
$$\sigma_{p1} = \sigma_{ave} + R = 12.77 MPa$$

$$\sigma_{p2} = \sigma_{ave} - R = -8.77 MPa$$

$$2\theta_{p1x} = \tan^{-1}\left(\frac{4}{10}\right) = 21.8^{\circ}$$

sketch of a **properly oriented stress element**

(c) Maximum in plane shear stress



$$\sigma_s = \sigma_{ave} = 2MPa$$

$$\tau_{\max} = R = 10.77MPa$$

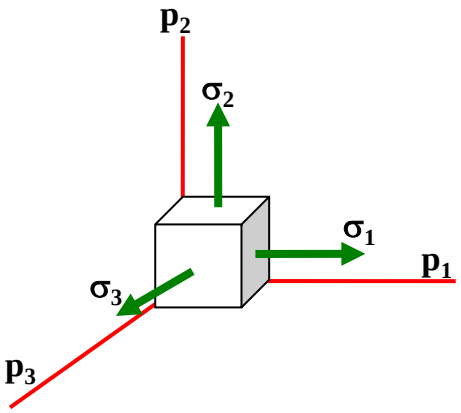
$$2\theta_{s1x} = 2\theta_{p1x} + 90^\circ$$

sketch of a **properly oriented stress element**

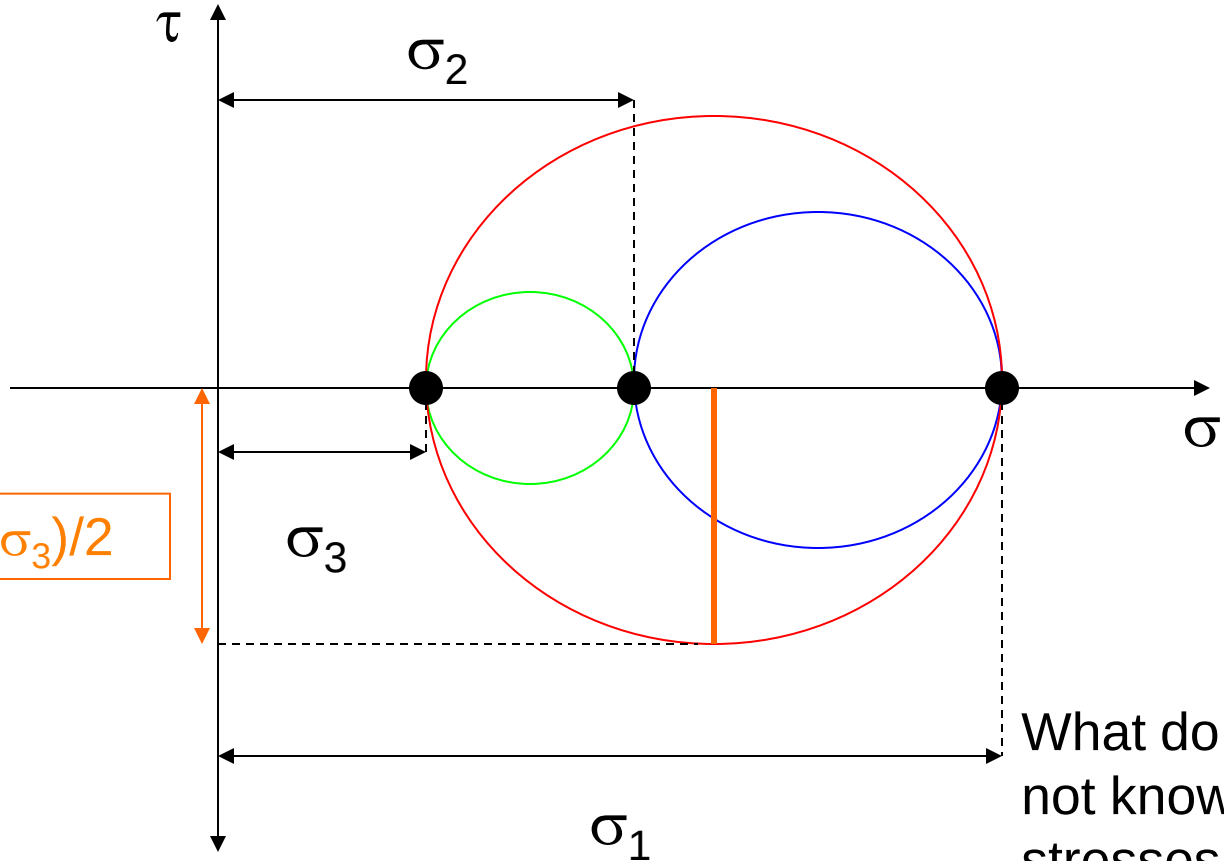
Three dimensional Mohr's circle

Assume we know the principal stresses:

$$\sigma_1 \equiv \sigma_{max}, \sigma_2 \equiv \sigma_{int}, \sigma_3 \equiv \sigma_{min}$$



τ^{abs}_{max}?



p_3-p_2

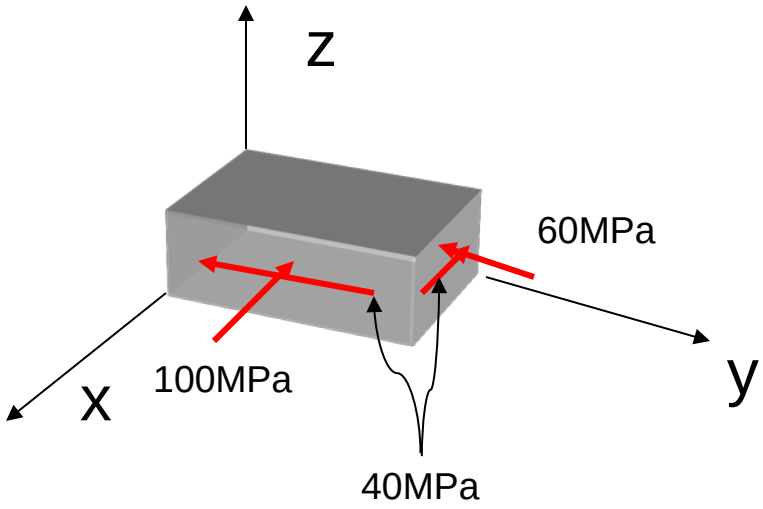
p_2-p_1

p_1-p_3

$\tau^{abs}_{max} = (\sigma_1 - \sigma_3)/2$

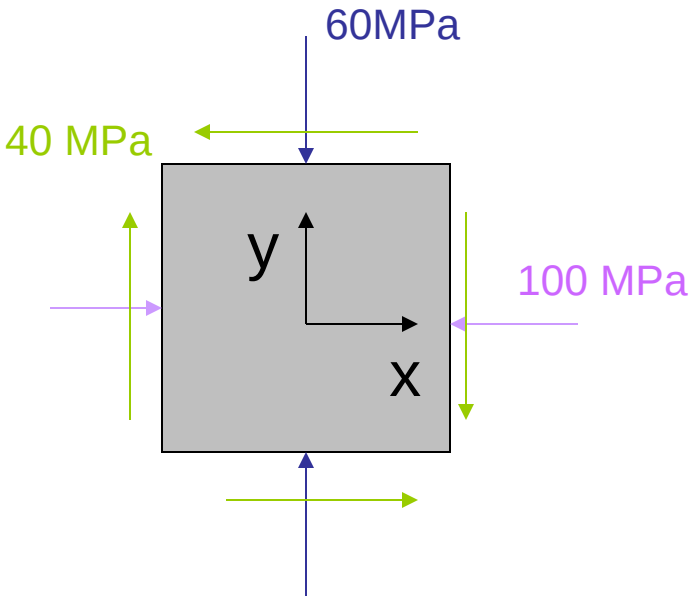
What do we do if we do not know the principal stresses

Example 3: For the given stress state (a) Sketch Mohr's circle for three dimensional stresses (b) determine the absolute maximum shear stress.



$$\sigma_x = -100MPa$$
$$\sigma_y = -60MPa$$
$$\sigma_z = 0$$
$$\tau_{xy} = -40MPa$$
$$\tau_{zy} = 0$$
$$\tau_{zx} = 0$$

Plane stress



(a) Sketch Mohr's circle for three dimensional stresses

$\sigma_x = -100MPa$

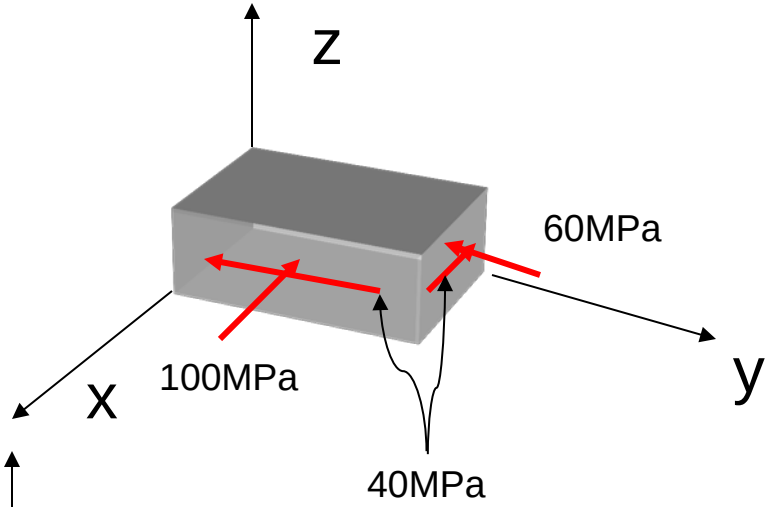
$\sigma_y = -60MPa$

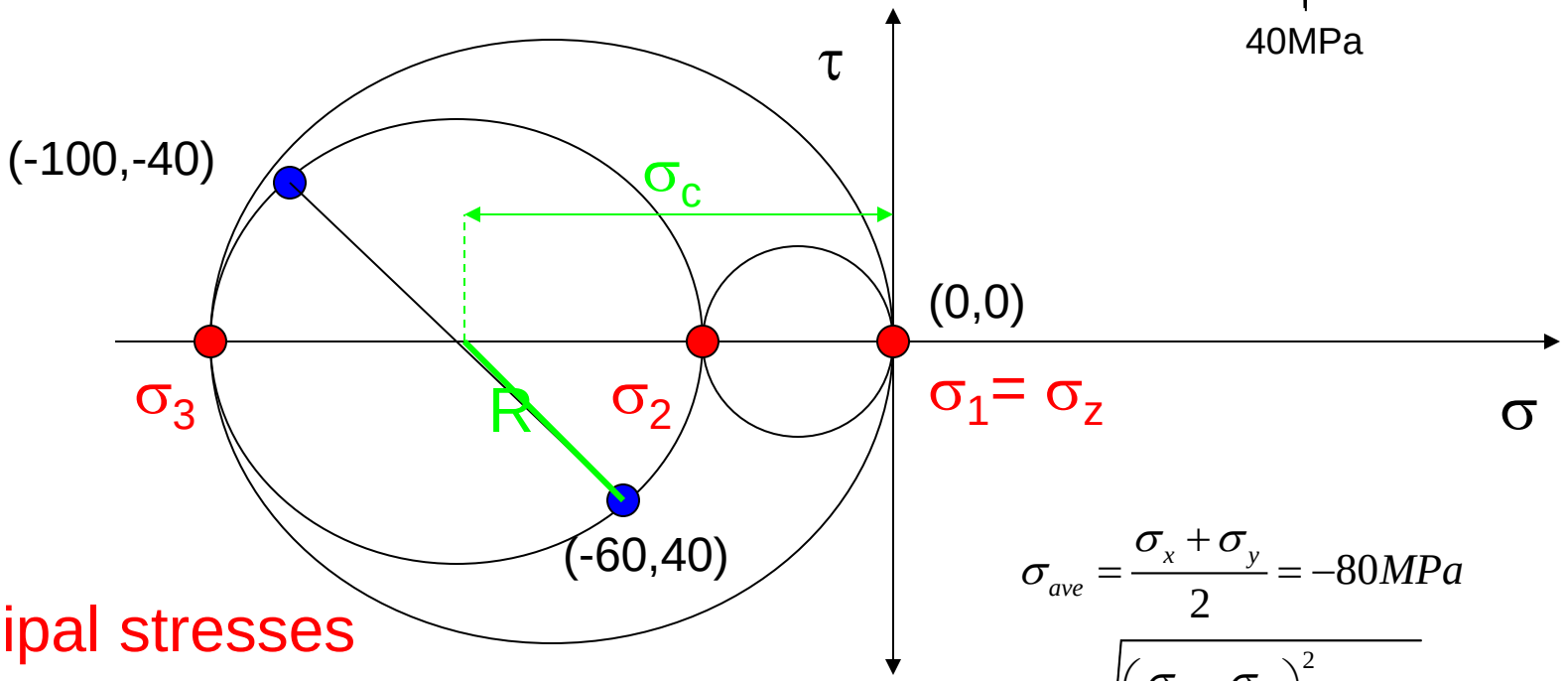
$\sigma_z = 0$

$\tau_{xy} = -40MPa$

$\tau_{zy} = 0$

$\tau_{zx} = 0$





principal stresses

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -80MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = 44.721MPa$$

(a) Find the absolute maximum shear stress $\tau_{\max}^{abs} = \dfrac{\sigma_1 - \sigma_3}{2}$

$\sigma_x = -100MPa$

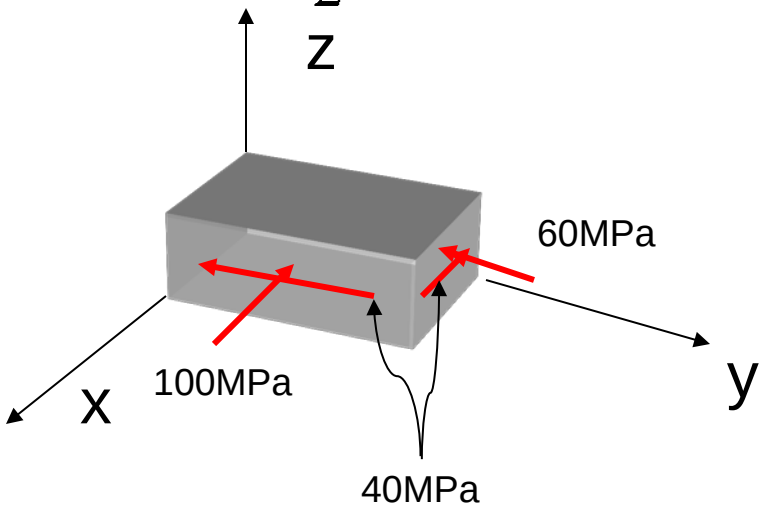
$\sigma_y = -60MPa$

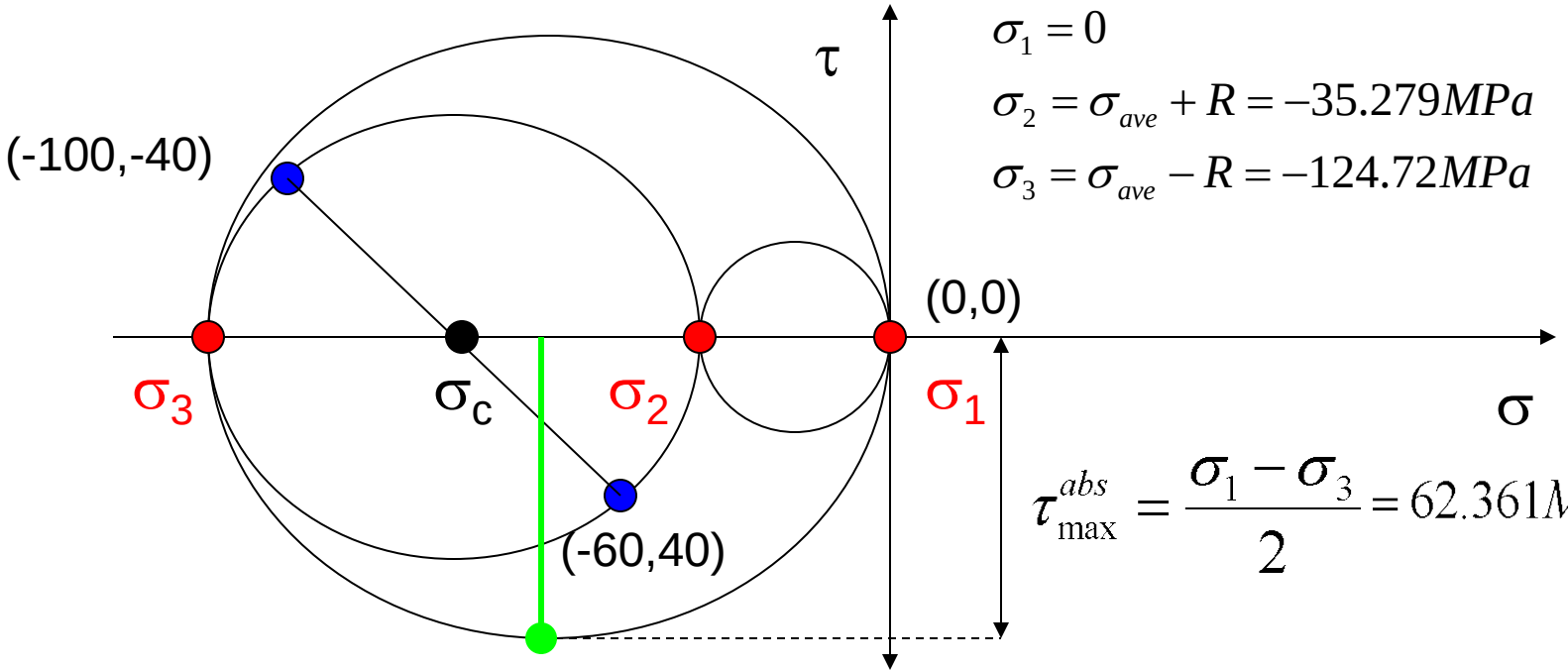
$\sigma_z = 0$

$\tau_{xy} = -40MPa$

$\tau_{zy} = 0$

$\tau_{zx} = 0$





$\sigma_1 = 0$

$\sigma_2 = \sigma_{ave} + R = -35.279MPa$

$\sigma_3 = \sigma_{ave} - R = -124.72MPa$

$\tau_{\max}^{abs} = \dfrac{\sigma_1 - \sigma_3}{2} = 62.361MPa$

Example 4: For the given stress state (a) Sketch Mohr's circle for three dimensional stresses (b) determine the absolute maximum shear stress.

$\sigma_x = 0$

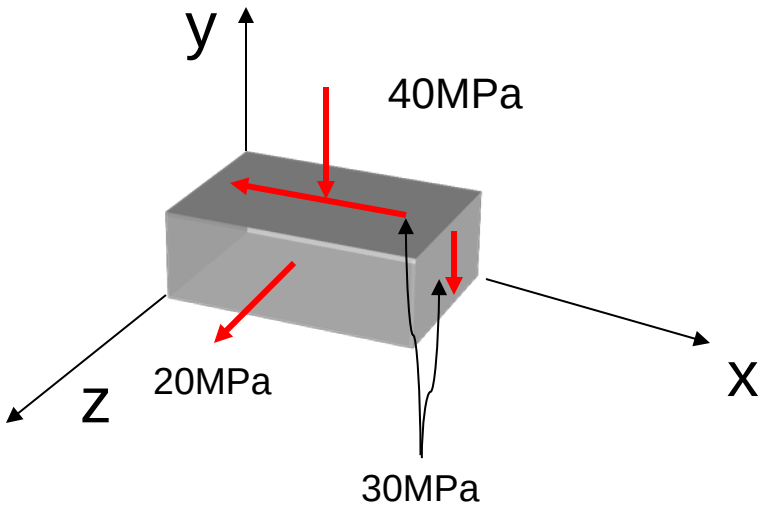
$\sigma_y = -40MPa$

$\sigma_z = 20MPa$

$\tau_{xy} = -30MPa$

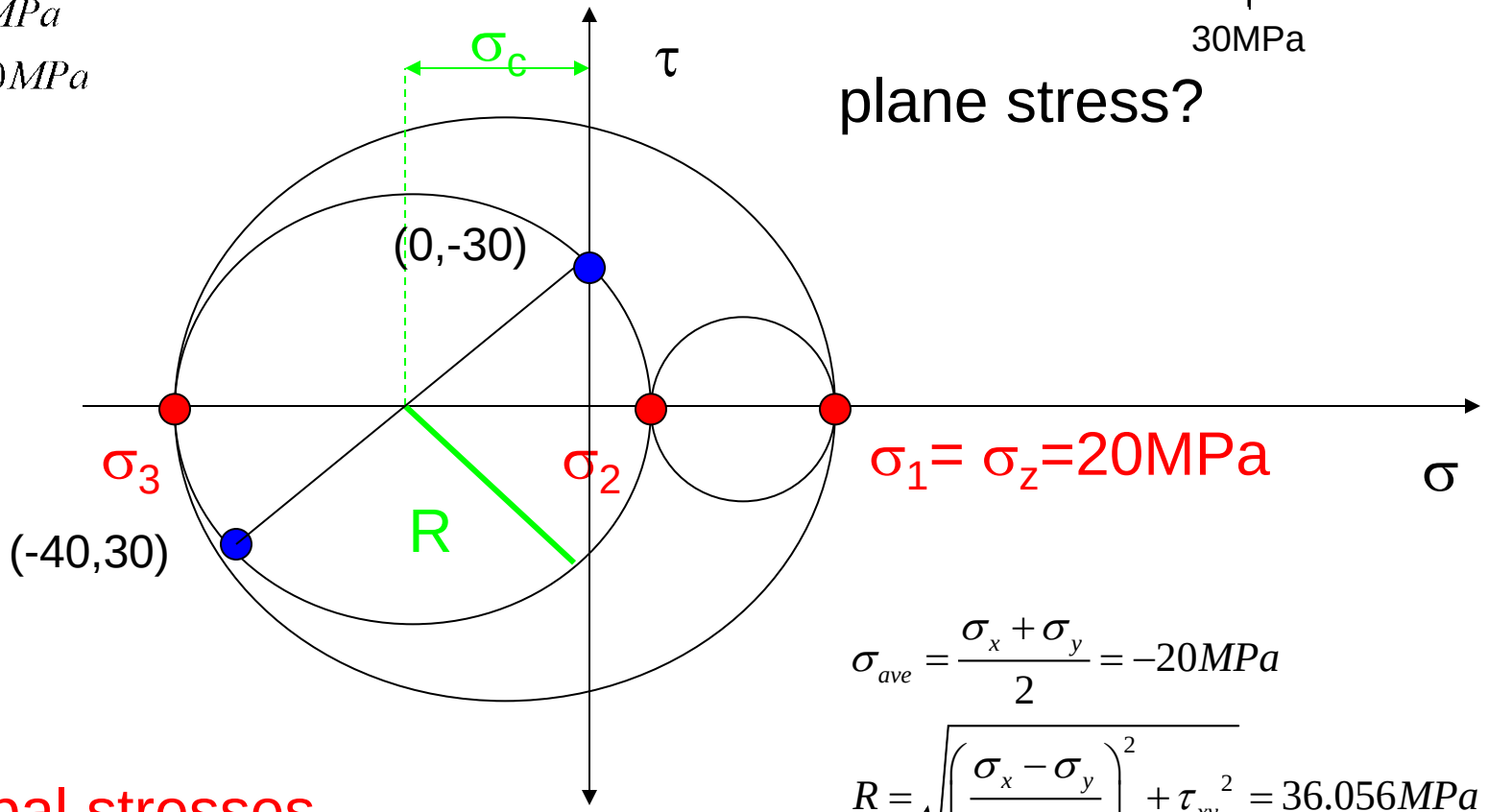
$\tau_{zy} = 0$

$\tau_{zx} = 0$



plane stress?

(a)



principal stresses

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -20MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = 36.056MPa$$

(b) determine the absolute maximum shear stress.

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -20MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = 36.056MPa$$

$\sigma_x = 0$

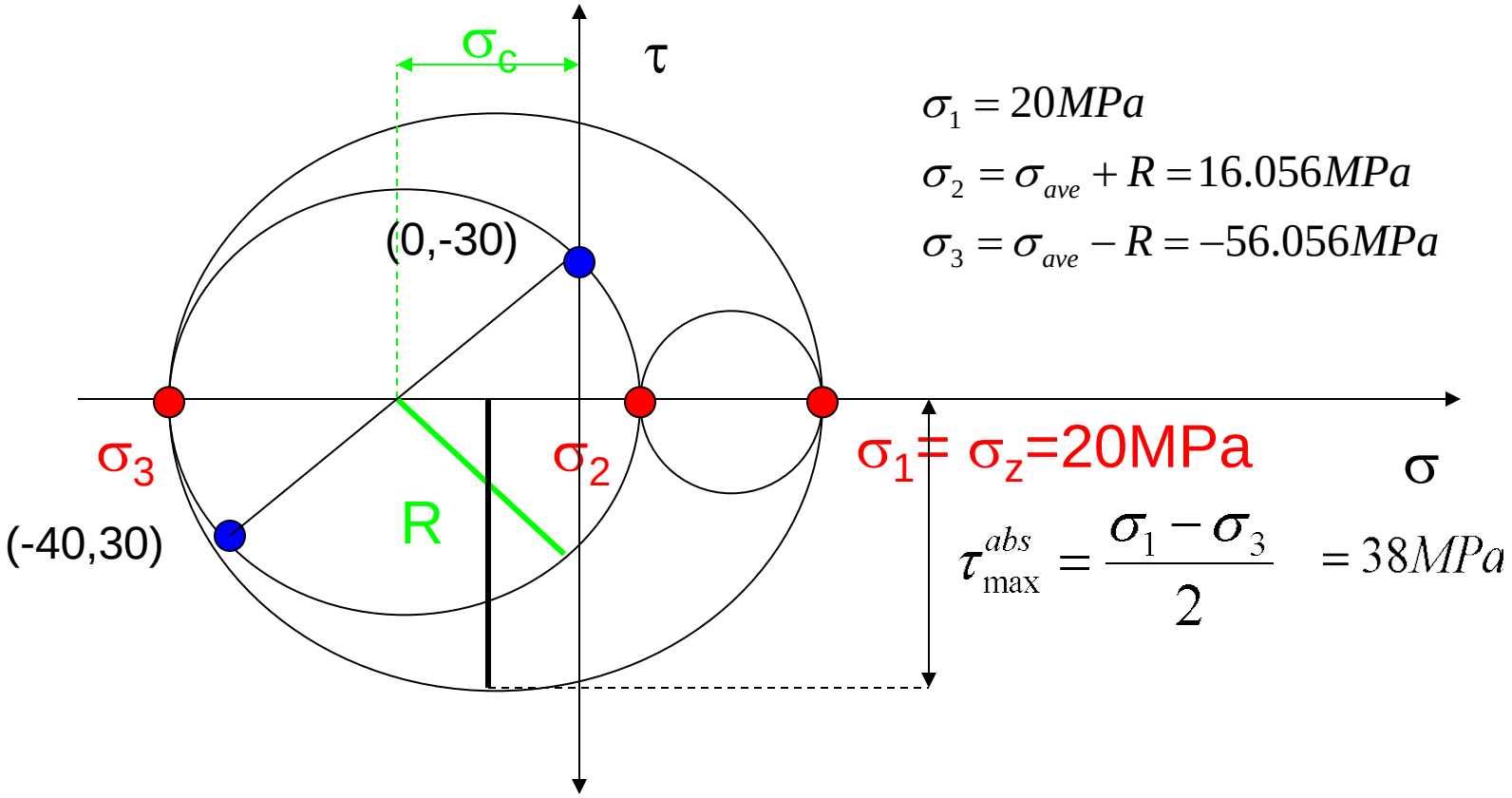
$\sigma_y = -40MPa$

$\sigma_z = 20MPa$

$\tau_{xy} = -30MPa$

$\tau_{zy} = 0$

$\tau_{zx} = 0$



$\sigma_1 = 20MPa$

$\sigma_2 = \sigma_{ave} + R = 16.056MPa$

$\sigma_3 = \sigma_{ave} - R = -56.056MPa$

$$\tau_{\max}^{abs} = \frac{\sigma_1 - \sigma_3}{2} = 38MPa$$