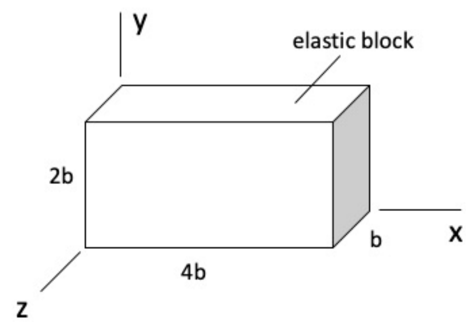
ME 323: Mechanics of Materials **Homework Set H05**

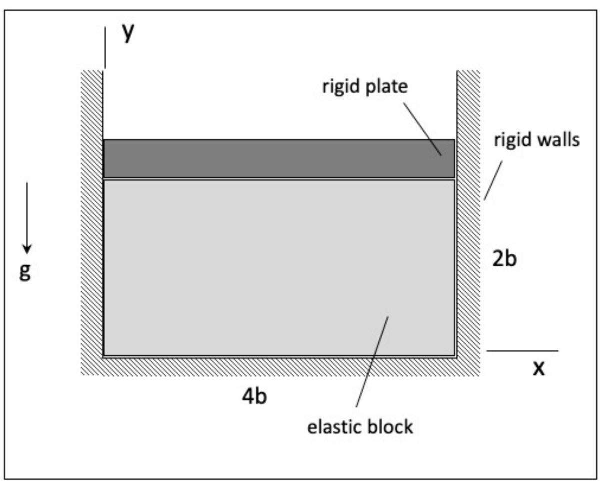
Summer 2024 **Assigned/Due: June 14/June 18**

A cuboid-shaped block having (*x,y,z*) dimensions of (*4b, 2b, b*) is made up of an elastic material having a Young's modulus of *E*, a Poisson's ratio of ν and a coefficient of thermal expansion of α. The weight of the block can be considered to be negligible.

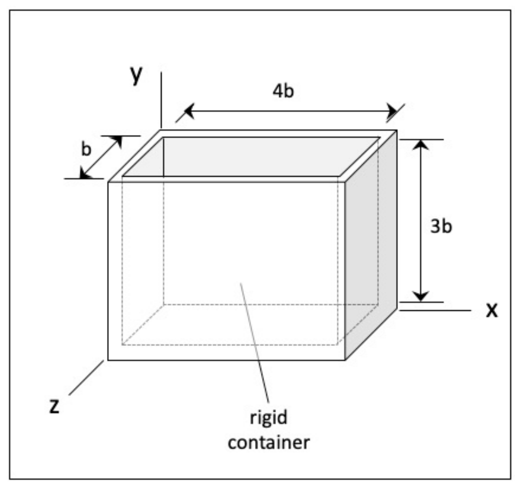


a) The block is snugly placed between two smooth, rigid, vertical walls, as shown below, with no restraints placed on the z-faces of the block. A rigid plate having a weight of W is placed on top of the block, and the temperature of the block is held fixed. Determine the (*x,y,z*) components of normal stress and strain, and the (*x,y,z*) dimensions of the block resulting from the weight of the plate.

b) The block is now snugly placed within a rigid container with smooth surfaces, as shown below. The temperature of the block is uniformly increased by an amount of ΔT. Determine the (*x,y,z*) components of normal stress and strain, and the (*x,y,z*) dimensions of the block resulting from the temperature increase of the material.



Part (a)



Part (b)

Hooke's Law in 3D

(1) $\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha\Delta T$

(2) $\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$

(3) $\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha\Delta T$

(a) Knowns: $\Delta T = 0$

$$\left.\begin{aligned} &(4)\quad \varepsilon_x = 0 \\ &(5)\quad \sigma_y = -\frac{W}{4b^2} \\ &(6)\quad \sigma_z = 0 \end{aligned}\right\} \varepsilon_y, \varepsilon_z, \sigma_x \text{ are UNKNOWN}$$

$(1),(4),(5),(6) \Rightarrow \quad 0 = \frac{1}{E}\left[\sigma_x - \nu\left(-\frac{W}{4b^2}+0\right)\right] \Rightarrow \sigma_x = -\frac{\nu W}{4b^2}$

$(2) \Rightarrow \quad \varepsilon_y = \frac{1}{E}\left[-\frac{W}{4b^2} - \nu\left(-\frac{\nu W}{4b^2}+0\right)\right] = \frac{W}{4Eb^2}(-1+\nu^2)$

$(3) \Rightarrow \quad \varepsilon_z = \frac{1}{E}\left[0 - \nu\left(-\frac{\nu W}{4b^2} - \frac{W}{4b^2}\right)\right] = \frac{W\nu}{4Eb^2}(1+\nu)$

(b) <u>Knowns</u>

$(7)\ \varepsilon_x = 0$

$(8)\ \varepsilon_z = 0$

$(9)\ \sigma_y = 0$

$\sigma_x, \sigma_z, \varepsilon_y$ are UNKNOWN

$(1) \Rightarrow \quad 0 = \frac{1}{E}\left[\sigma_x - \nu(0+\sigma_z)\right] + \alpha\Delta T \Rightarrow \sigma_x - \nu\sigma_z = -\alpha\Delta T E \quad (10)$

$(3) \Rightarrow \quad 0 = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x - 0)\right] + \alpha\Delta T \Rightarrow -\nu\sigma_x + \sigma_z = -\alpha\Delta T E \quad (11)$

- Multiply (10) by ν and add to (11):

$$(-\nu^2+1)\sigma_z = -\alpha\Delta T E(\nu+1) \Rightarrow \sigma_z = -\alpha\Delta T E\left(\frac{1+\nu}{1-\nu^2}\right) = -\left[\frac{\alpha\Delta T E}{1-\nu}\right]$$

- Multiply (11) by ν and add to (10):

$$(1-\nu^2)\sigma_x = -\alpha\Delta T E(1+\nu) \Rightarrow \sigma_x = -\alpha\Delta T E\left(\frac{1+\nu}{1-\nu^2}\right) = -\left[\frac{\alpha\Delta T E}{1-\nu}\right]$$

$(2) \Rightarrow \quad \varepsilon_y = \frac{1}{E}\left[0 - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$

$$= \frac{2\nu}{E}\left(\frac{\alpha\Delta T E}{1-\nu}\right) + \alpha\Delta T = \left(\frac{1+\nu}{1-\nu}\right)\alpha\Delta T$$