SOLUTION

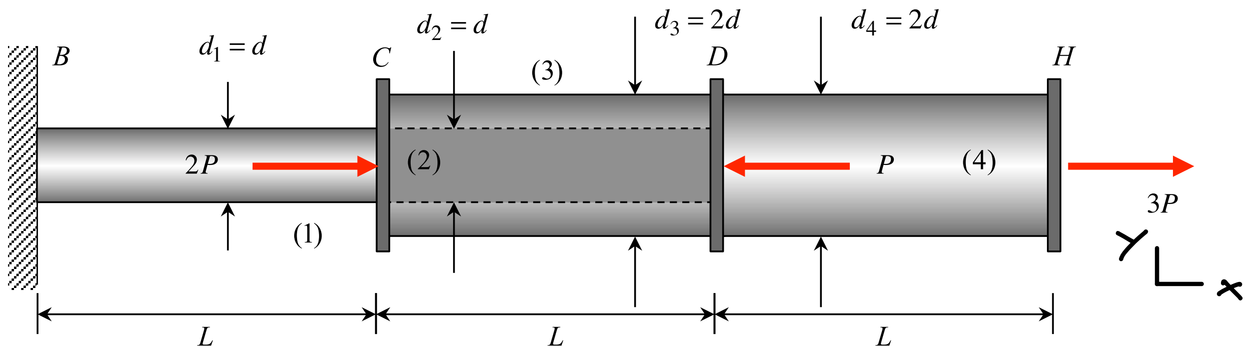
**ME 323: Mechanics of Materials** — **Homework Set H06**

**Summer 2024** — **Assigned/Due: June 17/June 19**

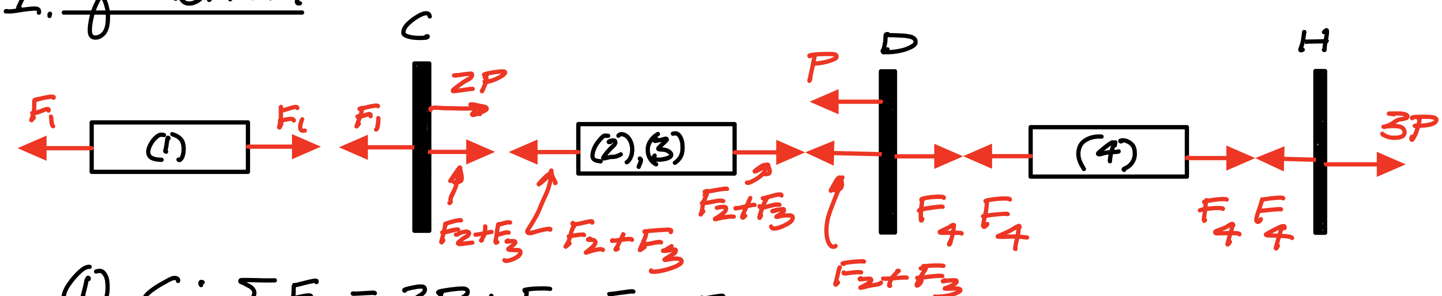
A rod is made up of elements (1), (2), (3) and (4), with each element having a length of $L$, and the elements have outer diameters of $d_1$, $d_2$, $d_3$ and $d_4$, respectively. Element (2) is a core inside the tube element (3), as shown in the figure, and elements (1), (2)/(3) and (4) are connected in series. The elements have Young's moduli of: $E_1 = E_2 = E$ and $E_3 = E_4 = 2E$. C, D and H represent rigid connectors for the rod elements. Loads of $2P$, $P$ and $3P$ act on connectors C, D and H, in directions shown on the figure below.

a) Determine the displacement of connector H.

b) Determine the stress in each element of the rod.




1. Equilibrium




(1) $C: \ \sum F_x = 2P + F_2 + F_3 - F_1 = 0$

(2) $D: \ \sum F_x = -P - F_2 - F_3 + F_4 = 0$

(3) $H: \ \sum F_x = 3P - F_4 = 0$

2. Load/Deformation

(4) $e_1 = \dfrac{F_1 L_1}{E_1 A_1} = \dfrac{F_1 L}{E\pi(d/2)^2} = \dfrac{4F_1 L}{\pi E d^2}$

(5) $e_2 = \dfrac{F_2 L_2}{E_2 A_2} = \dfrac{F_2 L}{E\pi(d/2)^2} = \dfrac{4F_2 L}{\pi E d^2}$

(6) $e_3 = \frac{F_3 L_3}{E_3 A_3} = \frac{F_3 L}{(2E)\left[\pi\left(\frac{2d}{2}\right)^2 - \pi\left(\frac{d}{2}\right)^2\right]} = \frac{2 F_3 L}{3\pi E d^2}$

(7) $e_4 = \frac{F_4 L_4}{E_4 A_4} = \frac{F_4 L}{(2E)\ \pi(2d/2)^2} = \frac{F_4 L}{2\pi d^2}$

3. Compatibility

(8) $e_2 = e_3 \Rightarrow \frac{4 F_2 L}{\pi E d^2} = \frac{2 F_3 L}{3\pi E d^2} \Rightarrow F_2 = \frac{1}{6} F_3$

4. Solve

$(3) \Rightarrow F_4 = 3P$

$(2) \,\&\, (8) \Rightarrow -P - \frac{1}{6} F_3 - F_3 + F_4 = 0$

$\hookrightarrow F_3 = \frac{6}{7}\left[-P + F_4\right] = \frac{12}{7} P$

$\Rightarrow F_2 = \frac{1}{6}\left(\frac{12}{7} P\right) = \frac{2}{7} P$

$(1) \Rightarrow F_1 = 2P + F_2 + F_3 = 2P + \frac{2}{7}P + \frac{12}{7}P = 4P$

Part a)

$(4) \Rightarrow e_1 = \frac{16}{\pi} \frac{PL}{Ed^2}$

$(5) \Rightarrow e_2 = \frac{8}{7\pi} \frac{PL}{Ed^2} = e_3$

$(7) \Rightarrow e_4 = \frac{3}{2\pi} \frac{PL}{Ed^2}$

$u_H = e_1 + e_2 + e_4 = \left(16 + \frac{8}{7} + \frac{3}{2}\right) \frac{PL}{\pi E d^2} = \frac{261}{14\pi} \frac{PL}{Ed^2} \longleftarrow u_H$

Part b)

$\sigma_1 = \frac{F_1}{A_1} = \frac{4P}{\pi(d/2)^2} = \frac{16 P}{\pi d^2} \longleftarrow \sigma_1$

$\sigma_2 = \frac{F_2}{A_2} = \frac{\frac{2}{7} P}{\pi(d/2)^2} = \frac{8}{7\pi} \frac{P}{d^2} \longleftarrow \sigma_2$

$\sigma_3 = \frac{F_3}{A_3} = \frac{\frac{12}{7} P}{(3\pi/4) d^2} = \frac{48}{21\pi} \frac{P}{d^2} \longleftarrow \sigma_3$

$\sigma_4 = \frac{F_4}{A_4} = \frac{3P}{\pi(2d/2)^2} = \frac{3}{\pi} \frac{P}{d^2} \longleftarrow \sigma_4$