SOLUTION

ME 323: Mechanics of Materials | ***Homework Set H07***

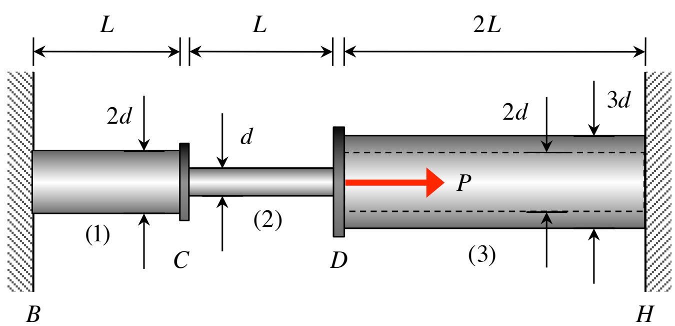
Summer 2024 | ***Assigned/Due: June 18/June 20***

PART A – 10 points

A three-segment rod is constructed as shown below. Segments (1) and (2) have a length of L, whereas segment (3) has a length of $2L$. Segments (1) and (2) have solid, circular cross sections with diameters of $2d$ and d, respectively, whereas segment (3) is a tube with outer and inner diameters of $3d$ and $2d$, respectively. Segments (1) and (2) are joined by a rigid connecter at C, and segments (2) and (3) are joined by a rigid connector at D. Ends B and H of the rod are fixed to rigid walls. All three segments are made of the same material, with E being the Young's modulus of the material. A force P acts on connector D.

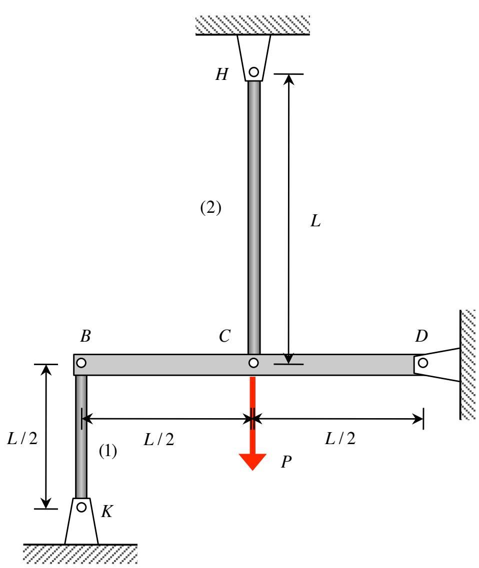
a) Determine the stresses in each of the three segments of the rod.

b) Determine the displacements of connectors C and D.

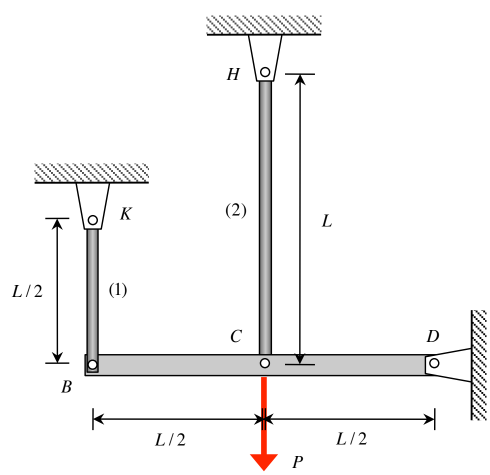


PART B – 4 points

Consider the two structures below, (i) and (ii). In each case, let F_1 and F_2 represent the axial loads carried by members (1) and (2), with the sign conventions that $F_i > 0$ and $e_i > 0$ for the ith member being in tension. For each structure, write down the *compatibility equation* relating the elongations e_1 and e_2.

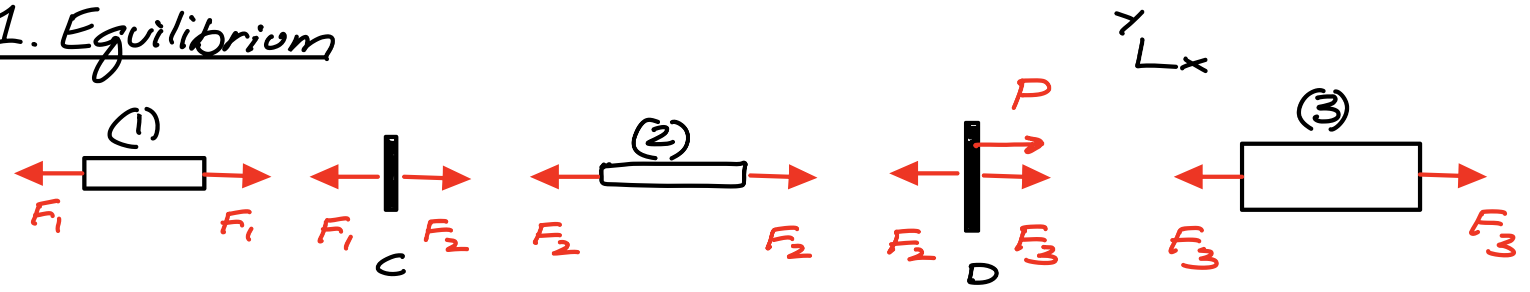


Structure (i)



Structure (ii)

1. Equilibrium



(1) C: $\sum F_x = -F_1 + F_2 = 0 \Rightarrow F_1 = F_2$

(2) D: $\sum F_x = F_3 - F_2 + P = 0 \Rightarrow F_2 = F_3 + P$

2. Force/elongation

(3) $e_1 = \dfrac{F_1 L_1}{E_1 A_1} = \dfrac{F_1 L}{E\pi(2d/2)^2} = \dfrac{F_1 L}{\pi E d^2}$

(4) $e_2 = \dfrac{F_2 L_2}{E_2 A_2} = \dfrac{F_2 L}{E\pi(d/2)^2} = \dfrac{4F_2 L}{\pi E d^2}$

(5) $e_3 = \dfrac{F_3 L_3}{E_3 A_3} = \dfrac{F_3(2L)}{E[\pi(3d/2)^2 - \pi(2d/2)^2]} = \dfrac{8F_3 L}{5\pi E d^2}$

3. Compatibility

$u_C = \cancel{u_B}^{0} + e_1$

$u_D = u_C + e_2 = e_1 + e_2$

(6) $u_H = u_D + e_3 = e_1 + e_2 + e_3 = 0$

4. Solve

(3)–(6) $\Rightarrow \dfrac{F_1 \cancel{L}}{\cancel{\pi E d^2}} + \dfrac{4F_2 \cancel{L}}{\cancel{\pi E d^2}} + \dfrac{8F_3 \cancel{L}}{5\cancel{\pi E d^2}} = 0$

(7) $\hookrightarrow F_1 + 4F_2 + \frac{8}{5}F_3 = 0$

(1),(2),(7) $\Rightarrow F_3 + P + 4(F_3 + P) + \frac{8}{5}F_3 = 0$

$\hookrightarrow F_3 = -\dfrac{5P}{5 + 8/5} = -\dfrac{25}{33}P$ (C)

$\therefore$

$F_2 = F_3 + P = \dfrac{8}{33}P$ (T)

$F_1 = F_2 = \dfrac{8}{33}P$ (T)

Part a)

From this, we have:

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(8/33)P}{\pi\left(\frac{2d}{2}\right)^2} = \frac{8}{33\pi}\,\frac{P}{d^2}\,(T) \longleftarrow \sigma_1$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{(8/33)P}{\pi(d/2)^2} = \frac{32}{33\pi}\,\frac{P}{d^2}\,(T) \longleftarrow \sigma_2$$

$$\sigma_3 = \frac{F_3}{A_3} = \frac{(-25/33)P}{\left[\pi\left(\frac{3d}{2}\right)^2 - \pi\left(\frac{2d}{2}\right)^2\right]} = -\frac{20}{33\pi}\,\frac{P}{d^2}\,(C) \longleftarrow \sigma_3$$

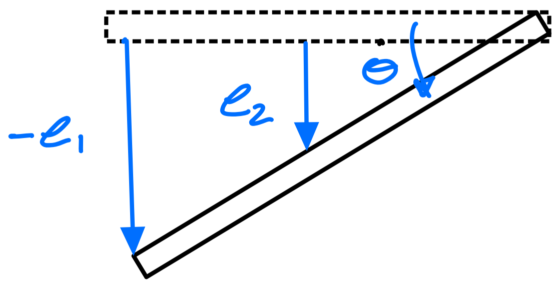
Part b)

$$u_C = \cancel{u_B}^{\,0} + e_1 = \frac{F_1 L}{\pi E d^2} = \frac{8}{33\pi}\,\frac{PL}{Ed^2}$$

$$u_D = u_C + e_2 = \frac{8}{33\pi}\,\frac{PL}{Ed^2} + \frac{4\left(\frac{8}{33}P\right)}{\pi E d^2} = \frac{40}{33\pi}\,\frac{P}{Ed^2}$$

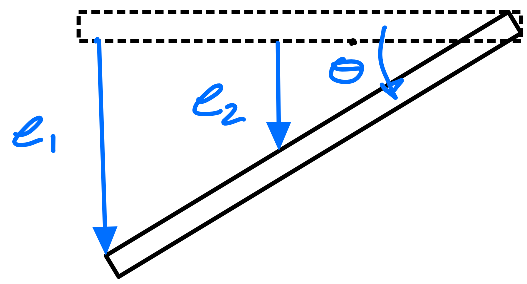
Part B

Structure I



$$\left.\begin{aligned} -e_1 &= L\theta \\ e_2 &= \tfrac{L}{2}\theta \end{aligned}\right\} \Rightarrow e_1 = -2e_2$$

Structure II



$$\left.\begin{aligned} e_1 &= L\theta \\ e_2 &= \tfrac{L}{2}\theta \end{aligned}\right\} \Rightarrow e_1 = 2e_2$$