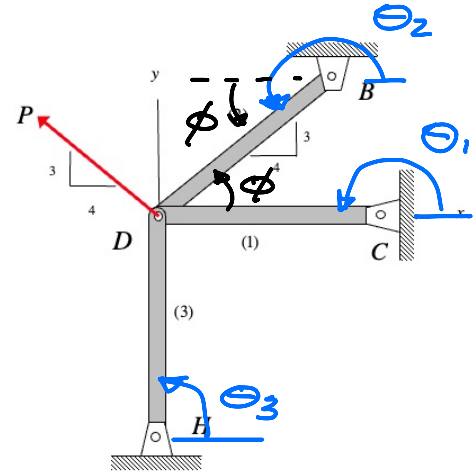
Solution

ME 323: Mechanics of Materials **Homework Set H08**

Summer 2024 **Assigned/Due: June 19/June 21**

A truss is constructed using three identical members (each of length L, cross-sectional area A and made up of a material having a Young's modulus of E). A load P acts on joint D.

Determine the stresses in each of the three segments of the truss.



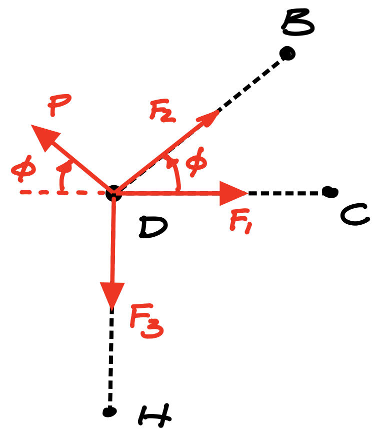
Leave your answers in terms of, at most, P, L, E and A.

$\cos\phi = \frac{4}{5}$ $\qquad \theta_1 = 180°$

$\sin\phi = 3/5$ $\qquad \theta_2 = 180° + \phi$

$\qquad \theta_3 = 90°$

1. Equilibrium: method of joints



(1) D: $\sum F_x = F_2\cos\phi + F_1 - P\cos\phi = 0$

(2) $\sum F_y = F_2 \sin\phi + P\sin\phi - F_3 = 0$

2 equations / 3 unknowns $(F_1, F_2, F_3) \Rightarrow$ indeterminate

2. Load/elongation

(3) $e_1 = \frac{F_1 L}{EA}$

(4) $e_2 = \frac{F_2 L}{EA}$

(5) $e_3 = \frac{F_3 L}{EA}$

3. Compatibility

(6) $e_1 = u_D \cancelto{-1}{\cos\theta_1} + v_D \cancelto{0}{\sin\theta_1} = -u_D$

(7) $e_2 = u_D\cos\theta_2 + v_D \sin\theta_2 = -\frac{4}{5}u_D - \frac{3}{5}v_D$

(8) $e_3 = u_D \cancelto{0}{\cos\theta_3} + v_D \cancelto{1}{\sin\theta_3} = v_D$

4. Solve

(9) (3) & (6) $\Rightarrow -\frac{F_1 L}{EA} = u_D$

(10) (5) & (8) $\Rightarrow \frac{F_3 L}{EA} = v_D$

(4), (7), (9), (10) $\Rightarrow \frac{F_2 \cancel{L}}{\cancel{EA}} = -\frac{4}{5}\left(-\frac{F_1 \cancel{L}}{\cancel{EA}}\right) - \frac{3}{5}\left(\frac{F_3 \cancel{L}}{\cancel{EA}}\right)$

$$(11) \hookrightarrow F_2 = \frac{4}{5}F_1 - \frac{3}{5}F_3$$

$$(1) \Rightarrow F_1 = P\cos\phi - F_2\cos\phi = \frac{4}{5}(P - F_2)$$

$$(2) \Rightarrow F_3 = P\sin\phi + F_2\sin\phi = \frac{3}{5}(P + F_2)$$

$$(11) \Rightarrow F_2 = \frac{16}{25}(P - F_2) - \frac{9}{25}(P + F_2)$$

$$\hookrightarrow F_2 = \frac{7}{50}P \; (T)$$

$$\Rightarrow F_1 = \frac{4}{5}\left[P - \left(\frac{7}{50}P\right)\right] = \frac{162}{250}P \quad (T)$$

$$\Rightarrow F_3 = \frac{3}{5}\left[P + \left(\frac{7}{50}P\right)\right] = \frac{171}{250}P \quad (T)$$

$$\therefore \begin{cases} \sigma_1 = \frac{162}{250}\frac{P}{A} & (T) \\ \sigma_2 = \frac{7}{50}\frac{P}{A} & (T) \\ \sigma_3 = \frac{171}{250}\frac{P}{A} & (T) \end{cases}$$