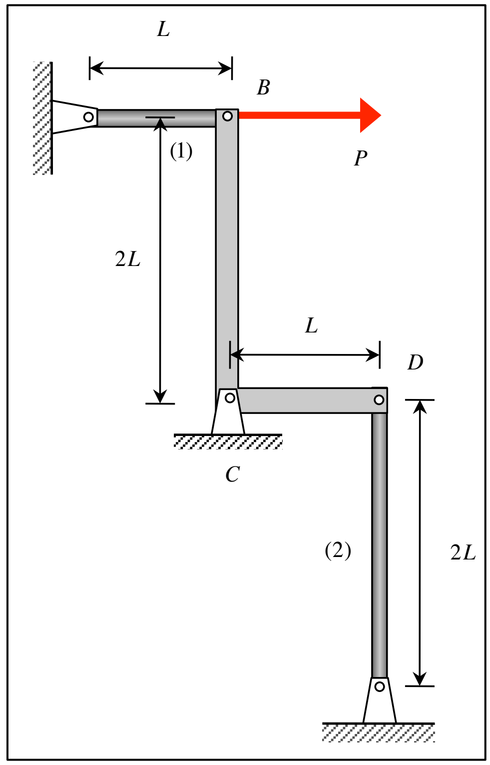
A rigid, L-shaped bar BCD is pinned to ground at C. Two circular cross-section elastic members (1) and (2), each having a Young's modulus of E, coefficient of thermal expansion α and diameter d, are connected between the ends of the bar and ground, as shown below. The elastic members and sections of the bar are either vertically oriented or horizontally oriented. A horizontal force P is applied to joint B. In addition, the temperatures of the elastic members (1) and (2) are *increased* by amounts of ΔT and $2\Delta T$, respectively. Following the four steps below, you are asked to determine the stress in each of the elastic members.

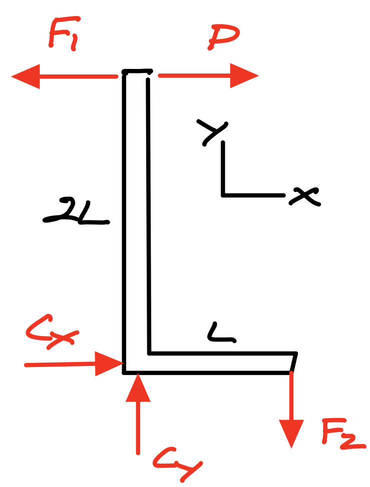


1. ***Equilibrium***. Draw the free body diagrams (FBD) of member BCD. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate?
2. ***Force/elongation equations***. Write down the force/elongation equations for members (1) and (2).
3. ***Compatibility***. Write down the appropriate compatibility equation(s) relating the elongations of rods (1) and (2).
4. ***Solution***. Solve your equations above for the loads carried by the two members. From these, determine the stress in those members. Write your answers in terms of α, ΔT, A and b.

1. Equilibrium

$$\Sigma M_c = F_1(2\cancel{L}) - F_2(\cancel{L}) - P(2\cancel{L}) = 0$$

(1) $\to 2F_1 - F_2 = 2P$



2. Load/deformation: $A = \pi(d/2)^2 = \frac{\pi}{4}d^2$

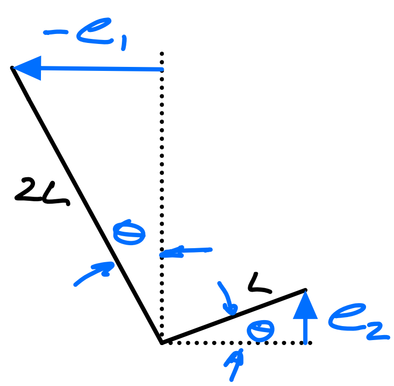
(2) $e_1 = \dfrac{F_1 L}{EA} + \alpha \Delta T L$

(3) $e_2 = \dfrac{F_2(2L)}{EA} + \alpha(2\Delta T)(2L)$

3. Compatibility

$$\sin\theta = \frac{-e_1}{2L} = \frac{e_2}{L}$$

(4) $\to \quad e_1 = -2e_2$



4. Solve

(2)-(4): $\dfrac{F_1 \cancel{L}}{EA} + \alpha\Delta T\cancel{L} = -2\left[\dfrac{2F_2\cancel{L}}{EA} + 4\alpha\Delta T\cancel{L}\right]$

(5) $\hookrightarrow F_1 = -4F_2 - 9\alpha \Delta T E A$

(1) & (5): $2[-4F_2 - 9\alpha\Delta TEA] - F_2 = 2P$

$\hookrightarrow F_2 = -\left[\dfrac{2P + 18\alpha TEA}{9}\right] = -\dfrac{2P}{9} - 2\alpha\Delta TEA$

$\therefore \hookrightarrow F_1 = -4\left[-\dfrac{2P}{9} - 2\alpha\Delta TEA\right] - 9\alpha\Delta TEA$

$= \dfrac{8}{9}P - \alpha\Delta TEA$

$\therefore \begin{cases} \sigma_1 = \dfrac{F_1}{A} = \dfrac{8}{9}\dfrac{P}{A} - \alpha\Delta TE \\ \sigma_2 = \dfrac{F_2}{A} = -\left[\dfrac{2}{9}\dfrac{P}{A} + 2\alpha\Delta TE\right] \end{cases}$

σ_1

σ_2

w/ $A = \dfrac{\pi}{4}d^2$