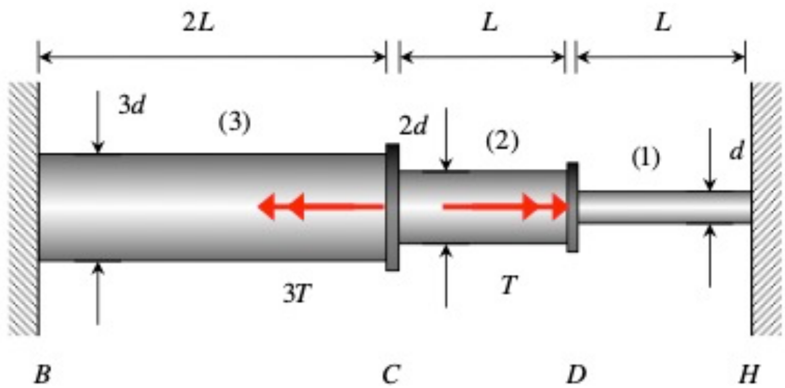
ME 323: Mechanics of Materials **Homework Set H11**

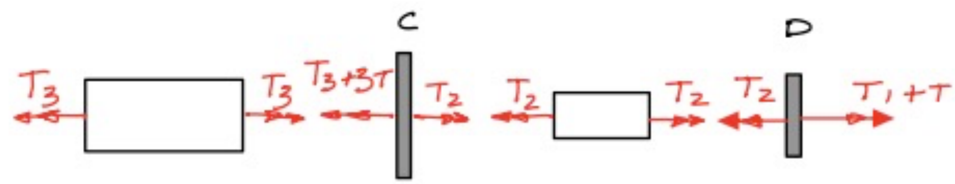
Summer 2024 **Assigned/Due: June 24/June 26**

A shaft is made up of solid elements (1), (2) and (3) with circular cross-sections. All elements are made up of the same material having a Young's modulus of E and Poisson's ratio of ν. Torques of $3T$ and T act on rigid connectors C and D, respectively.

1) ***Equilibrium***. Draw free body diagrams (FBDs) of connectors C and D. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate?
2) ***Torque/rotation equations***. Write down the torque/rotation equations for elements (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the rotations of elements (1), (2) and (3).
4) ***Solution***. Solve your equations above for the torques carried by the three elements. Also, determine the maximum shear stresses in the shaft. At which location(s) does this maximum shear stress exists? Write your answers in terms of T and d.



1. Equilibrium



$$(1)\ C: \Sigma M = T_2 - T_3 - 3T = 0$$
$$(2)\ D: \Sigma M = -T_2 + T_1 + T = 0$$

$\Big\}$ 2 equations/3 unknowns $(T_1, T_2, T_3) \Rightarrow$ INDETERMINATE

2. Torque/rotation: $G = \frac{E}{2(1+\nu)}$

$$(3)\ \Delta\phi_1 = \frac{T_1 L_1}{G I_{p1}} = \frac{T_1 L}{G(\pi d^4/32)} = \frac{32}{\pi}\frac{T_1 L}{G d^4} \ ; \quad I_{p1} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

$$(4)\ \Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{p2}} = \frac{T_2 L}{G\pi d^4/2} = \frac{2}{\pi}\frac{T_2 L}{G d^4} \ ; \quad I_{p2} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2} d^4$$

$$(5)\ \Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{p3}} = \frac{T_3(2L)}{G\ 81\pi d^4/32} = \frac{64}{81\pi}\frac{T_3 L}{G d^4} \ ; \quad I_{p3} = \frac{\pi}{2}\left(\frac{3d}{2}\right)^4 = \frac{81\pi}{32} d^4$$

3. Compatibility

$$\phi_C = \cancel{\phi_B}^{\,0} + \Delta\phi_1$$

$$\phi_D = \phi_C + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2$$

$$(6) \quad \phi_H = \phi_D + \Delta\phi_3 = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3 \overset{\text{wall}}{=} 0$$

4. Solve: 6 equations / 6 unknowns ($T_1, T_2, T_3, \Delta\phi_1, \Delta\phi_2, \Delta\phi_3$)

$$(3)-(6): \quad \frac{32}{\cancel{\pi}} \frac{T_1 \cancel{L}}{\cancel{Gd^4}} + \frac{2}{\cancel{\pi}} \frac{T_2 \cancel{L}}{\cancel{Gd^4}} + \frac{64}{81\cancel{\pi}} \frac{T_3 \cancel{L}}{\cancel{Gd^4}} = 0$$

$$(7) \quad \hookrightarrow 2592 T_1 + 162 T_2 + 64 T_3 = 0$$

$$(8) \quad (2): \quad T_1 = T_2 - T$$

$$(9) \quad (1): \quad T_3 = T_2 - 3T$$

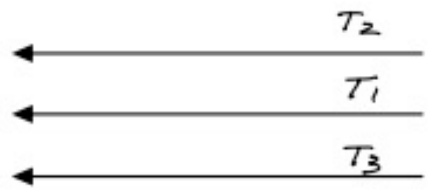
$$(7)-(9): \quad 2592(T_2 - T) + 162 T_2 + 64(T_2 - 3T) = 0$$

$$(2592 + 162 + 64) T_2 = (2592 + 192) T$$

$$\hookrightarrow T_2 = \frac{2784}{2818} T$$

$$(2): \quad T_1 = T_2 - T = -\frac{34}{2818} T$$

$$(1): \quad T_3 = T_2 - 3T = -\frac{5670}{2818} T$$



$$\left.\begin{aligned}
\tau_{1,max} &= \frac{T_1 \rho_{1,max}}{I_{P_1}} = \frac{T_1 (d/2)}{\frac{\pi}{32} d^4} = \frac{16}{\pi} \frac{T_1}{d^3} \\
\tau_{2,max} &= \frac{T_2 \rho_{2,max}}{I_{P_2}} = \frac{T_2 \left(\frac{2d}{2}\right)}{\frac{\pi}{2} d^4} = \frac{2}{\pi} \frac{T_2}{d^3} \\
\tau_{3,max} &= \frac{T_3 \rho_{3,max}}{I_{P_3}} = \frac{T_3 \left(\frac{3d}{2}\right)}{\frac{81\pi}{32} d^4} = \frac{16}{27\pi} \frac{T_3}{d^3}
\end{aligned}\right\}$$