ME 323: Mechanics of Materials — ***Homework Set H12***

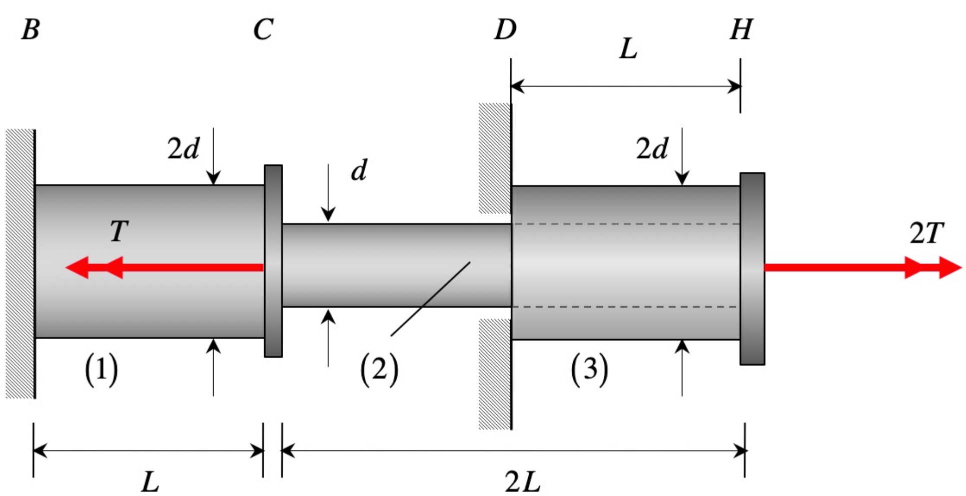
Summer 2024 — ***Assigned/Due: June 25/June 27***

A shaft is consists of three circular cross-sectioned elements, with each element being made up of the same material with a shear modulus of G.

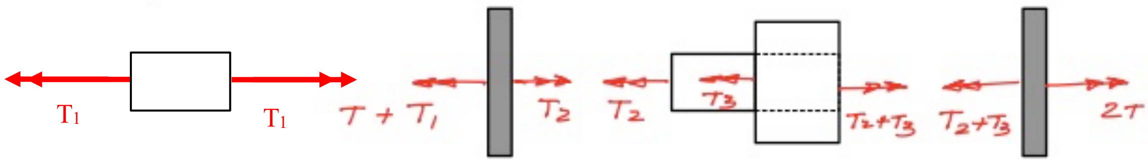
- Element (1) is a solid shaft of length L and outer diameter of $2d$. This element is attached to a fixed wall at B and to a rigid connecter C.
- Element (2) is a solid shaft of length $2L$ and outer diameter of d. This element is attached to rigid connecters C and H. Note that element (2) passes through a hole in the wall at D; element (2) is NOT connected to the wall at D.
- Element (3) is a tubular shaft of length L, outer diameter of $2d$ and inner diameter d. This element is attached to a fixed wall at D and a rigid connecter H.

Torques of T and $2T$ act at connectors C and H, respectively, as shown in the figure below.

1) ***Equilibrium***. Draw free body diagrams (FBDs) of connectors C and H. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate?
2) ***Torque/rotation equations***. Write down the torque/rotation equations for elements (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the rotations of elements (1), (2) and (3).
4) ***Solution***. Solve your equations above for the torques carried by the three elements. Also, determine the maximum shear stresses in the shaft. At which location(s) does this maximum shear stress exists? Write your answers in terms of T and d.



1. Equilibrium



$$\left.\begin{aligned} &(1)\quad C:\ \Sigma M = T_2 - T_1 - T = 0 \\ &(2)\quad H:\ \Sigma M = -T_2 - T_3 + 2T = 0 \end{aligned}\right\}$$

two eqns/three unknowns ⇒ INDETERMINATE

2. Torque/rotation

$$(3)\quad \Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{p1}} = \frac{T_1 L}{G(\pi d^4/2)} = \frac{2}{\pi}\frac{T_1 L}{G d^4} \qquad ;\ I_{p1} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2} d^4$$

$$(4)\quad \Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{p2}} = \frac{T_2(2L)}{G(\pi d^4/32)} = \frac{64}{\pi}\frac{T_2 L}{G d^4} \qquad ;\ I_{p2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

$$(5)\quad \Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{p3}} = \frac{T_3 L}{G(15\pi d^4/32)} = \frac{32}{15\pi}\frac{T_3 L}{G d^4} \qquad ;\ I_{p2} = \frac{\pi}{2}\left[\left(\frac{2d}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right] = \frac{15\pi}{32} d^4$$

3. Compatibility

$$\phi_C = \overset{(wall)}{\cancelto{0}{\phi_B}} + \Delta\phi_1 = \Delta\phi_1$$

$$\phi_H = \phi_C + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2$$

$$\phi_H = \overset{(wall)}{\cancelto{0}{\phi_D}} + \Delta\phi_3 = \Delta\phi_3$$

$$(6)\quad \Delta\phi_1 + \Delta\phi_2 = \Delta\phi_3$$

4. Solve: 6 eqns/6 unknowns $(T_1, T_2, T_3, \Delta\phi_1, \Delta\phi_2, \Delta\phi_3)$

$$(3)\text{-}(6):\quad \frac{2}{\cancel{\pi}}\frac{T_1 \cancel{L}}{\cancel{Gd^4}} + \frac{64}{\cancel{\pi}}\frac{T_2 \cancel{L}}{\cancel{Gd^4}} + \frac{32}{15\cancel{\pi}}\frac{T_3 \cancel{L}}{\cancel{Gd^4}} = 0$$

$$(7)\quad 30T_1 + 960T_2 = 32T_3$$

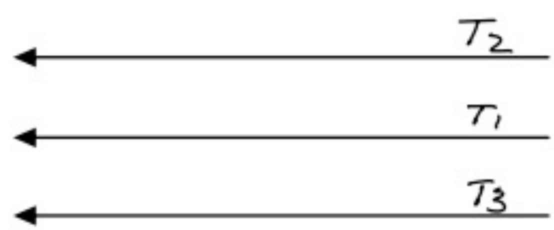
$$(1),(2),(7):\quad 30(T_2 - T) + 960T_2 = 32(-T_2 + 2T)$$

$$\rightarrow (30 + 960 + 32)T_2 = 30T + 64T$$

$$\rightarrow T_2 = \frac{47}{511}T$$

$$(1):\quad T_1 = T_2 - T = -\frac{464}{511}T$$

$$(2):\quad T_3 = -T_2 + 2T = \frac{975}{511}T$$



$$\tau_{1,max} = \frac{T_1 \rho_{1,max}}{I_{p1}} = \frac{T_1(2d/2)}{\pi d^4/2} = -\frac{928}{511\pi}\frac{T}{d^3}$$

$$\tau_{2,max} = \frac{T_2 \rho_{2,max}}{I_{p2}} = \frac{T_2(d/2)}{\frac{\pi}{32}d^4} = -\frac{612}{511\pi}\frac{T}{d^3}$$

$$\tau_{3,max} = \frac{T_3 \rho_{3,max}}{I_{p3}} = \frac{T_3\left(\frac{2d}{2}\right)}{\frac{15\pi}{32}d^4} = \frac{2088}{511\pi}\frac{T}{d^3}$$