SOLUTION

ME 323: Mechanics of Materials **Homework Set H14**

Summer 2024 **Assigned/Due: June 27/July 1**

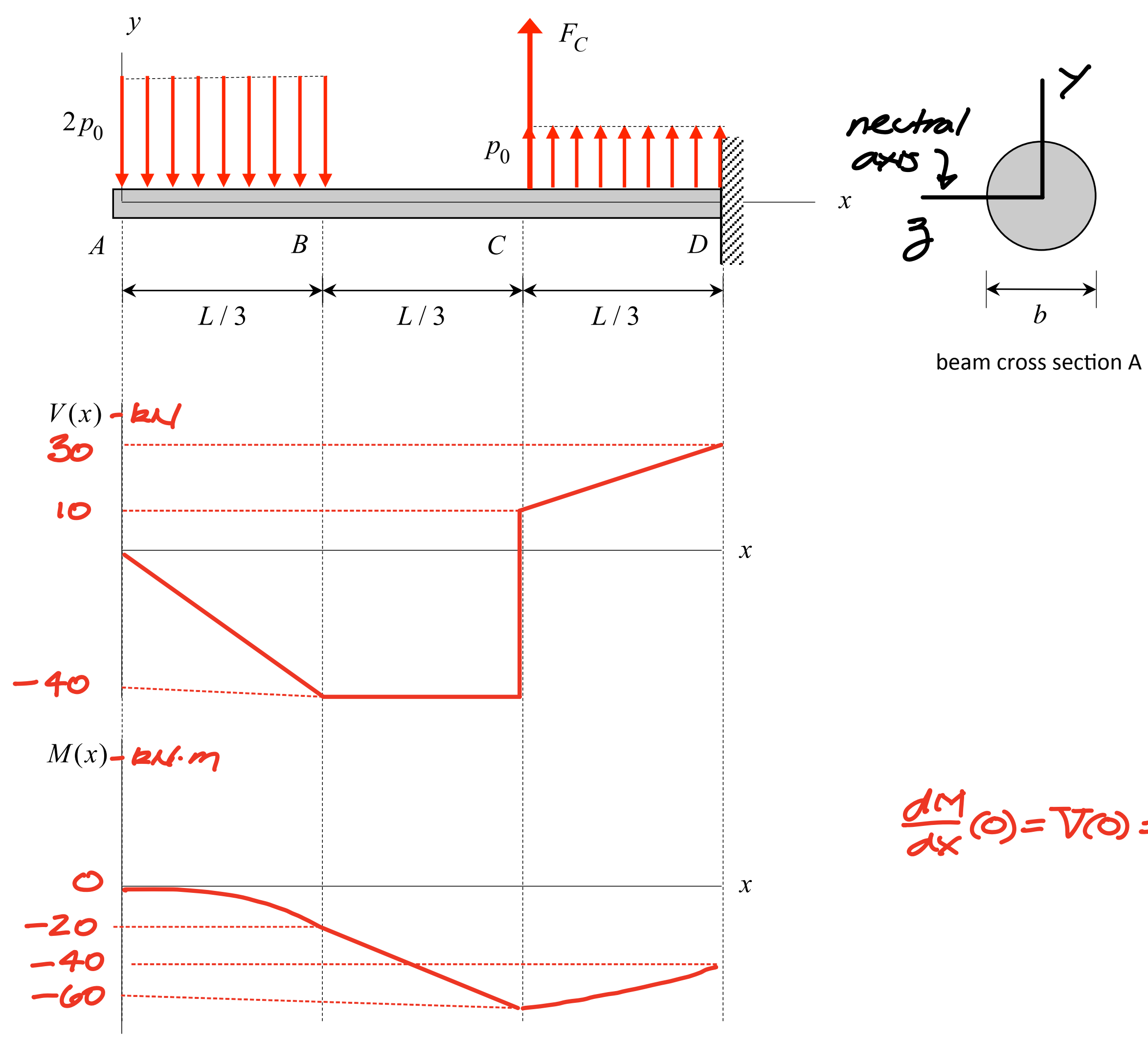
Consider the loading on the cantilevered beam shown below.

a) Sketch the shear force $V(x)$ and bending moment $M(x)$ distribution on the beam using the axes below. Provide details on your calculations.

b) Determine the location(s) along the beam at which the maximum magnitude normal stress exists and location(s) along the beam at which the maximum magnitude shear stress exists.

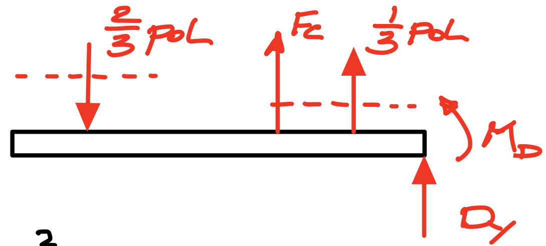
c) Consider a circular beam cross-section "A" shown. For this cross section, determine the maximum magnitude normal stress and its location on the cross section. Also, what is the shear stress at the neutral axis? Feel free to use the results from Example 10.11 of the lecture book in finding the neutral axis shear stress.

Use the following in your calculations: $L = 3\,m$, $p_0 = 20\,kN/m$, $F_C = 50\,kN$ and $b = 0.1\,m$.



$$\frac{dM}{dx}(0) = V(0) = 0$$

- External reactions



$$\sum M_D = M_D - \left(\tfrac{1}{3}p_0L\right)\left(\tfrac{L}{6}\right) - F_c\left(\tfrac{L}{3}\right) + \left(\tfrac{2}{3}p_0L\right)\left(\tfrac{5L}{6}\right) = 0$$

$$\hookrightarrow \; M_D = \tfrac{1}{3}F_cL - \tfrac{1}{2}p_0L^2 = \tfrac{1}{\cancel{3}}(50)(\cancel{3}) - \tfrac{1}{\cancel{2}}(\cancel{20}^{10})(3)^2 = -40\,\text{kN}$$

$$\sum F_y = -\tfrac{2}{3}p_0L + F_c + \tfrac{1}{3}p_0L + D_y = 0$$

$$\hookrightarrow \; D_y = \tfrac{1}{3}p_0L - F_c = \tfrac{1}{3}(20)(3) - 50 = -30\,\text{kN}$$

a) Shear force / bending moment

$V(x)$	$M(x)$
$V(1) = \cancel{V(0)}^{0} + \int_0^1 (-2p_0)\,dx$	$M(1) = \cancel{M(0)}^{0} + \int_0^1 V(x)\,dx$
$= -2(20)(1) = -40\,\text{kN}$	$= \tfrac{1}{2}(-40)(1) = -20\,\text{kN·m}$
$V(2^-) = V(1) + \int_1^2 \cancel{p(x)}^{0}\,dx = -40\,\text{kN}$	$M(2) = M(1) + \int_1^2 V(x)\,dx$
$V(2^+) = V(2^-) + F_c$	$= -20 + (-40)(1) = -60\,\text{kN·m}$
$= -40 + 50 = 10\,\text{kN}$	$M(3) = M(2) + \int_2^3 V(x)\,dx$
$V(3) = V(2^+) + \int_2^3 p_0\,dx$	$= -60 + (10)(1) + \tfrac{1}{2}(20)(1)$
$= 10 + (20)(3-2) \overset{\checkmark}{=} 30\,\text{kN}$	$\overset{\checkmark}{=} -40\,\text{kN·m}$
(Checks w/ reaction at D)	(Checks w/ reaction at D)

b) Maximum shear force @ $1\,m < x < 2\,m \Rightarrow$ max. shear stress there also

Maximum bending moment @ $x = 2m \Rightarrow$ max. normal stress there also

c) $|\sigma_{max}| = \dfrac{|M| y_{max}}{I}$ w/ $y_{max} = \dfrac{b}{2}$ and $I = \dfrac{\pi}{4}\left(\dfrac{b}{2}\right)^4 = \dfrac{\pi}{64}b^4$

$$= \frac{|M|(b/2)}{\frac{\pi}{64}b^4} = \frac{32|M|}{\pi b^3} = \frac{32(60)}{\pi(0.1)^3} = \frac{1920}{\pi}\,\text{MPa}$$

$$|\tau_{max}| = \frac{4}{3}\frac{|V|}{A} = \frac{4}{3}\frac{|V|}{\pi(b/2)^2} = \frac{16}{3\pi}\frac{|V|}{b^2} = \frac{16}{3\pi}\frac{40}{(0.1)^2} = \frac{64}{3\pi}\,\text{MPa}$$