ME 323: Mechanics of Materials
Summer 2023

Homework Set H15
Assigned/Due: June 28/July 2

Consider the shear force/bending moment diagrams for the beam shown below. The beam has a cross-section as shown to the right, with $b=0.2 \mathrm{ft}$.
a) Determine the location $Y_{O}$ of the centroid O of the beam cross-section and the second area moment $I_{O}$ about the neutral axis, $z$.
b) Determine the maximum magnitude normal stress along section AB of the beam.

c) Determine the shear stress at the neutral axis along section BC of the beam.


Let $O$ be the centroid of the beam cross-section.

$$
\begin{aligned}
& \underline{Y}_{0}=\frac{A_{1} Y_{1}+A_{2} Y_{2}}{A_{1}+A_{2}} \\
& \text { wm } \quad A_{1}=\frac{1}{2}(2 b)(b)=b^{2} \\
& A_{2}=(2 b)(b)=2 b^{2} \\
& \bar{Y}_{1}=\frac{2}{3} b \\
& Y_{2}=b+\frac{b}{2}=\frac{3 b}{2} \\
& \therefore I_{0}=\frac{\left(b^{2}\right)\left(\frac{(3 b)+\left(2 b^{2}\right)\left(\frac{(3 b}{2}\right)}{b^{2}+2 b^{2}}\right)}{} \\
& =\frac{11}{9} b \\
& I_{0}=I_{10}+I_{20} \\
& \omega / I_{10}=\frac{1}{36}(2 b) b^{3}+\frac{1}{2}(2 b)(b)\left[\frac{11}{4} b-\frac{2}{3} b\right]^{2} ; \text { P.A.T. } \\
& =\frac{59}{162} b^{4} \\
& I_{20}=\frac{1}{12}(2 b) b^{3}+(2 b)(b)\left[\frac{3}{2} b-\frac{11}{9} b\right]^{2} ; \text { P.A.T. } \\
& =\frac{52}{162} b^{4}
\end{aligned}
$$




$$
\therefore \quad I_{0}=I_{10}+I_{20}=\frac{37}{54} b^{4}
$$

(b)

$$
\begin{aligned}
M & =180 \text { Mip.ft } \\
\sigma_{d} & =-\frac{M Y_{d}}{I_{0}} \\
\omega / Y_{d} & =-Y_{0}=-\frac{11}{9} b \\
I_{0} & =\frac{37}{54} b^{4} \\
\therefore \quad \sigma_{d} & =\frac{M\left(\frac{11}{9} b\right)}{\frac{37}{54} b^{4}}=\frac{66}{37} \frac{M}{b^{3}}
\end{aligned}
$$

