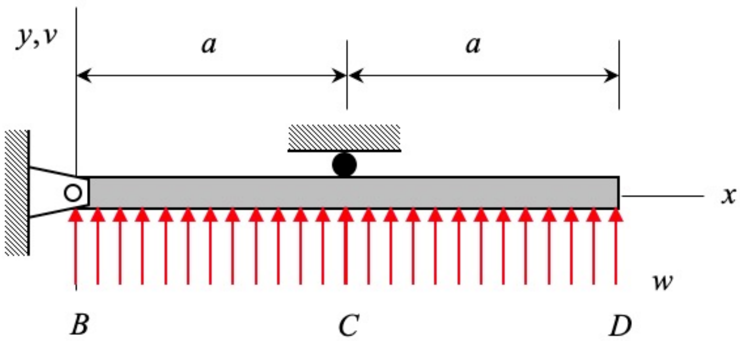
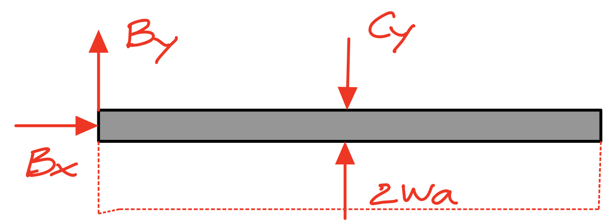
ME 323: Mechanics of Materials — **Homework Set H16**

Summer 2024 — **Assigned/Due: July 1/July 3**

Consider the loading on the beam shown below where the beam is supported by a pin joint at B and a roller support at C. Using integration techniques, determine the slope of the displacement at B and the displacement of end D of the beam.



External reactions



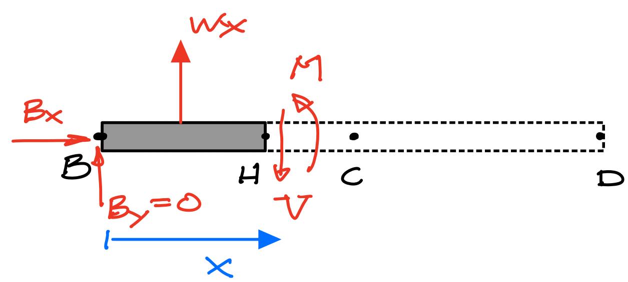
- $\sum M_B = -C_y \not{a} + (2wa)\not{a} = 0$

  $\hookrightarrow C_y = 2wa$

- $\sum F_y = B_y - C_y + 2wa = 0$

  $\hookrightarrow B_y = C_y - 2wa = 0$

Section BC



$$\sum M_H = M - wx\left(\frac{x}{2}\right) = 0$$

$$\hookrightarrow M(x) = \frac{1}{2}wx^2$$

$$\theta(x) = \theta(0) + \frac{1}{EI}\int_0^x M(x)\,dx$$

$$= \theta_B + \frac{1}{EI}\int_0^x \left[\frac{1}{2}wx^2\right]dx$$

$$= \theta_B + \frac{1}{EI}\left(\frac{1}{6}wx^3\right)$$

$$v(x) = \cancel{v(0)}^{\,0} + \int_0^x \theta(x)\,dx = \int_0^x \left[\theta_B + \frac{1}{EI}\left(\frac{1}{6}wx^3\right)\right]dx$$

$$= \theta_B x + \frac{1}{EI}\left(\frac{1}{24}wx^4\right)$$

$$\therefore \quad \theta(a) = \theta_B + \frac{1}{EI}\left(\frac{1}{6} w a^3\right) \triangleq \theta_c$$

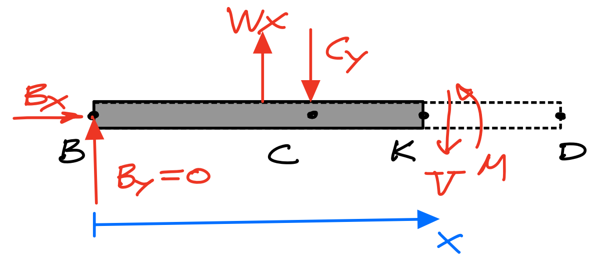
$$V(a) = \theta_B a + \frac{1}{EI}\left(\frac{1}{24} w a^4\right)$$

Since $V(a) = 0 \Rightarrow 0 = \theta_B a + \frac{1}{EI}\left(\frac{1}{24} w a^4\right)$

$$\hookrightarrow \quad \theta_B = -\frac{1}{EI}\left(\frac{1}{24} w a^3\right) \longleftarrow \theta_B$$

$$\& \quad \theta_c = \frac{1}{EI}\left(\frac{1}{8} w a^3\right)$$

<u>Section CD</u>



$$\Sigma M_K = M + C_y(x-a) - wx\left(\frac{x}{2}\right) = 0$$

$$\hookrightarrow \quad M(x) = \frac{1}{2} w x^2 - 2wa(x-a)$$

$$\theta(x) = \theta(a) + \frac{1}{EI}\int_a^x \left[\frac{1}{2} w x^2 - 2wa(x-a)\right] dx$$

$$= \theta_c + \frac{1}{EI}\left\{\frac{1}{6} w(x^3 - a^3) - 2wa\left[\frac{1}{2}(x^2 - a^2) - a(x-a)\right]\right\}$$

$$= \frac{w}{EI}\left\{\frac{1}{6} x^3 - a x^2 + 2a^2 x - \frac{19}{6} a^3\right\}$$

$$V(x) = \cancel{V(a)}^{\,0} + \int_x^a \theta(x)\, dx$$

$$= \int_x^a \frac{w}{EI}\left\{\frac{1}{6} x^3 - a x^2 + 2a^2 x - \frac{19}{6} a^3\right\} dx$$

$$= \frac{w}{EI}\left\{\frac{1}{24}(x^4 - a^4) - \frac{1}{3} a(x^3 - a^3) + \frac{1}{2} a^2(x^2 - a^2) - \frac{19}{6} a^3(x-a)\right\}$$

$$V_D = v(2a)$$