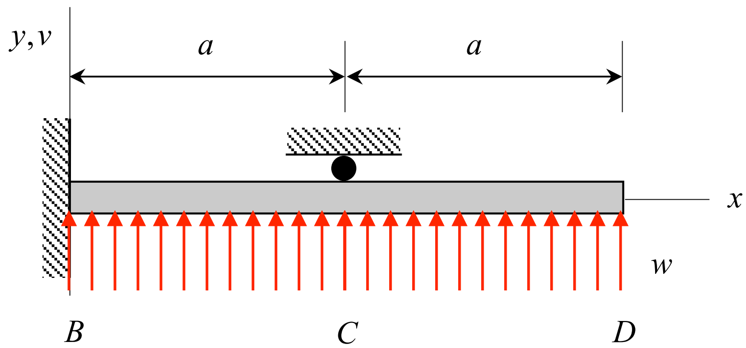
SOLUTION

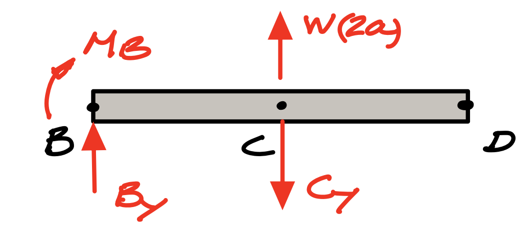
ME 323: Mechanics of Materials — **Homework Set H17**

Summer 2024 — **Assigned/Due: July 2/July 8**

Consider the loading on the propped-cantilevered beam shown below. Using integration techniques, determine the reactions on the beam at B and C.



1. Equilibrium – ext. reactions



$$\sum M_B = -C_y a + (2wa)a - M_B = 0$$

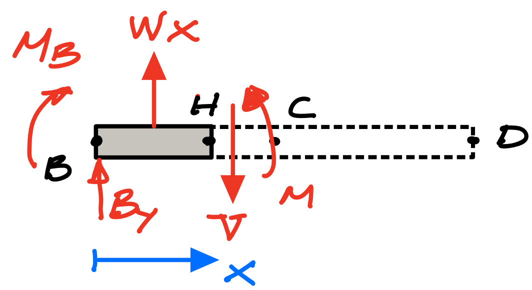
(1) $\hookrightarrow \quad M_B = 2wa^2 - C_y a$

$$\sum F_y = 2wa - C_y + B_y = 0$$

(2) $\hookrightarrow \quad B_y = C_y - 2wa$

2 equations / 3 unknowns $\Rightarrow$ INDETERMINATE

2. Load/deformation



- Section BC:

$$\sum M_H = -M_B + M - wx\left(\frac{x}{2}\right) - B_y x = 0$$

$$\hookrightarrow \quad M(x) = M_B + B_y x + \tfrac{1}{2} w x^2$$

$\therefore$

$$\theta(x) = \underbrace{\cancel{\theta(0)}}_{\substack{0 \\ \text{wall}}} + \frac{1}{EI}\int_0^x \left(M_B + B_y x + \tfrac{1}{2} w x^2\right) dx$$

$$= \frac{1}{EI}\left(M_B x + \tfrac{1}{2} B_y x^2 + \tfrac{1}{6} w x^3\right)$$

$$v(x) = \underbrace{\cancel{v(0)}}_{\substack{0 \\ \text{wall}}} + \int_0^x \frac{1}{EI}\left(M_B x + \tfrac{1}{2} B_y x^2 + \tfrac{1}{6} w x^3\right) dx$$

$$= \frac{1}{EI}\left(\tfrac{1}{2} M_B x^2 + \tfrac{1}{6} B_y x^3 + \tfrac{1}{24} w x^4\right)$$

3. Compatibility: enforce boundary conditions on deflection function

(3) $$V_c = V(a) = 0 = \frac{1}{EI}\left(\frac{1}{2}M_B a^2 + \frac{1}{6}B_y a^3 + \frac{1}{24}w a^4\right)$$

$$\hookrightarrow \quad 12M_B + 4B_y a + wa^2 = 0$$

4. Solve

(1) and (2) into (3):

$$12(2wa^2 - C_y a) + 4(C_y - 2wa)a + wa^2 = 0$$

$$\hookrightarrow (12a - 4a)C_y = 17wa^2$$

$$\hookrightarrow C_y = \frac{17}{8}wa \qquad C_y$$

$$(1) \Rightarrow M_B = 2wa^2 - \left(\frac{17}{8}wa\right)a = -\frac{1}{8}wa^2 \qquad M_B$$

$$(2) \Rightarrow B_y = \frac{17}{8}wa - 2wa = \frac{1}{8}wa \qquad B_y$$