ME 323: Mechanics of Materials **Homework Set H19**

Summer 2024 **Assigned/Due: July 8/July 10**

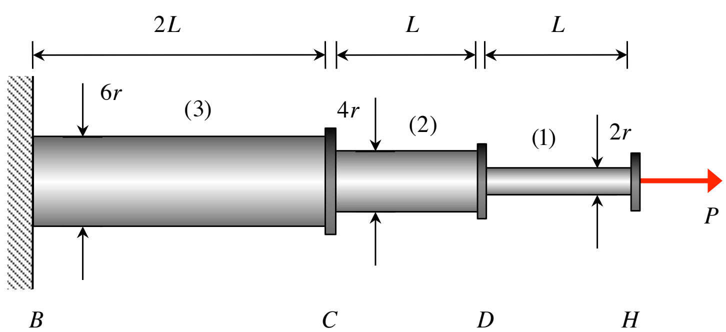
Consider the rod shown below with an axial force P acting at H. All three components of the shaft are made of a material having a Young's modulus of E.

a) Write down the strain energy in the rod in terms of the geometric and material properties of the rod, and the loading P.

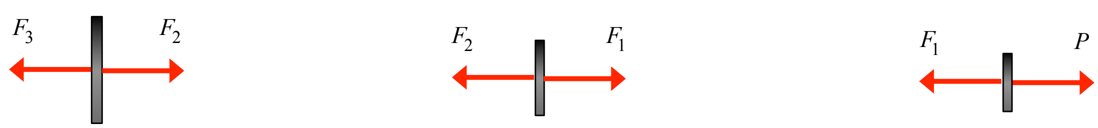
b) Write down the work done by P in terms of P and the horizontal displacement of H, u_H.

c) Using the work energy equation, determine the displacement u_H.

d) Using Castigliano's theorem, determine the displacement u_H.



Equilibrium



$\sum F_x = F_3 - F_2 = 0 \;\Rightarrow\; F_3 = F_2 = P$ $\quad\sum F_x = -F_2 + F_1 = 0 \;\Rightarrow\; F_2 = F_1 + P$ $\quad\sum F_x = -F_1 + P = 0 \;\Rightarrow\; F_1 = P$

Strain energy

$$U = U_1 + U_2 + U_3 = \frac{1}{2}\frac{F_1^2(2L)}{E\pi(3r)^2} + \frac{1}{2}\frac{F_2^2 L}{E\pi(2r)^2} + \frac{1}{2}\frac{F_3^2 L}{E\pi(r)^2}$$

$$= \frac{P^2 L}{2\pi E r^2}\left[\frac{2}{9} + \frac{1}{4} + 1\right] = \frac{53}{72}\frac{P^2 L}{\pi E r^2}$$

$$W = \frac{1}{2} P u_H$$

Using the work energy equation:

$$W = U \;\Rightarrow\; \frac{1}{2} P u_H = \frac{53}{72}\frac{P^2 L}{\pi E r^2} \;\Rightarrow\; u_H = \frac{53}{36}\frac{PL}{\pi E r^2}$$

Using Castigliano's theorem:

$$u_H = \frac{\partial U}{\partial P} = \frac{53}{36}\frac{PL}{\pi E r^2}$$